Astrophysics & Cosmology
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Lecture – 32
Distances and the Hubble Parameter

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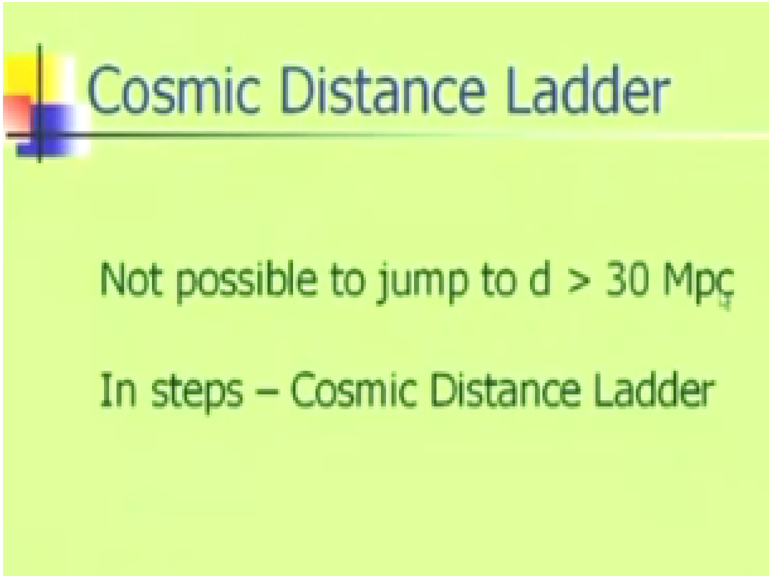

Welcome, let me remind you that we were discussing determination of the Hubble parameter and for that, you need to measure distances and redshifts independently and for this, we use the cosmic ladder and it is necessary to go beyond 30 mega parsec that is the point that we had made in the last lecture.

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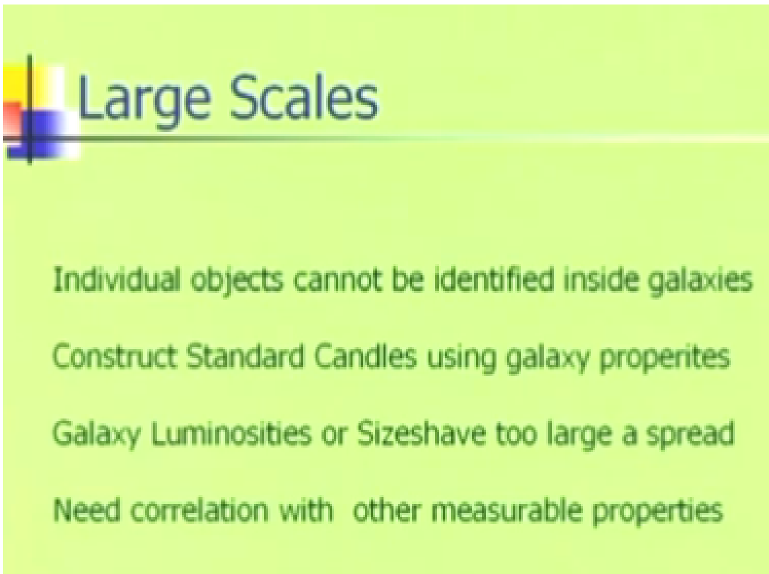

And at these distances, it is not possible to look for individual stars in galaxies, Cepheid variables will not work, one has to use individual galaxy properties and we were discussing correlations between galaxy properties and the luminosity or diameter, so that you can use the angular diameter, distance or the luminosity distance measurements okay.

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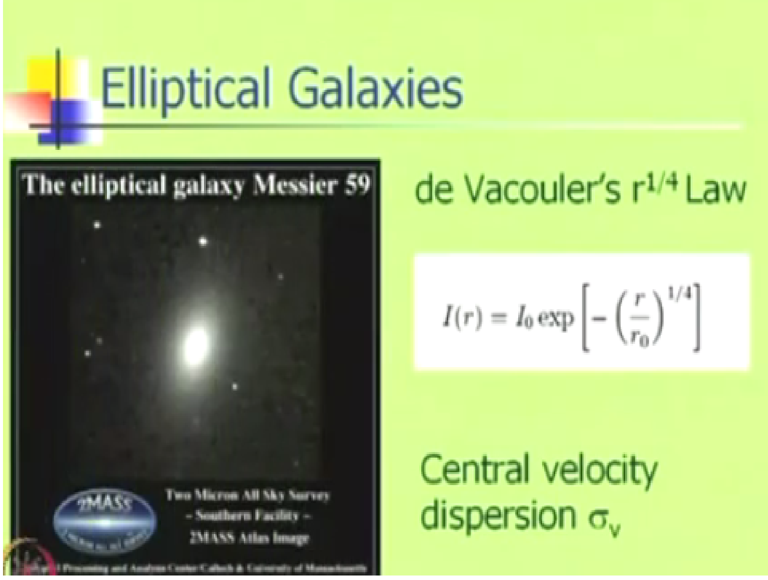

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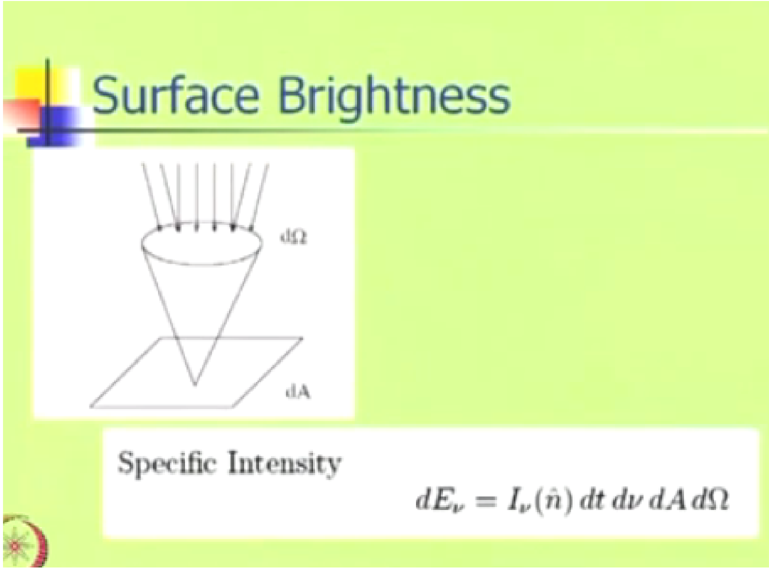

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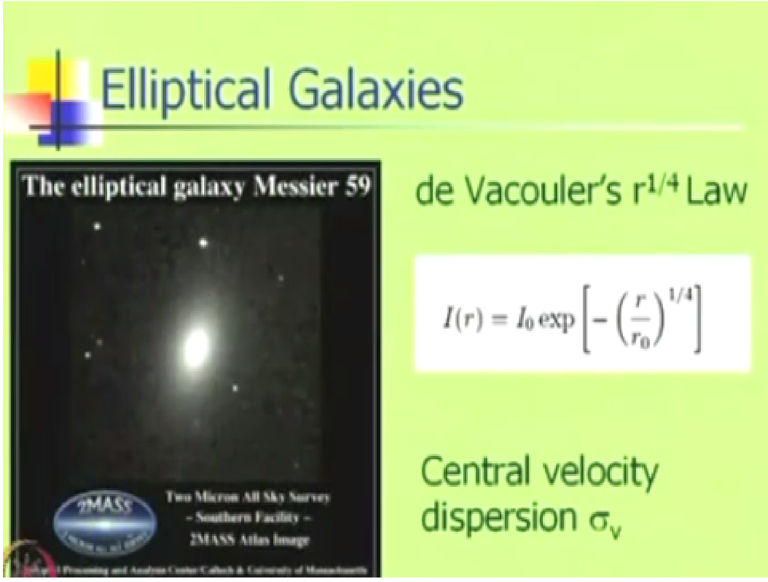

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And we discussed about the correlations that exist between; for example, the luminosity, the central brightness, surface brightness and the velocity dispersion for elliptical galaxies and for spiral galaxies; we have the Tully Fisher relation, which is a relation between the rotational velocity which you can measure and the luminosity which you cannot directly measure. So, if you can measure the rotational velocity either in optical or 21 centimetres or in infrared, you can then determine the luminosity, okay.

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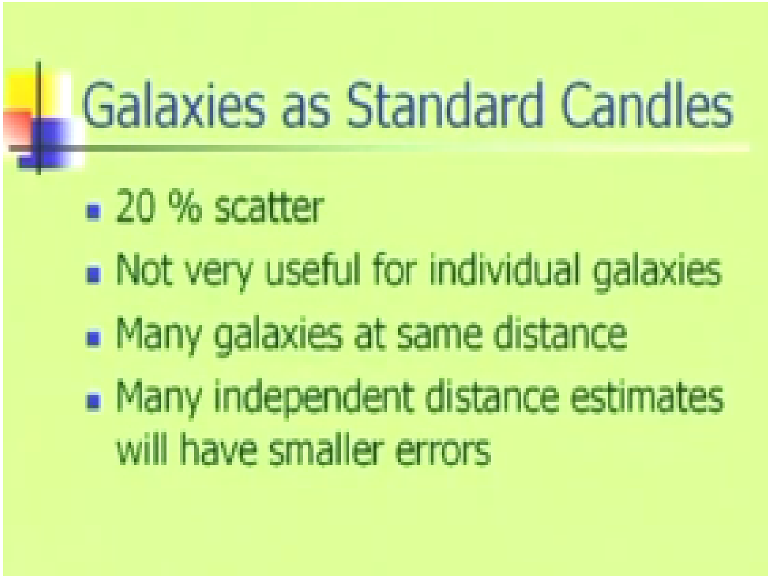

So, this is the basic idea and so this; these ideas the fact that there are correlations between the galaxy properties like luminosity or diameter and other things that you can observe. This allows us to use galaxies as standard candles or standard rulers. The problem is that; there is a 20% scatter in these relations, so these relations are not precise, there is a 20% uncertainty, which you do not want in your Hubble parameter, okay.

So, this uncertainty in the distance is also going to be of the order of 20% and it is not very useful to use these methods for individual galaxies. If you want to determine Hubble parameter it is not very useful because the Hubble parameter will have large uncertainties. Now, what one could do is; if you have many galaxies at the same distance for which you know the distance is the same.

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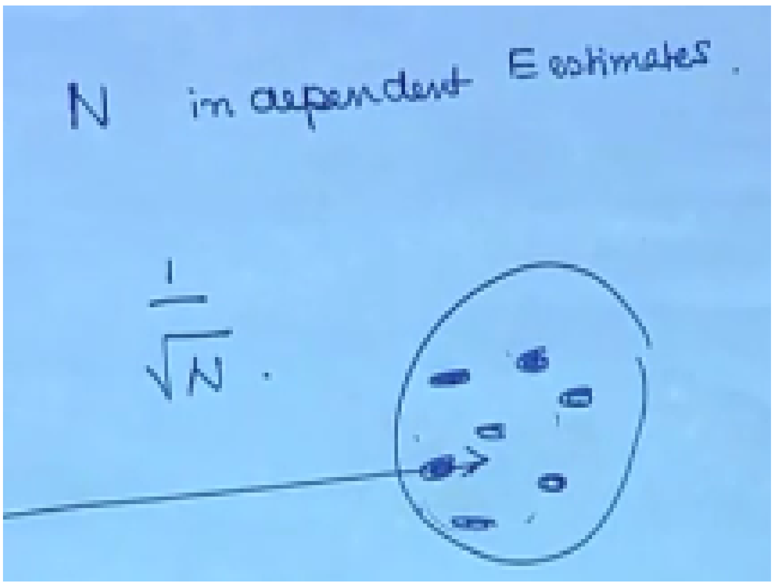

Then, if you have for example; N independent readings measurements; independent estimates, then the error will go down as 1/root N, this is a well-known thing, okay. If, I combine 2 measurements the error goes down as 1/root 2; the variance adds okay and then it gets divided by 2 the whole thing, okay. So, the variance adds up and the signal also adds up, so the signal to noise ratio will go as increases a factor of root 2.

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And this is the basic idea, so if you have many independent estimates of galaxies which are at the same distance, so you have many independent estimates of the same distance, you can then use these to reduce the error from 20% to lower and this is possible because there are certain things; objects, which are called clusters of galaxies which are very important in their own right also.

So, this shows you an image of a cluster of galaxies and a cluster is the part of space where you have many galaxies located close together and these galaxies are physically bound, so they are gravitationally bound okay. So, this shows you an image of a cluster, you can see that these extended things over here they are all galaxies; here, here, here, they are all individual galaxies okay. So, the way we imagine a cluster; a cluster is a collection of galaxies which are bound okay.

So, these galaxies are in Virgo equilibrium more or less okay, the formal bound structure. So, if I am observing from far away, I can assume that the distance to all of these galaxies is the same and I can then individually determine the distances to these galaxies and combine these to reduce the error that is the basic idea, okay. I will have; if there are N galaxies inside, I will have N; independent estimates of the distance to the cluster.

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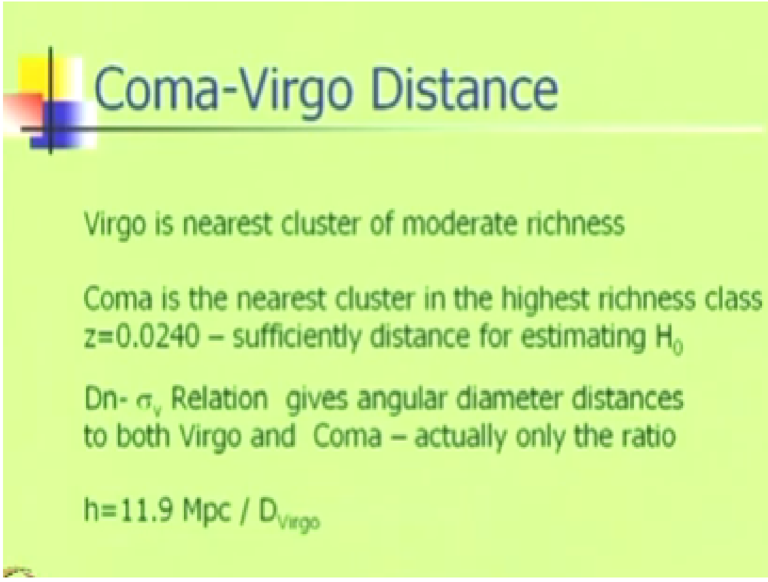

So, there are Virgo cluster is the nearest cluster; it is a cluster of moderate richness, so it does not have very, very, very many galaxies okay, so clusters of galaxies have been classified of varying being of different richness classes and Virgo is a moderate as of modest richness okay and it is not; it is still collapsing, so it is not fully bound as yet the thing there are it is still in the process of being formed okay.

So, this is the Virgo cluster and coma cluster is nearby nearest cluster which is of in the high richness class okay, so coma cluster; let us start our discussion with coma cluster. Coma cluster is at a redshift of 0.0240, so which is sufficiently far for us to determine H0, we saw that the errors will be < 10%, if the distance is > 0.01, so this definitely satisfies that criteria, the coma cluster is at a redshift of 0.204.

And one could use their Dn sigma relation to get the angular diameter distance of both the Virgo and the coma cluster okay but the Dn sigma relation itself is not calibrated, in the sense that we know that there is a relation between D and n sigma, you can measure the sigmas infer the Dn but we do not know what the relation and there will be some unknown, so we will not get the diameter, we will get something that is proportional to the diameter.

To determine the diameter, you need to calibrate this relation using a sample of galaxies, whose distances are known okay, that is the problem. So, this relation is not calibrated, we find that there is a relation between the angular diameter; between the diameter of the galaxy and the velocity dispersion but the distance; this relation is not calibrated okay. So, if you use this, you will get something that is proportional to the diameter.

So, you can use this to determine the ratio of the distances to Virgo and coma and once you know the distance to; so once you know the distance to coma, you can use this; you can measure the redshift also. The redshift is 0.024, so you can; the distance measurement and the redshift measurement will together give you the Hubble parameter but this will only give you the Hubble parameter in terms of the distance to Virgo.

And if you use this, then the value of small H which is Hubble parameter in 100 kilometres per second per mega parsec turns out to be; 11.9 mega parsec by D Virgo, so the distance to Virgo is unknown because this method only gives us the ratio of the distances, okay. I hope the difficulty is clear, so we know that such a relation does exist but we do not know its calibration.

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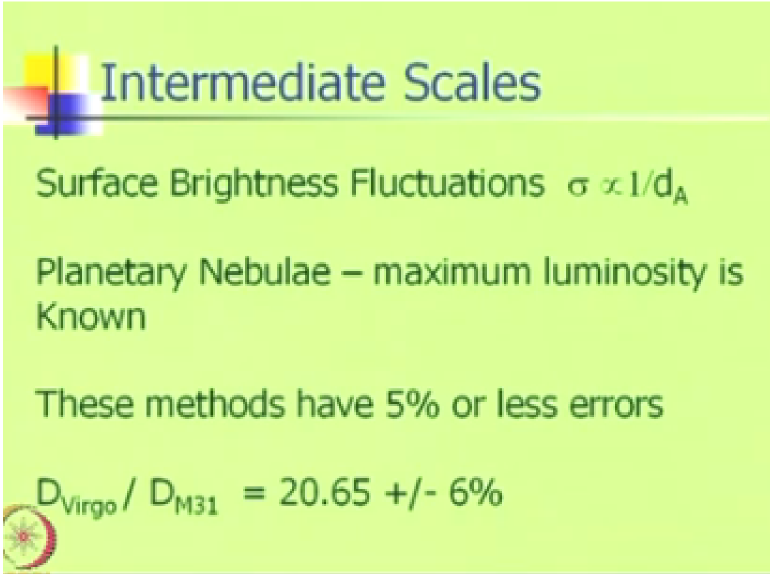

So, we have to calibrate this by relation D and sigma relation by determining the distance to Virgo. So, how to determine the distance to Virgo? That is the basic problem, so one can determine the distance to Virgo using a variety of methods. So, these are intermediate distances, Virgo is quite close to us and one of them is the method of surface brightness fluctuations. This is a very clever method. Let me explain this to you.

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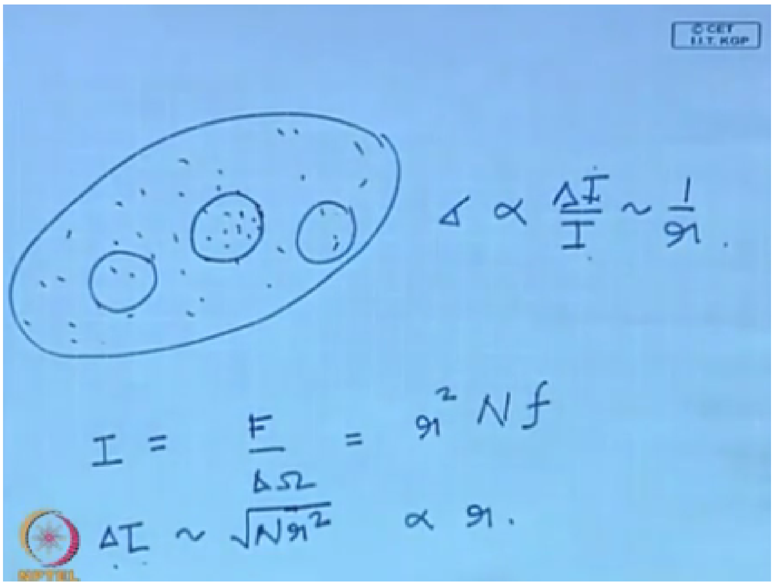

Again imagine a galaxy; this is an image of a galaxy and the galaxy we know has stars, so inside the galaxy, there are discrete stars and the light that we receive from a galaxy is from these discrete starts. Now, when I measure this specific intensity from the galaxy what we do is; we will measure the light coming from an angle; solid angle delta omega, so this will be the flux from a solid angle delta omega.

This will give me the specific intensity and the flux of the light coming; the flux from this region will be the flux from this region will be = the area subtended by this solid angle into r square, so it will essentially be = r square times the flux from the number of stars in this region, number density of stars per unit area into the flux from individual stars, let us call that f.

Now, you see the specific intensity from this therefore, comes from discrete points and it then follows that the number of stars in this region will have Poisson fluctuations which, so the variance of the specific intensity will be proportional to root N; the standard deviation in the specific intensity will be proportional to root N*r square; N is let us assume same. The number density of stars per unit area of a galaxy is same from galaxy to galaxy.

So, the specific intensity fluctuations will be proportional to r that is the crux of this argument or the relative variance in the specific intensity that is delta I/ I will scale as 1/r, okay because the further away the galaxy is more stars I will have in the same solid angle and more the stars see if it were a continuum of stars there would be no fluctuations at all. The fluctuations come because it is discrete.

So, if I put the same area here, I will get a different number of stars, the number will not be exactly the same. If I put another same area here, the number will have fluctuations; these fluctuations are there because the stars are discrete, okay. This is the Poisson fluctuation and this varies the variance; the standard deviation of this is proportional to the square root of the number.

So, the fractional fluctuation is proportional to 1/ r so, if you can measure the surface brightness fluctuations and these will vary as one by the angular diameter distance that is the basic point. So, you can use this to determine the angular diameter distances to different galaxies. This is one method that can be used, okay. You cannot use this method very far away because the galaxy becomes too small.

The second method is that you can use planetary nebulae, we have learned what planetary nebulae are; this planetary nebula is a stage in the evolution of a star where the outer shell becomes extremely large, so it has a high luminosity and there are spectral emission lines which are typical of planetary nebulae, so they can be identified and the maximum luminosity of a planetary nebula is known.

So, if you can identify a planetary nebula, the outer shell is expanding and if you can determine the maximum luminosity of that this is known, so if once you can do this then you know you have a standard candle okay. So, these methods have around 5% error and now okay with; so you can use these methods and determine the distance to Virgo and the distance to Virgo in terms of M 31, which is a nearby galaxy.

So, you have to apply the same method to both galaxies inside the window cluster and this galaxy M 31, which is nearby which will allow us then to determine the distance to Virgo cluster in terms of the distance to M 31, again these methods are not calibrated, there is no absolute calibration. All that we know is that; these fluctuations are proportional to 1 by angular diameter distance, okay this also.

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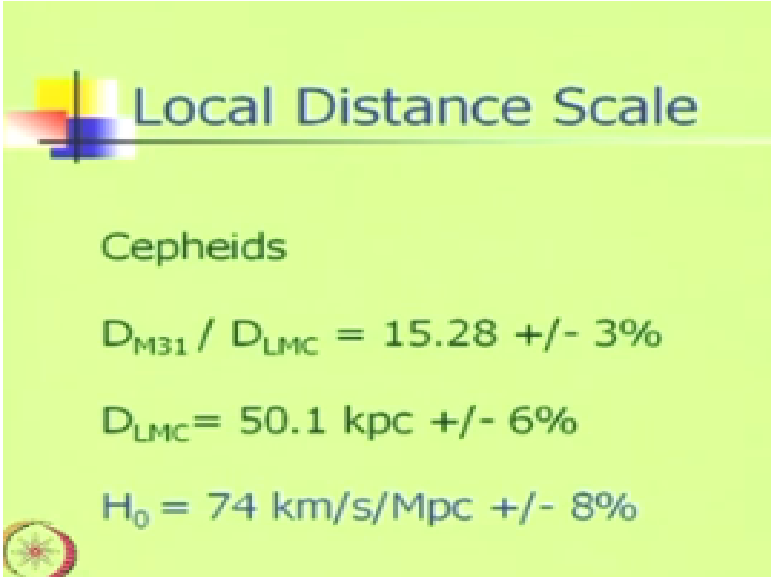

So, you have to determine the calibration, so you can; these methods are uncalibrated, so if you can apply them to 2 different galaxies at 2 different distances they will give me the ratio of the distances okay and this gives us a ratio that the distance to Virgo by distance to M 31 is 20.65. Now, once you have reached M 31, you can then use this Cepheid variable to determine this ratio of the distance to between M 31 and the large magellanic cloud.

And this comes out to be 15.28+ -3% and the distance to LMC can be determined from a variety of methods. The most accurate of these is from the supernova which was seen there recently in 1987 and these methods or you could use the Cepheid variables stars again to determine the distance to LMC and if you can determine the open cluster apply the open cluster fitting or some such thing to LMC,

Then you have a calibration of the Cepheid period luminosity relation okay, so finally you are led to a value of the Hubble parameter from this which is 74 kilometres per second per mega parsec + -8% okay where around 70 and these there are other methods which I shall not be discussing which also yield value of around 70 kilometres per second per mega parsec.

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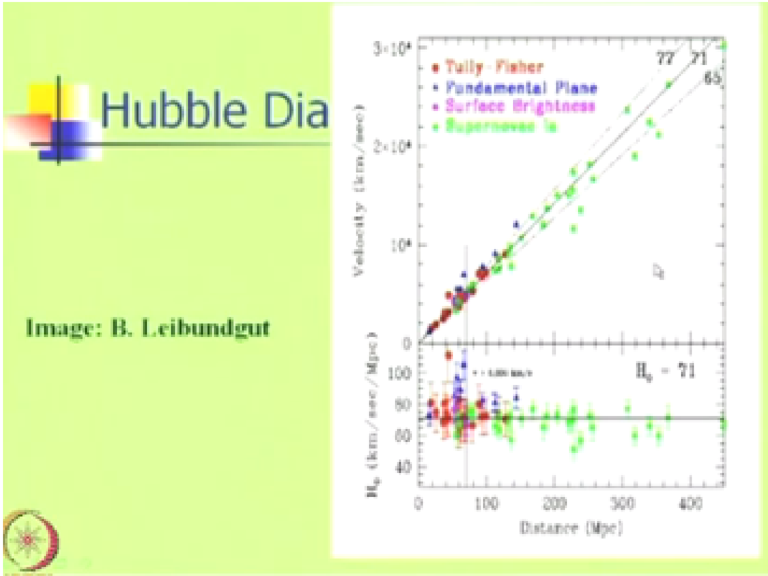

So, H has a value of around 0.7, okay, so this picture is a modern Hubble diagram. The picture had showed you right at the start distance versus redshift or distance versus velocity is what is called a Hubble diagram. This is a modern Hubble diagram and this has a variety of data which has gone into this okay, so let me tell you what these data are, so first you have the Tully Fisher relation, which are shown in red.

You have the fundamental plane for elliptical galaxies, which are shown in blue and you have the surface brightness fluctuations which are there at lower edge shifts are shown in this violet colour; purple colour and finally there is something, which I have not talked about which are the supernovae 1a, which are shown in green and the supernova one a go to very large distances or redshifts.

So, the things that we were talking about till now redshift of 0.02 is around 60 mega parsec, 0.024 will be around 75 mega parsec okay, so that is somewhere within 100. If you go beyond 100 distances which is beyond a redshift of 0.02, beyond the coma cluster, none of the earlier methods work the method, which gives good results beyond those distances beyond 100 mega parsec is based on supernovae 1a.

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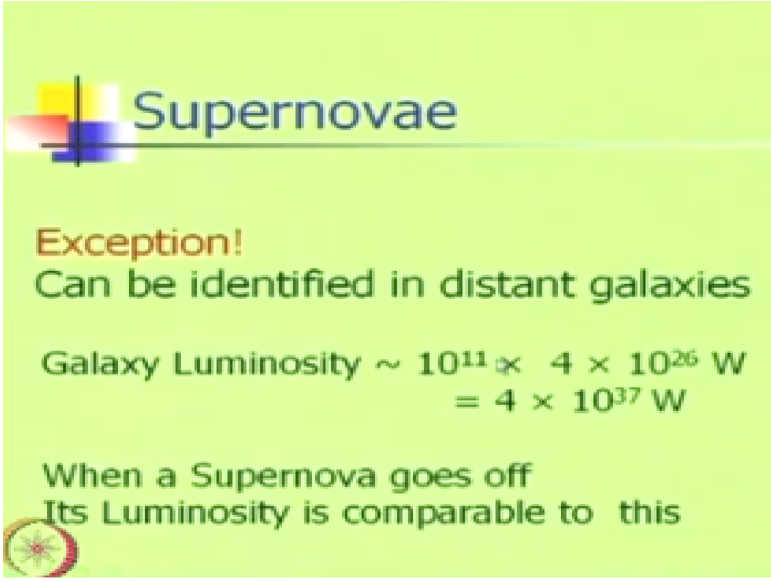

So, let me now embark upon this, so we have already learned about supernovae this is an explosion and they can even be identified in distant galaxies. Why? Well, a galaxy has luminosity of the order of 4 * 10 to the power 37 watts. This is easy to estimate, you know the number of stars in a galaxy 10 to the power 11 and you know the luminosity of a single star 4 * 10 to the power of 26 watts.

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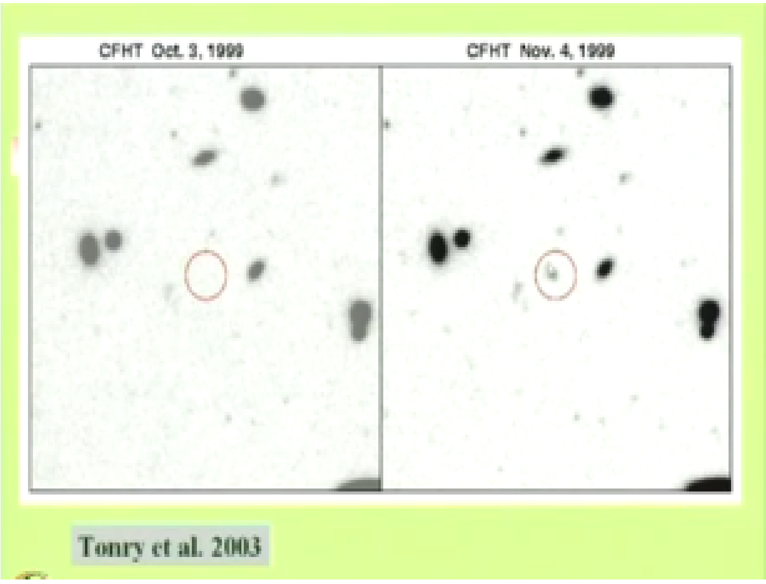

So, you multiply these 2 and you find the typical luminosity of a galaxy is in this range, 4 * 10 to the power 37 watts. The luminosity of a supernovae can be comparable to that of the entire galaxy and the supernovae goes off, so they are easy to identify okay and so you can identify them even in distant galaxies that is the key point okay. So, this shows you 2 images of the sky, the image on the left, the image over here is before the supernovae went off.

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You do not see anything any source in that part of the sky and after the supernovae went off you see a very faint source away okay, so without the supernovae the galaxy is too faint to be seen but the supernovae is so bright that it makes a galaxy visible in the image okay. So, you have one identification of a supernovae here, this shows you another image of a supernovae here, it is so bright okay.

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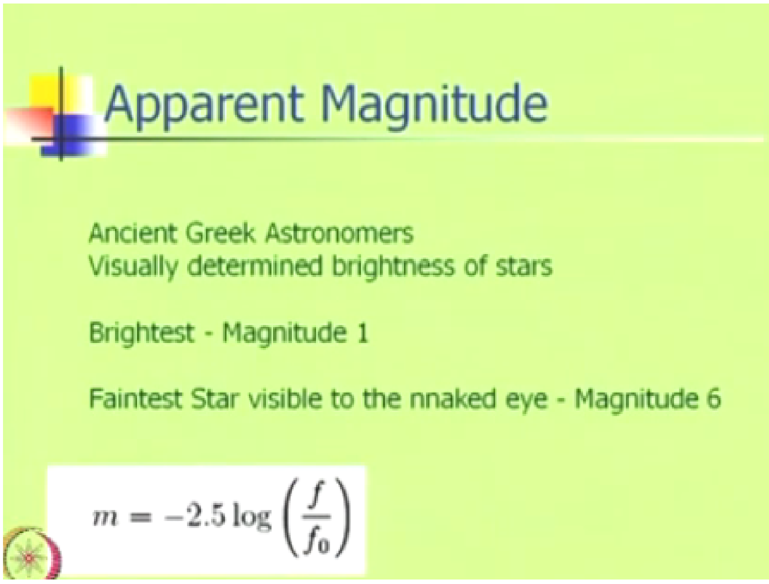

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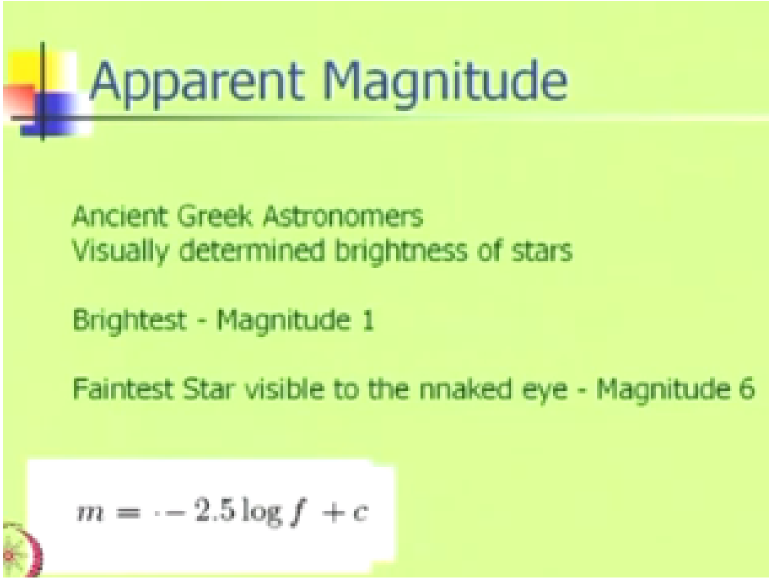

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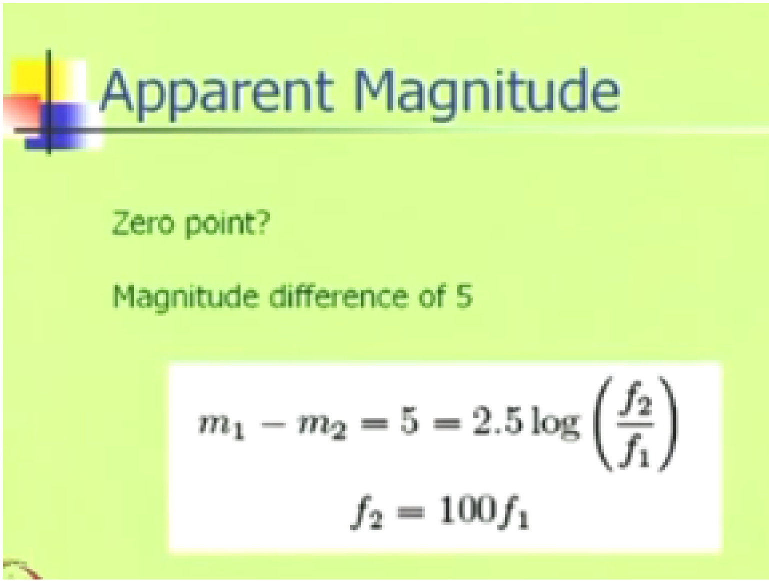

And we are particularly interested in a particular type of supernovae which is called 1a, okay. Before we go into this discussion of supernovae, let me just briefly remind you that we had talked about magnitudes and we had talked about absolute magnitudes. Absolute magnitude is the magnitude that an object will have if you place it at a distance of 10 parsecs okay.

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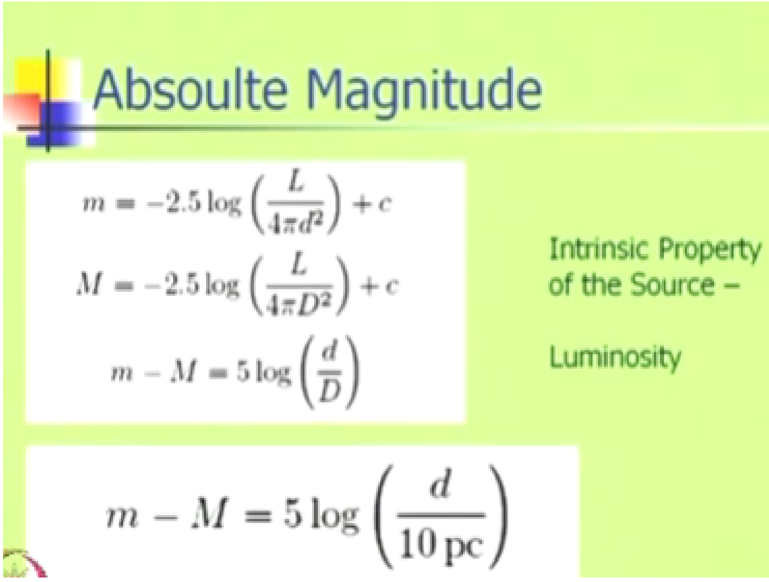

And so; if you know the distance it is possible, if you know the apparent magnitude and the absolute magnitude to an object, you can determine the distance that is if you know the luminosity and the flux you can determine the distance. Well in terms of magnitudes the difference m - Capital m is called the distance modulus and it is related to the distance as 5; log the distance in units of 10 parsec okay.

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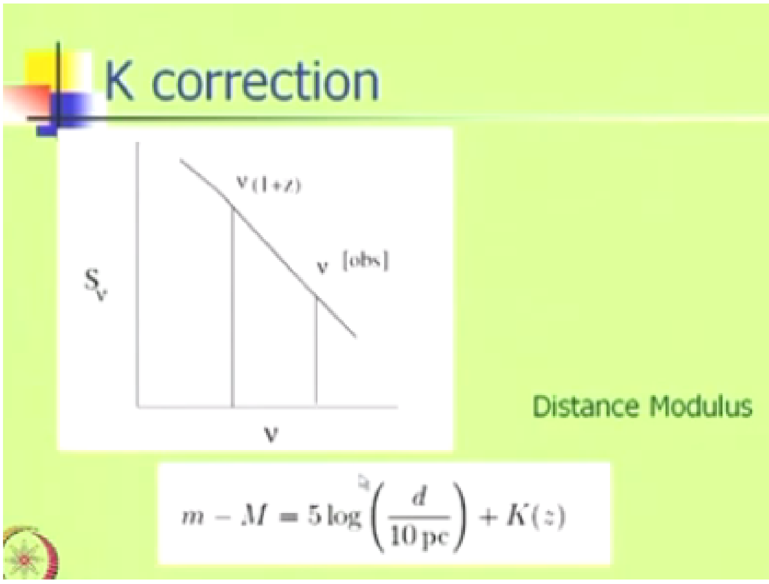

And all these supernovae observations are discussed in terms of this distance modulus. Basically, cosmological observations are largely discussed in terms of distance modulus. There is a point which you have to remember, which I have not mentioned till now it this is called the K correction. If you are dealing for example with galaxies it is necessary to apply something more called the K correction. What is the K correction?

The K correction accounts for the following fact, when I look at a galaxy at an observed frequency, so I am doing my observation at this frequency, I am actually probing the galaxy at another frequency which is nu * 1 + z, because in the rest frame of the galaxy the light was emitted at a different frequency. Now, if you remember that when we talk about the luminosity of the galaxy and the apparent magnitude, the luminosity will usually be in the same frequency as that I am carrying out my observation.

So, I am comparing here the apparent magnitude at a frequency with the luminosity at the same frequency but what I am observing is not the luminosity at that frequency I am observing the luminosity at a different frequency, it gets red shifted, so this has to be accounted for by this and that is accounted for through this factor called the K correction okay. So, K corrections are usually calculated assuming that the galaxy has a spectrum which is a power law.

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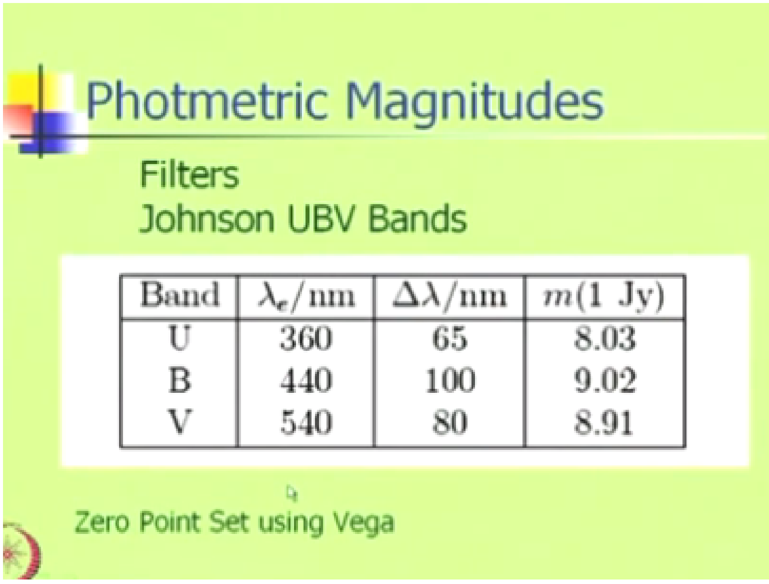

Band	λ_e/nm	$\Delta\lambda$/nm	m(1 Jy)
U	360	65	8.03
B	440	100	9.02
V	540	80	8.91

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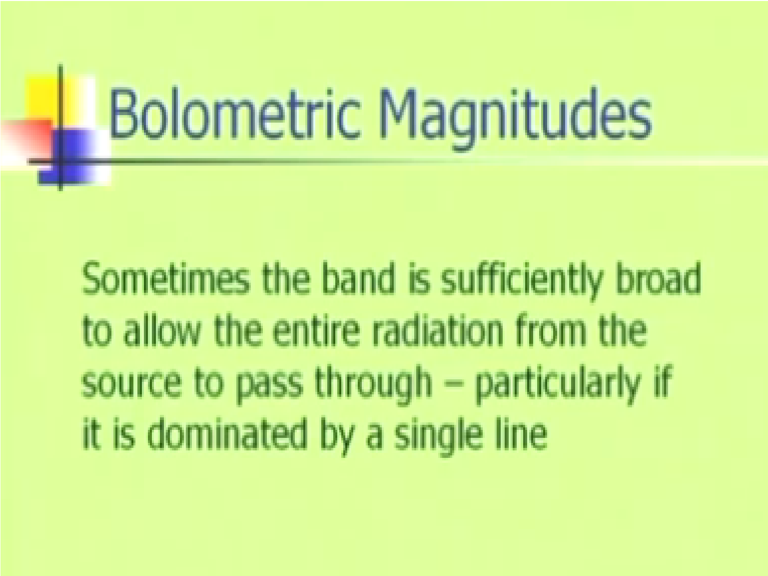

So, they will make a model for the galaxy something which is typical of galaxies and then you can get a K correction, so it will correct for the fact that you are comparing the luminosity at a different frequency and with the magnet flux at a different frequency okay and we have done discussed all of this, so that is the point the distance modulus gives us and it quantifies the distance in some logarithmic unit okay.

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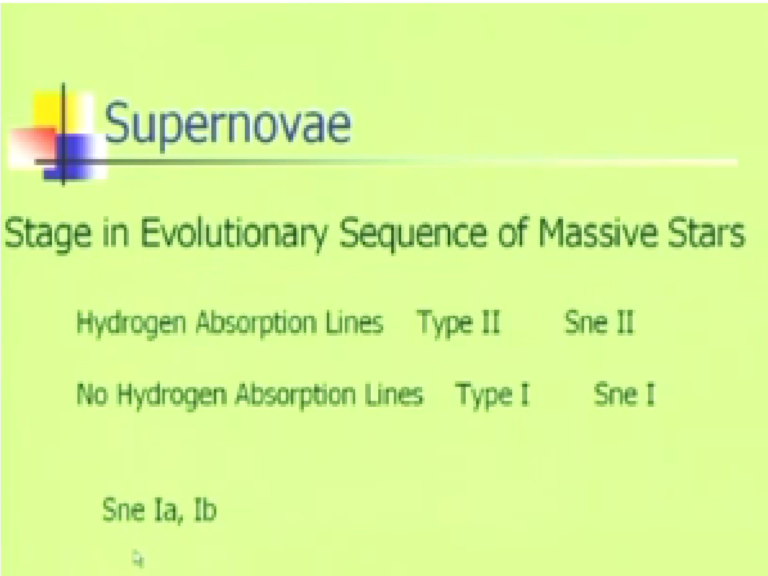

Now, what are supernovae? Let me again briefly remind you of this, in the evolution of massive stars and we have already learned about this when we were discussing stellar evolution. Supernovae are denoted by Sne and they are classified into 2 type I and type II. The type II supernovae have hydrogen absorption lines they are called Sne II and supernovae where you do not see any hydrogen absorption lines they are classified as type I.

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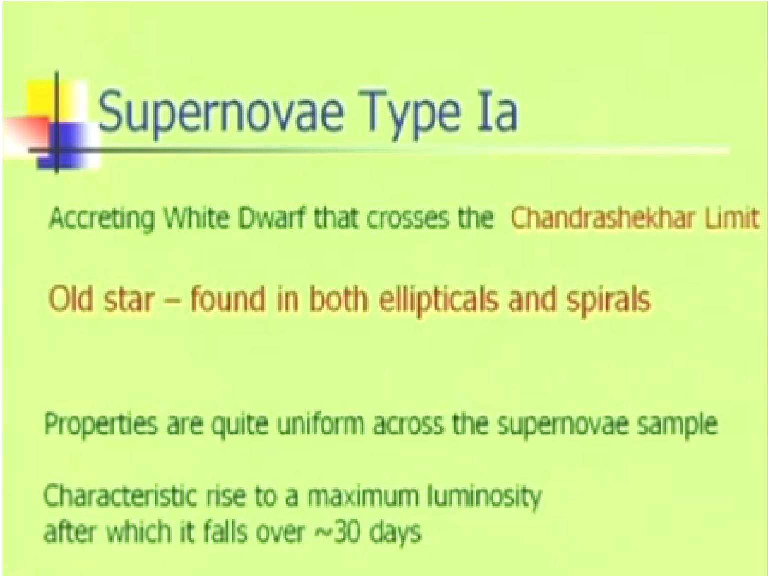

So, they are referred to as Sne I; one again is divided into 2; there is Ia and Ib. Now, supernova of type 1a is of particular interest in cosmology and we have already learned that these supernovae are white dwarfs which are in binary orbit around a star, so this is a binary system where one of the stars has already evolved into a white dwarf, the other is still in the main sequence; the smaller star is still in the main sequence.

And the white dwarf obviously is below the Chandrasekhar limit, it starts to accrete matter from the other star and as a consequence it crosses the Chandrasekhar limit and once it crosses the Chandrasekhar limit the degeneracy pressure is not sufficient and to support the star, so it collapses and you have a supernovae. This is a supernova of type 1a. This is what is seen as supernovae of type 1a.

And these are old stars because both the stars are pretty old because one of them has already become a white dwarf, so these are found in both ellipticals and spiral galaxies. I have mentioned earlier that elliptical have old stars only, spirals have gas, so star formation is taking place and you find a mixture of young stars and old stars in spirals not all stars are there both in elliptical and spiral.

So, you can see supernova of type 1a both in in elliptical and spirals and their properties are quite uniform across the entire supernovae one sample that is the crucial point we would like to use them as standard candles and they have a characteristic rise, so if there is the light curve is if you look at the flux coming as a function of time that is what is called the light curve and the light curve shows a characteristic rise to a maximum luminosity and then it falls off.

And the whole thing occurs over a period of around 30 days okay. So, this is supernovae of type 1a. This is a picture which shows you a galaxy NGC 4526, which has a supernovae, the supernovae is 1994 D that is the name given to the supernovae. It occurred in the year 1994 and this is the supernovae and this is supernovae of type 1a, it is an image of supernovae of type 1a.

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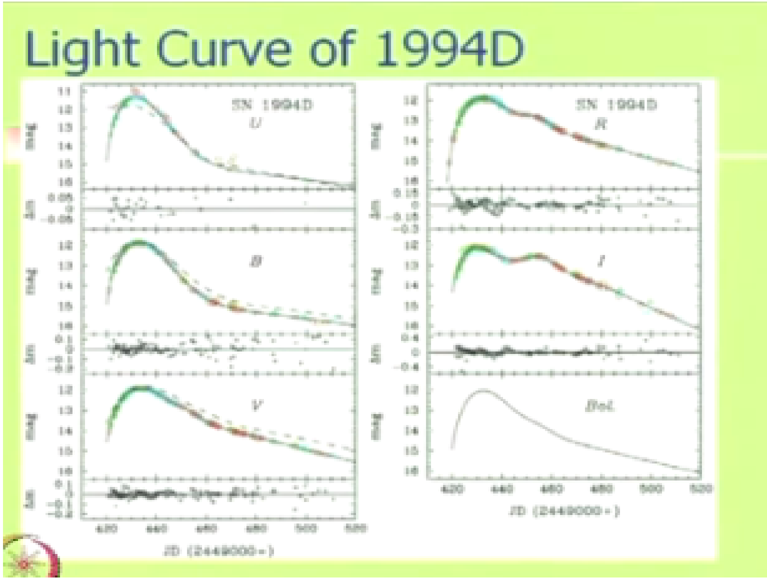

You know it is a supernova of type 1a by looking at the spectrum and these are the light curves of these supernovae in different bands. We have already learned about the photometric bands, so these are the light curves in the U band, B band, V band etc, so you see that there is a rise in the flux and then the flux falls off and this whole thing is of the order of several days; few 10s of days.

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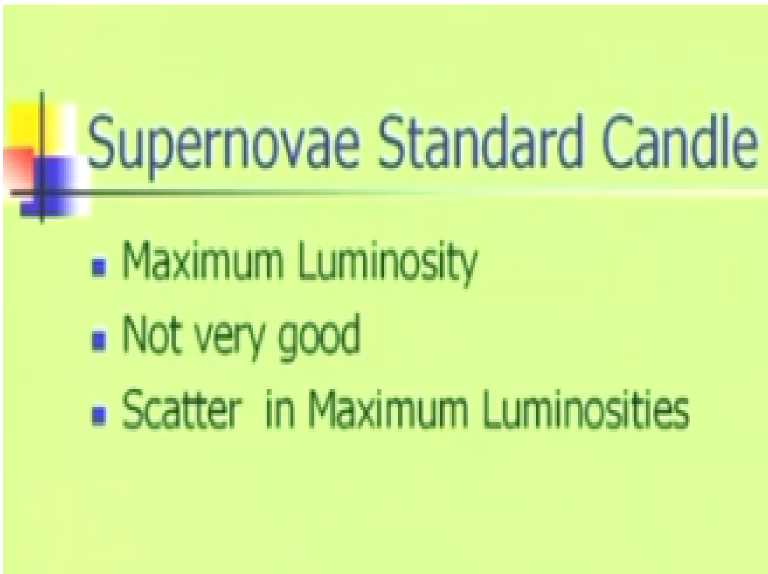

Now, the question is how do we use these as standard candles? Because for getting estimates of distances we need standard candles, if you have a standard candle and you can measure the flux then you know the distance okay. So, if you use the maximum luminosity; one idea is to use the maximum luminosity the peak luminosity of the supernovae, well if you do this it is not a very good standard candle.

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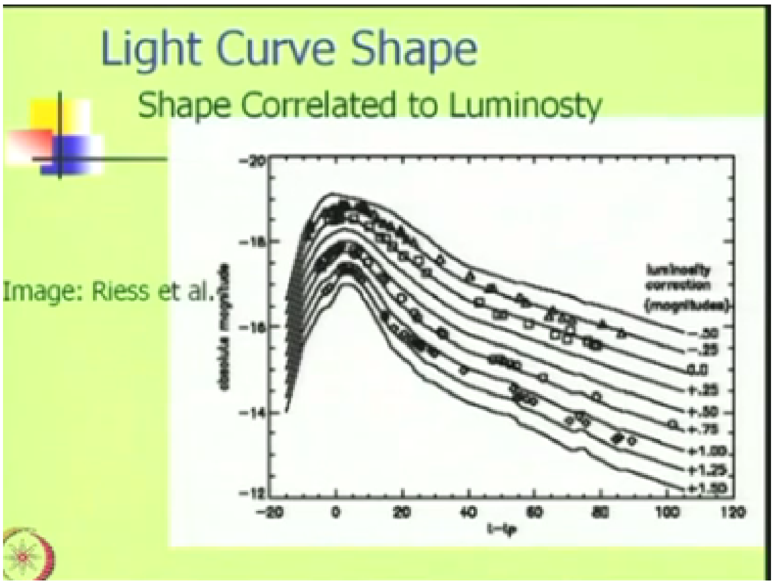

Because there is a considerable amount of scatter in the maximum luminosity, so if the distance that you infer will not be correct because the luminosity itself is not the same for all so maximum luminosity is not the same across the entire sample. Well, let me show you the light curve shapes, well the way people address this problem is to consider the not only the maximum luminosity but to consider the entire light curve.

So, this picture shows you the light curve for many supernovae of different luminosities. A question that will arise is; how do we know the luminosity of supernovae? You can only measure the flux. To do this; you need to find supernovae in galaxies whose distances can be determined by some other method right, you measure the flux, so you need to find galaxies in supernovae whose distances can be determined by some other method.

For example, galaxies where you also have Cepheid variable stars, so you can independently get the distance to the galaxy take the measured flux and convert that to luminosity okay, so this is how you will calibrate your distance; the luminosities. So, suppose you have done that and you have these 2 supernovae in galaxies whose distances are known, this shows you the light curve of several such supernovae.

The point to note is that the maximum luminosity is related to the shape of the curve; the shape of the curve shows a systematic behaviour, there is a correlation between the shape of the curve and the luminosity. The ones which are which correspond to more luminous supernovae the rate at which the light curve falls off is slower whereas the ones which correspond to fainter ones; fainter supernova their light curve falls off faster.

So, it has been found that this; okay so, the shape of the curve is correlated with the luminosity and the shape can be determined irrespective of the distance because the shape will not change where you put the galaxies okay, you can measure the flux and determine the shape of the light curve. If you know the shape of the light curve, you can tell what luminosity and what is the luminous; the peak luminosity of the supernovae, okay.

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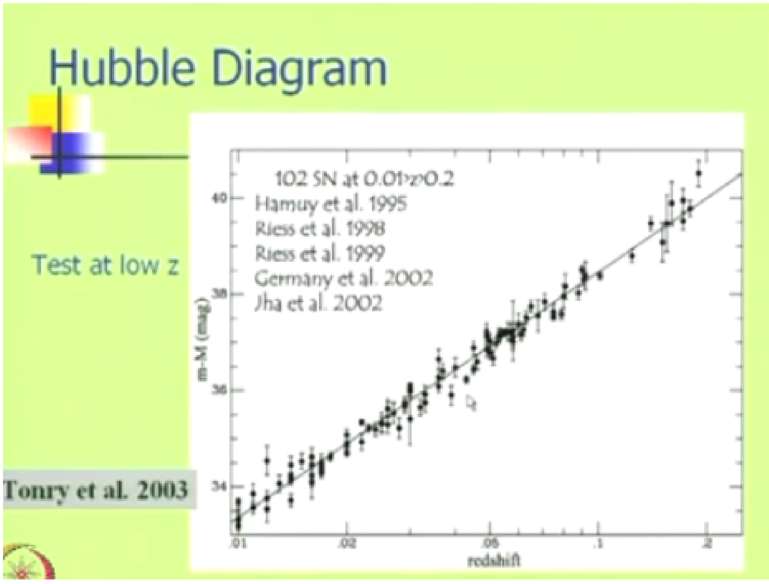

People found that such a correlation does hold and the basic idea is that you do this; so you look at the shape and use the shape to determine the peak luminosity and this shows you a Hubble diagram which has been determined based on these luminosities okay and you see that this is a Hubble diagram at low redshift. The redshift; the maximum redshift in this diagram is 0.2, so at low redshift, you expect the distance to scale linearly with the redshift.

And if provided the distances have been estimated correctly right, so here this is; these are distances which have been; so here the distances have been estimated based on the observed fluxes. You have supernovae; you observe the supernovae shape, you infer the luminosity based on the supernovae, the peak luminosity of the supernovae based on the shape, you measure the flux, so you know the distance.

And that distance here is plotted against a redshift. If my luminosity estimates are correct then they will lie on a straight line which is what we see over here, right. So, this indicates that the correlation that I talked mentioned is works okay and not only does it work it, so this is a compilation of data by several people not only does it work, it can also be used to determine the Hubble parameter because the slope of this curve will give us the Hubble parameter.

So, I do not have the value; the precise value here but it comes out to be close to 70; 70 + something little more than 70, okay which is consistent with the estimates for the other estimate I told you about okay. So at loaded ships, the distance luminosity; the Hubble diagram comes out to be a straight line as expected which indicates that the method works, it also gives us a handle on the Hubble parameter. Now, it is the high redshift, which are interested.

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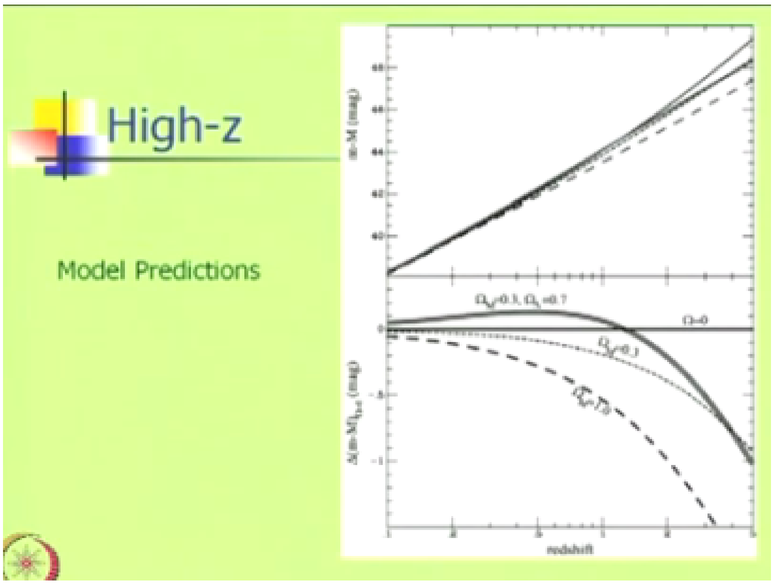

Because at high redshift, the relation between the luminosity distance; these are standard candles, so you will get the luminosity distance. At high redshift, that this relation between the luminosity distance and redshift depends on the value of the cosmological parameters, so it depends on what constitutes your universe and these curves will be different for different constituents of the universe (()) (33:36), okay.

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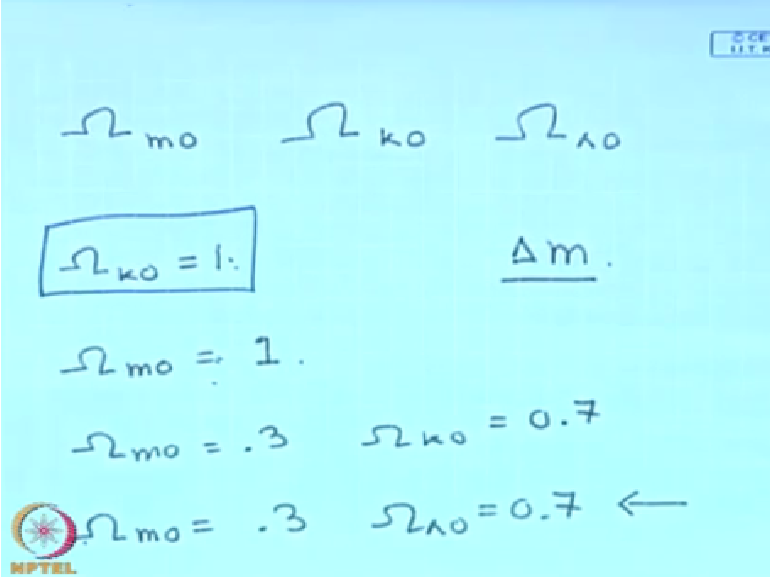

So, let me explain the significance of these curves, let me spend a little time doing that. The reference curve over here shown in the black line; solid black line is a model which has no matter so, okay this picture is a model; considers a model which has 3 variables; omega matter not, omega curvature not and omega lambda not and the reference line in this picture is a model which has omega curvature0 = 1 and rest are 0.

There is no matter there is no cosmological constant or dark energy okay. This is the reference line and which is the one here, it is a reference line here okay. Now, the thick line; the double the double line here okay, the dashed; let us look at the dashed one first. The dashed curve over here is a model which has omega matter0 = 1. A model which has only matter nothing else and the matter is the density of the matter is exactly = the critical density, especially flat.

That is Einstein de sitter model okay, that is a dashed curve here which shows you how the luminosity distance varies with the redshift and then you have the dotted curve over here; the dotted curve has omega matter0 = 0.3 and omega curvature 0.7, cosmological constant is 0 and finally that is the dotted line over here and finally you have a model where omega matter0 is 0.3 and omega lambda0 is 0.7, which is the one showed in double line.

So, this is the one shown over here, luminosity distance was a redshift. Now, this is what is shown in the upper panel. The lower panel what it shows you is the difference in the distance modulus, see the main point is that the luminosity difference turns out to be different in these 4 models and the luminosity distance is maximum in this range for the model which has; for this model okay.

Now, the distance can be quantified using the distance modulus right, that is something we have discussed, so the distance can be quantified using the distance modulus. So, you calculate the distance modulus for all these models, take this as the standard reference and then you look at the difference in the distance modulus between the different models with reference to this okay that will essentially be the ratio of the distances, log of that.

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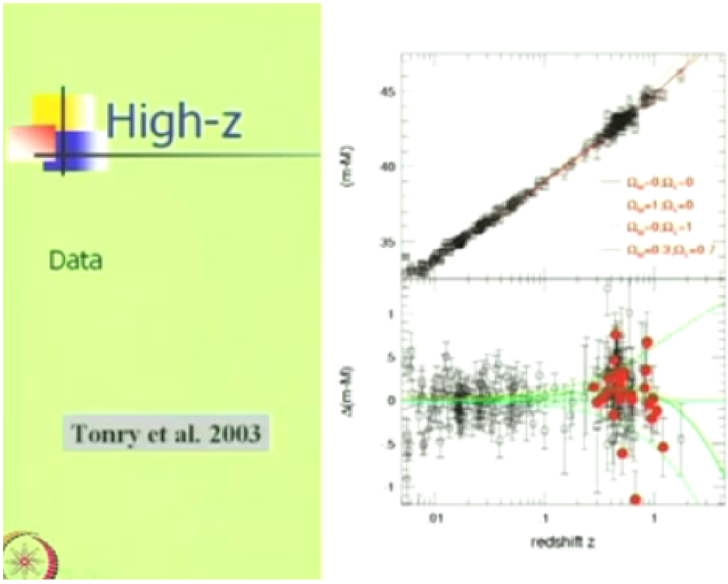

It will be the ratio of the luminosity distance in the 2 models log of that, a difference in the distance modular is a ratio of the distances okay log of that. So, this is what is being plotted here fine. So, let us look at the data now, so this shows you the data at including high redshift okay, this is as of 2003, so the data extends up to a redshift of more than 1, somewhat more than 1 between less than; so <1; sorry it does not go more than 1, it goes to value quite just above point 1, where these different, you see the different curves start to differ somewhere.

This is a different scale all together this is a logarithmic scale okay, so the differences are going to be extremely small at a redshift of around 0.1, but if you have more data which go up to a redshift of 1, then you can discriminate between the different models which are shown over here and if you plot them it is the differences are more noticeable if you plot the curves in terms of the difference in distance modulate.

If you plot the curves in terms of this difference in this of the difference moduli, the data is more clearer, now here, you see the error bars are extremely large because you have taken individual supernovae and the distance estimate to an individual supernovae is not very has large errors in it, the large error comes in the scatter of this relation. So, we can predict the peak luminosity from the shape but there will be an uncertainty which gets reflected in the uncertainty in the distance modulus okay.

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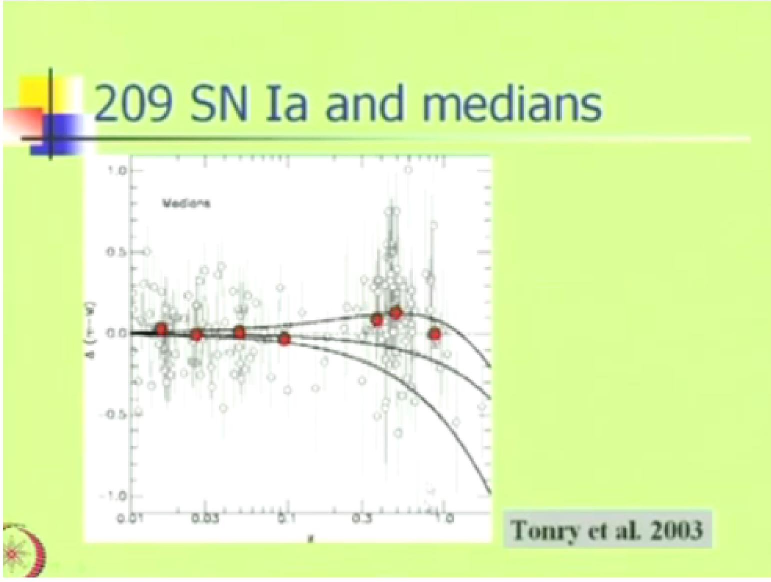

But if you bind these data okay, so you can bind these data and look at the median data, so here the data has been 3926 and one has now look taken the median value of the distance modulus. It is quite clear that the observation favours a model which has omega matter 0.3 and omega lambda 0.7, okay. The model which has just omega matter = 1, it is not consistent with the data neither is the model where omega matter is 0.3 and omega lambda is 0.7.

Now, there is a model where there is no matter at all it is all curvature free expansion okay, so you require a model with some amount of cosmological constant and to explain the data. So, this was the; these are 209 supernovae 1a, this is the reference 2003, not very new pretty old okay. Now, so you can now consider a class of cosmological models where there are 3 parameters.

You see the Hubble parameter does not come into this picture at all because you are looking at the difference of distance moduli, so the Hubble parameter will cancel out, you are looking at the ratio of distances; Hubble parameter does not come in the picture. So, you can; suppose we consider a model with 3 parameters for my cosmology okay. Then these observations fitting my model to the data in these observations I can put constraints on the model parameters.

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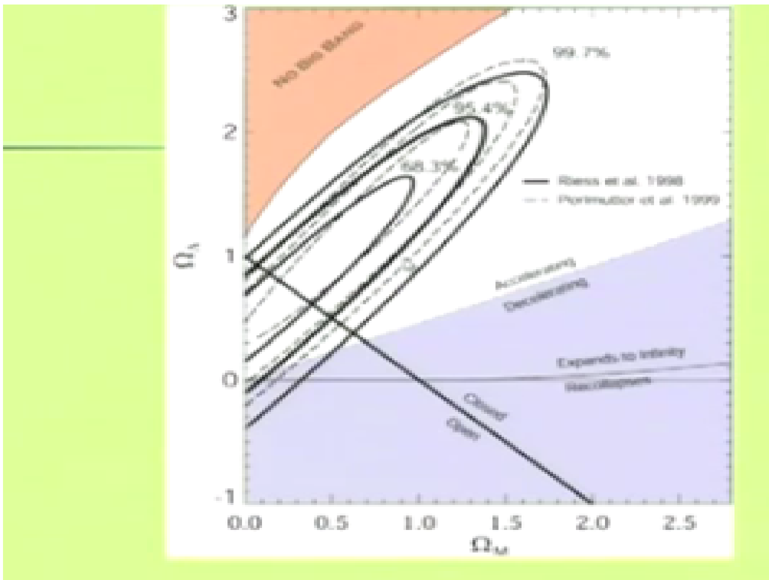

How can I put constraints on the model parameters? There are 3 parameters in my model; the amount of matter at present, the amount of curvature at present, the amount of cosmological constant at present, the sum of these 3 should be 1, so it is quite clear that any 2 of them will do. So, the 2 which have been shown here are this and this okay and these can take any value as shown in this figure.

So, this is a figure based on this okay same data suppose, we use it now to constrain the modules okay. So, this along the x axis, you have omega matter along the y axis, you have omega lambda present okay and if you want the universe to be spatially flat, then omega curvature should be 0, the sum of these 2 should be 1, which is the solid line over here. If you go below this line, you are in a model which is open.

If you go below the solid line in this diagram you are in a model where which is open, so this omega curvature0 will be positive, if you go above this line you are in a model where omega curvature 0 is negative, so the curvature is; omega curvature0 is negative, so the space is closed okay, K is 1 in my metric, fine. So, we have a clear understanding of this. This is a solid line which is spatially flat which corresponds to models which are especially flat.

Now, what does this? How do we interpret this picture? Let me explain this to you. Suppose, I take some combination of values of omega matter and omega lambda here, I take this point, this will be a combination of omega matter and omega lambda, I use these values of omegas to calculate dL. We know how to do that, so I use these values to calculate dL as a function of z, which we have discussed how to do.

Suppose, now I take dL as a function of z and see if it is consistent with this data, the data has got the distances to different supernovae; measured distances, right. The data has got the measure distances to different supernovae; this is a distance modulus to different supernovae as a function of redshift, right. We know how to calculate distance modulus, take d and divide by 10 parsec take log of that, that will give me this and multiplied by 5, it will give me a distance modulus, right.

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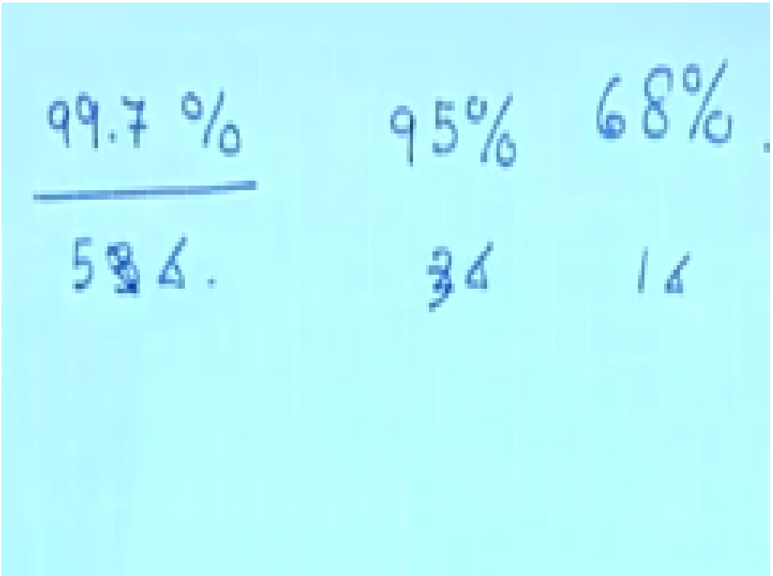

So, I have the data for the distance modulus from observations, my model will predict something how good is my fit or what is the possibility likelihood? What is the confidence with which I can say that this model is allowed or the model is ruled out okay, so these are confidence intervals what these confidence intervals tell us is that the innermost, the outermost curve over here.

The outermost curve over here says that there is 99.7 % probability with 99.7% confidence, I can say that the values of Omega matter and Omega lambda in my universe lie within this range okay. Essentially, if I take a value outside this, it will not be able to fit this data. The shape of the difference in distance moduli curve will be quite different okay, these are so confidence intervals basically, okay.

So, how you calculate them is; you take a model and look at the; compare the predicted dL versus z against the measure dL versus z and see how good is the fit, on the goodness of fit you can come to a confidence interval. So, with 99.7% confidence, you can say that my universe has values of omega matter and omega lambda inside this range okay, with 95% confidence, you can say that it is within a smaller range over here and with 68% confidence.

You can say that it should live within this okay, so 95; that is 95% and 65%, if my errors are Gaussian then this corresponds to 1 sigma, this corresponds to 2 sigma, this corresponds to 3 sigma, sorry; 1, 3 sigma and 5 sigma okay.

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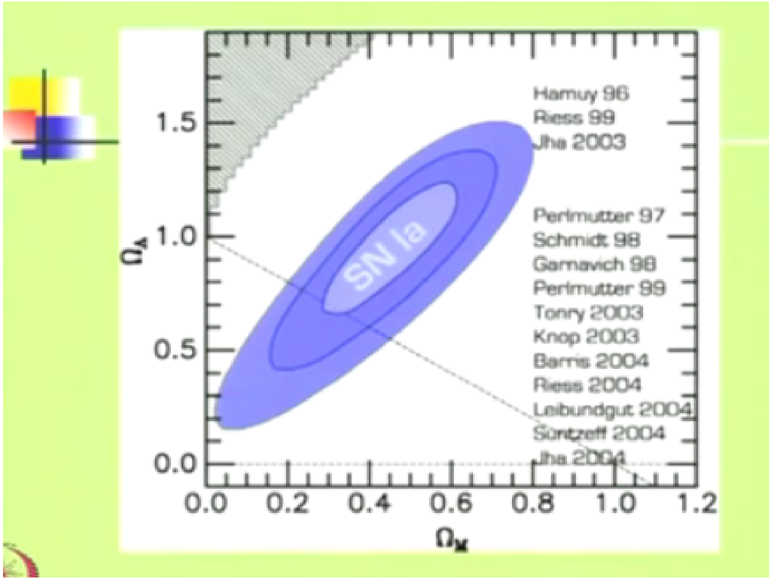

So, you see what we learn from this is that the supernovae observations can play a very important role in determining, what is the cosmological model that describes our universe? That is the crucial crux of the whole discussion okay and all of these models okay, this is a same curve from other some other data, so again you see this is the 68% confidence level; confidence region, the 95% confidence region and the 99.7% confidence region in a curve of; you know in the space of omega matter and omega lambda.

And the solid line is the region which the; where the universe is spatially flat. The point to note over here is that the supernovae observations indicate with a reasonably good level of confidence that the omega lambda has to be there, so my universe is a indicate with the reasonable level of confidence that the our universe has something which has got a matter component at present which has got negative pressure okay, how does this?

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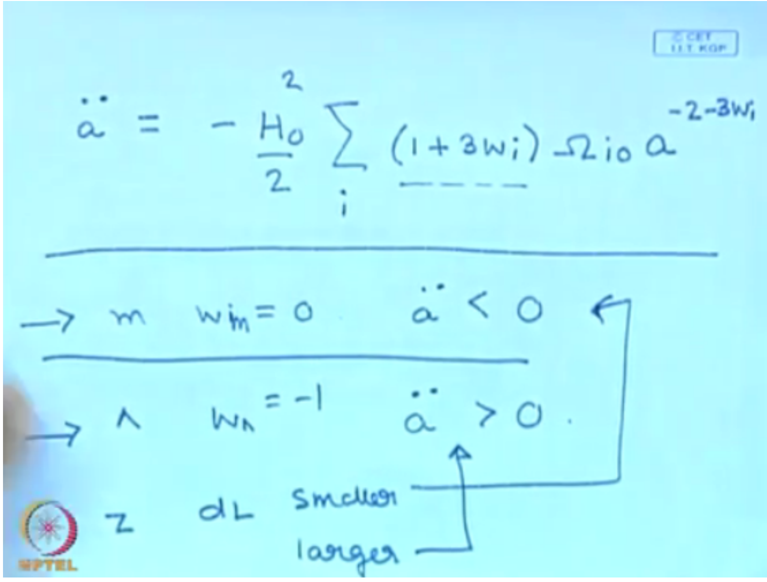

So, let me just briefly discuss this once more. we have learned that the basic equation in cosmology is this, oh sorry no; so suppose, we have a model which has got matter and cosmological constant then our various components and then the equation is that this is = –H0 square/2 some of the contribution from each component 1+ 3wi * omega i0 * a to the power –2 -3wi, okay.

Now, in this equation; if I have ordinary matter then this term will be positive because wi is 0 for matter; wi is 0, which means that a double dot will be negative because this is a positive term, this is a positive term, this is the universe is going to be decelerating, whereas if I have a cosmological constant, the pressure is negative; w is -1, so this is going to be -2, so it is going to make a contribution which will this term will be negative.

So, that will make a contribution a double dot is and >0, okay that is the crux of the whole thing. Now, what happens if my universe; how is it different if my universe is accelerating or decelerating, that is the next thing. If my universe is decelerating then the, whereas if my universe is accelerating, the distances are smaller in this model as compared to a model where my universe is accelerating okay.

Given the redshift; same redshift my distances are smaller in this model dL is smaller in this model and dL is largely relatively in this model which is accelerating that is quite intuitively quite clear okay, so the luminosity distance for the same redshift the luminosity distance is smaller if the universe is decelerating, it is larger if the universe is accelerating. Observations show that if I assume this model, a model which has only where there is only matter; the matter that I see or some dark matter is the only thing that is there the universe.

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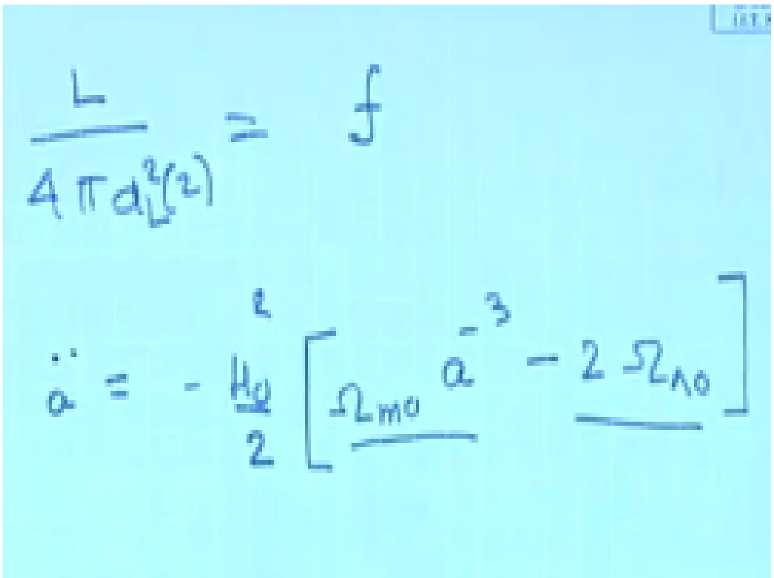

If I take such a model and I use this then the; I know the luminosity of the supernovae I can predict the flux, so I can predict how faint the supernovae should be; it turns out that it is actually fainter than this what this model will predict. The supernovae turn out to be fainter than what this model predicts, so you have to somehow make the distance is larger and the only way you can make the distance is larger is to make by making the universe accelerate okay.

So, this is; what leads us to a picture where the universe has; is now undergoing a phase of expansion where it is accelerating okay, so the universe is now in a phase of expansion where it is accelerating that is how we interpret these observations okay. Now, you may ask the question what is the universe always accelerating, obviously not because as becomes smaller, see if you write down the equation for a double dot; if I write this down for a model which is omega matter and omega lambda0, this is what we get.

The curvature does not make any contribution here it is just the constant of integration when I integrate this, right, so let us just look at this equation. This equation will; the universe will decelerate when this term dominates and it will accelerate when this term dominates okay, so there will be a transition from an decelerated phase to an accelerated phase depending on which of these 2 terms dominate okay.

So, let me okay; let me stop the discussion here and resume in the next lecture, time has run out, so let me stop here.