Astrophysics & Cosmology
Prof. Somnath Bharadwaj
Department of Physics and Meteorology
Indian Institute of Technology – Kharagpur

Lecture - 31
Distances (Contd.)

Welcome. Let me remind you of what we have been discussing distances in the context of cosmology and let me start of by recapitulating the things that we have discussed in the last class. So, we started off by looking at the cosmological line element.

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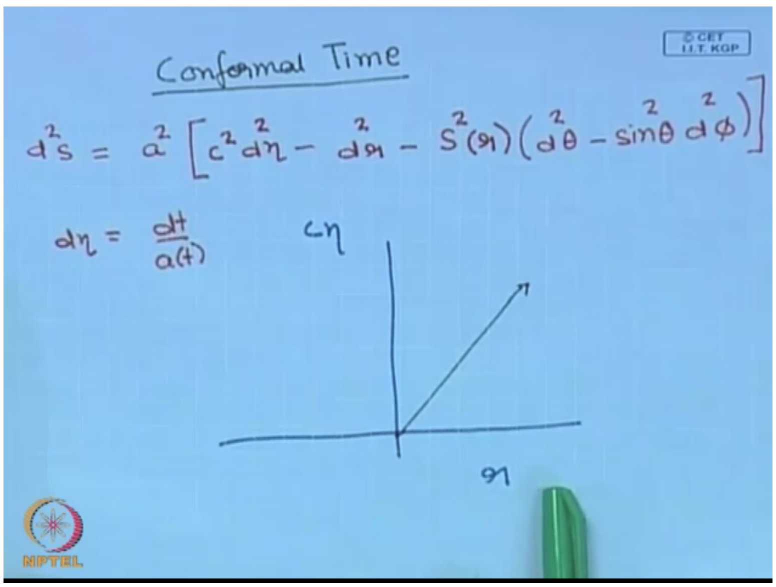

In terms of the conformal time eta and the advantage of the conformal time is that the propagation of light in comoving coordinates looks like straight line if I draw it on a space time diagram where I use eta instead of usual cosmic time. That is the big advantage of using this conformal time. And let me also remind you that there are 2 different functions over here which come into the line element one is the r and the other is s.

If the matric is specially flat then r and s are same otherwise they are 2 distinctly different functions if the space is curved

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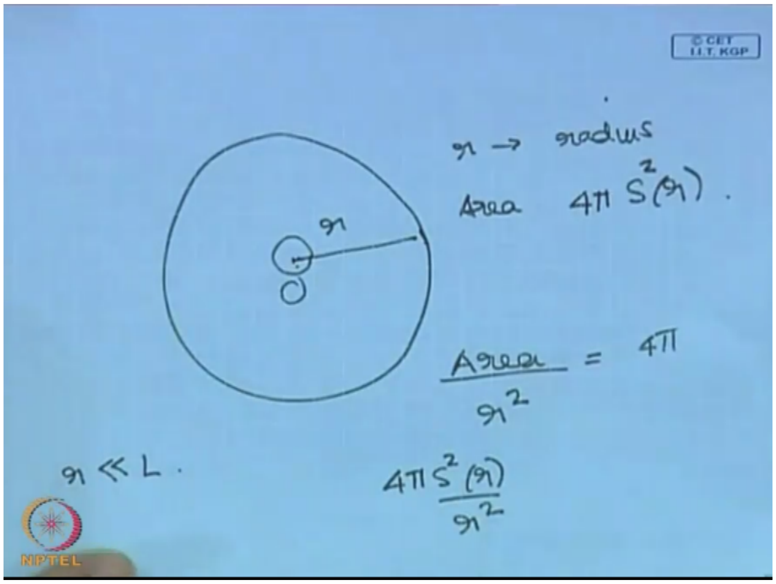

And the significance of this I had explained in the last class r is the distance. So, let us take a another point over here a galaxy let us say over here. r is the comoving coordinate to the galaxy so if I consider of sphere of radius r the surface area of that sphere is given by 4 pi into this square. Because this is the thing that multiples these solid angular terms. So the surface area of the sphere is given by 4 pi s square in general and the distance is r.

So, the ratio of the surface are to the radius square is not a constant. It is a function of r that is the way the curvature of space manifests itself and this ratio becomes a constant when the radius is much smaller than the radius of curvature, comoving radius. You recovered the value 4 pi. But if the space is curved as you approach the length scale the comoving curvature length scale you have significant deviation in this ratio from the value 4 pi.

So, there are 2 things over here this is the radial distance. This is the transverse distance and the physical situation that we considered let us move on to that.

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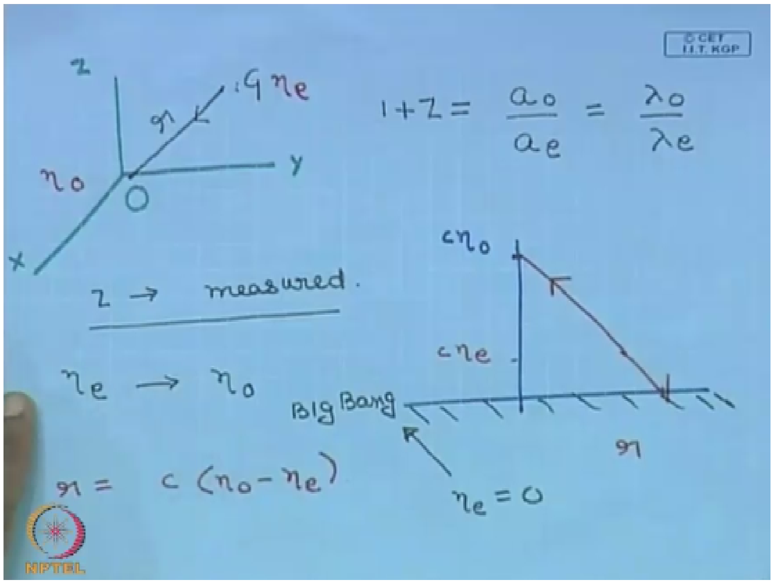

So were considering a physical situation where there is a galaxy from which we are receiving radiation and the radiation is red shifted. The observed red shift is z. And we have seen that this red shift will be the ratio of the scale factors between the epoch when the light was emitted and the light was observed and the comoving distance to this galaxy = c into the difference in the conformal time between these 2 events. And we worked out how to calculate this.

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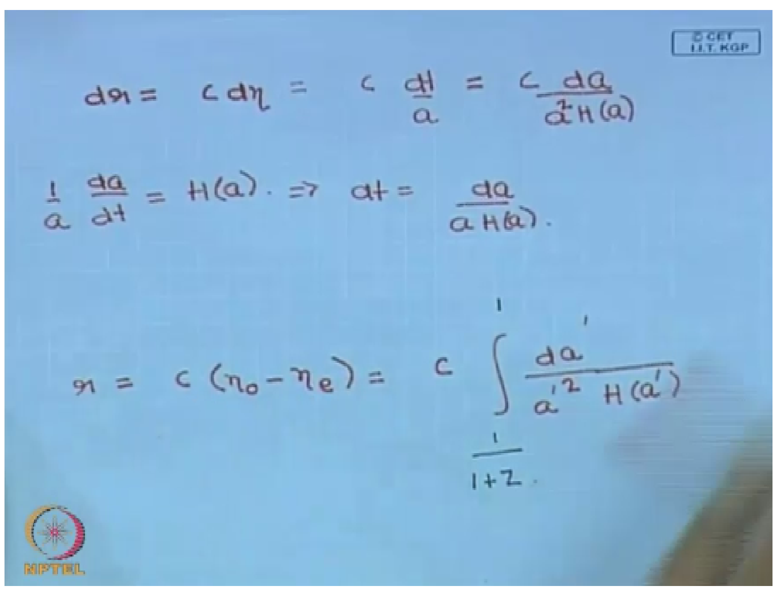

So to calculate this you have to carry out integral. This integral allows us to determine the comoving distance to any source whose red shift is known. Usually in cosmology red shift is the observed quantity everything else has to be determined from the red shift usually. So, if the red shift is known you can use this formula to determine the comoving distance also the difference in conformal time.

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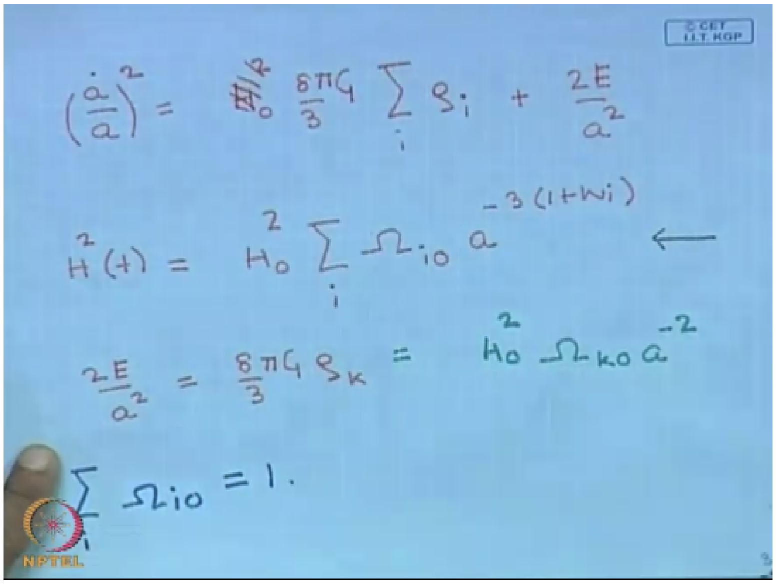

And the way to use this formula you have the Hubble parameter as a function of scale factor here. So, one has to use the dynamics of the Universe which depends on the constituents of the Universe to determine this. And we had worked out one particular example. The example that we had worked out is where omega matter is 1.

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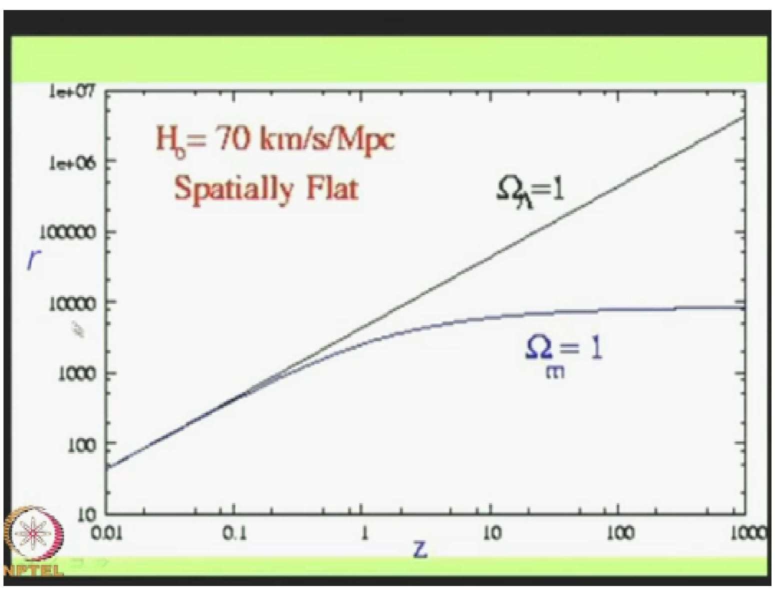

There is only matter in the Universe. And for this modal we had seen that the comoving distance goes as 1-1/ square root 1+z*2C/H0. Let me show you what this looks like. We had drawn this in the last class let me show you here in the graph what this looks like. So, you see that it is increases initially as expected according to Hubble's law this we had checked. We have seen that when red shift is extremely small the is the same as Hubble's law.

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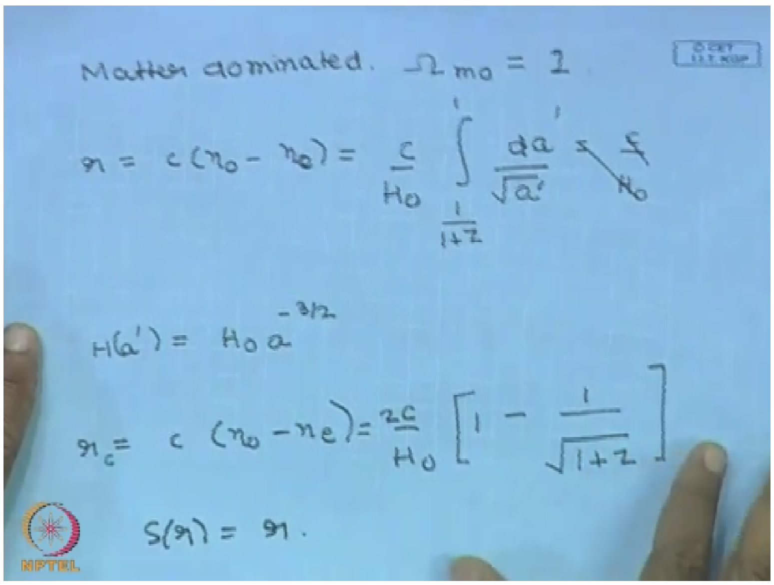

We had worked this out in the last class and then what happens is that as we will go to larger red shifts there will be deviations from the linear relation because the dependence is different and finally you find that at very large red shift it saturates which is a constant. And we had also discussed the implication of this. This implies that red shift infinity corresponds to a finite distance.

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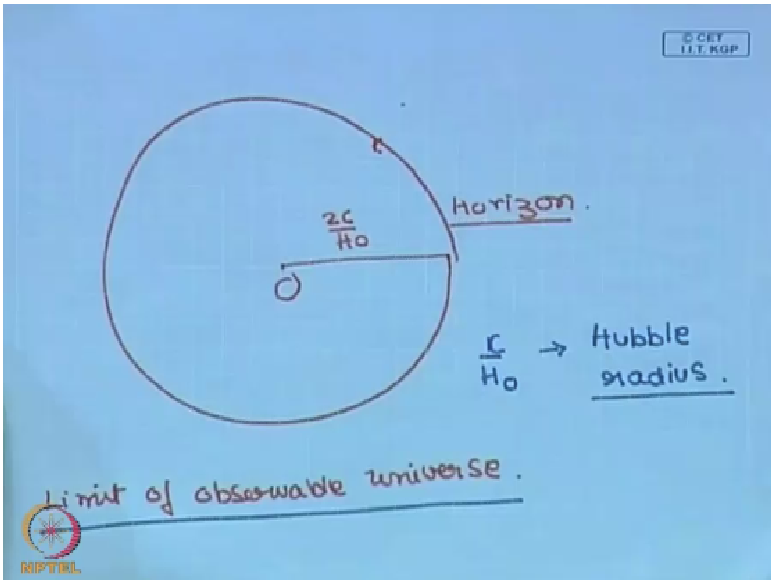

That finite distance is the horizon that is the furthest distance that we can be see, observe at present. A photon which was emitted at the big bang from this surface will reach us now. And we had also introduced a length scale called the Hubble radius C/H0 and in this particular cosmological modal the horizon is twice the Hubble radius and we can make an estimate of how much this is. So, let me quickly do.

This is a very important number let us just quickly make an estimate. So, the Hubble radius C/H0, 3*10 to power 8 meter per second which we can write as 10 to power 5 kilometers per second.

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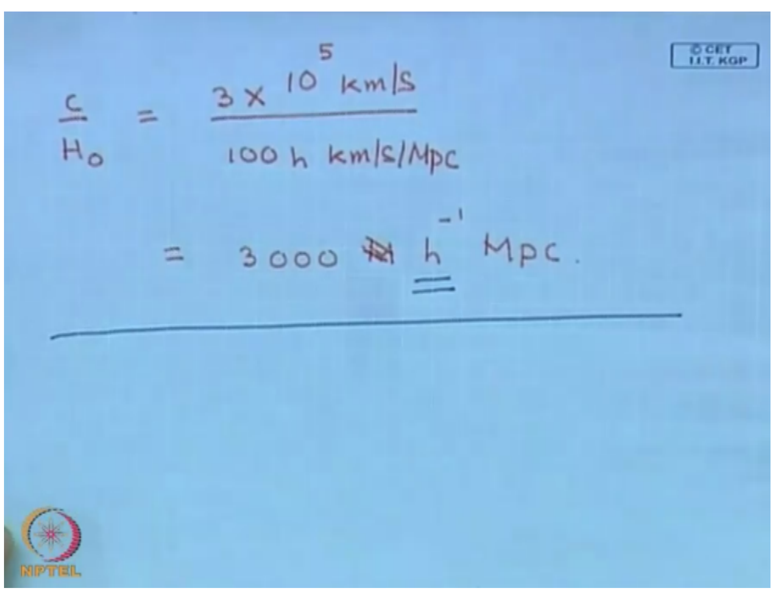

That is the speed of light divided by 100h kilometers per second per Mpc. So, we see that the Hubble radius has a value which is 3000 here we have 100, here we have 10 to the power 5. So, it is 3000 h inverse Mpc. And the horizon in this modal is twice this 6000 megeparsec h inverse megeparsec. So, the furthest distance that you can see in this omega matter =1 cosmological modal is 6000 h inverse megeparsec that is the size of the horizon.

Another point to note which I have mentioned earlier also that the entire distance scale crucial hinges on this factor h inverse on the Hubble parameter. If the Hubble parameter is smaller all the length scale are larges. Hubble parameter is larger all the length scale inversely related. So, this is just a numerical estimate now let us go back to this picture. So, here I have drawn this function for you so the function that I am showing over here is essentially this.

Where we have worked out the integral which allows us to calculate. We have worked out this integral for the particular cosmological modal where Omega matter is 1. In this picture I also show you another –the units here are megeparsec. I have chosen Hubble parameter H 70 and models are both spatially Flat. I have also shown the comoving distance r for another cosmological modal which has only cosmological constant and no matter.

You see the difference so the way the comoving distance varies with red shift is crucially dependent on the cosmological modal that we have. Through the dependence of the Hubble parameter H on the scale factor. The scale factor has different dependence on a for different cosmological models. The other thing that is important which I should also show you now is that you have another function which is S.

You have 2 functions coming into the picture so we have one more function which is S. Let me also show you how S behaves with red shift because given the red shift you can determine r and for the r you can determine S. If the modal is specially flat S and r are exactly same. But if the model has curvature then they will be different.

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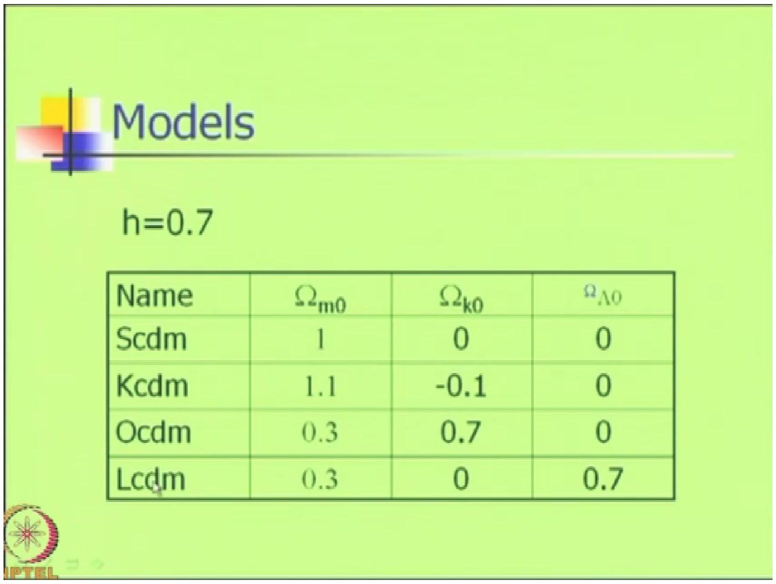

So, the entire discussion subsequent discussion is going to be in the context of these 4 cosmological models. We have given them names. So, I will request you to keep this in mind that there are 4 cosmological model in the context of which we are going to have the discussion. The first one is the standard cold dark matter, cosmological modal. And this has omega matter not equal 1.

So, the entire Universe is filled with dark matter. I have told you that there is evidence for dark matter and this is a modal where assume that the entire Universe is filled with this matter so that it is exactly = the critical density. We do not see so much matter in the Universe so let us assume that it is dark and this is the modal. This is another modal where there is more density then the critical density.

And in addition to this the universe –if there is more density then the critical density the potential energy is going to be negative which will make the Universe collapse and this modal is going to be specially curved. Omega curvature is negative so it is going to be specially curve. So, the sum total of these 2 has to be one so this is the contribution from curvature. So, this is a modal which I am following Kcdm curvature. It has some curvature which is positive.

The third modal is open Cdm. This is cosmological modal where omega matter. The matter contribution pressure less matter is .3. There are observations which indicate that this is the matter contribution at present is around 30 percent of the critical density total matter contribution. So, the rest of it we are assuming here is in curvature and this is a 4th model where again omega matter is .3 and the rest of it is in cosmological constant lambda .7.

This is called lambda cdm, Lcdm. Let me also mention that this is the currently favoured cosmological modal. Observation seem to favour the last one. So, let us now see what is the behavior of this function S.

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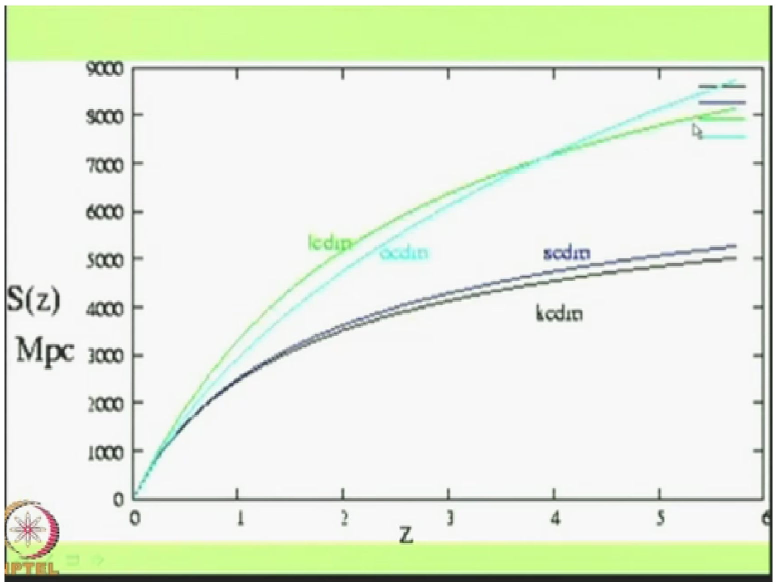

How does it behave as a function of red shift for these 4 different cosmological models? This picture shows you exactly that. So, you see in this modal in the curve one which is positively curved the function S is basically sign and for large red shift this will grow slower then the linear. Sign function it grows slower than the linear function as sin theta. For small theta it is linear and then it becomes slower. So, you see this modal falls below all the other models.

The modal over here falls below all the other models kcdm and then we have the standard cdm which is completely better dominated has only matter in it. And here we have the open cdm and the lambda cdm models. The value is zr comparable but again you see that at low red shift the lambda cdm modal dominates whereas at it is reversed at larger end shifts.

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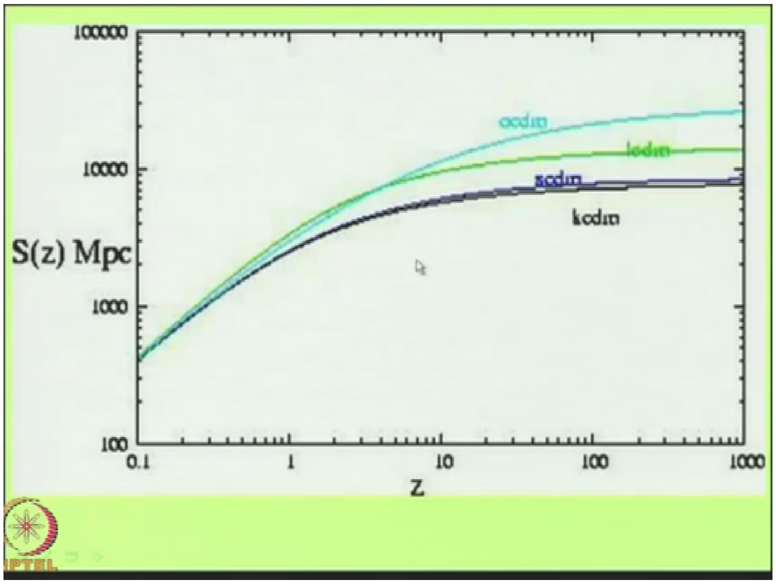

And this shows you the same function plotted over a large red shift range in a log scale. It is the same function but plotted over a large red. If you want to display some function over a large red shift range obviously you cannot use a linear scale. You will only see the behavior over a small part of it. So, this shows you the same function over a large red shift range all the way to red shift 1000 and this shows you the behavior of the different functions.

So, you see if you look at around redshift 1000 the open cdm modal the S z functioning increases much more is much higher than the other 3 models. Remember this is a log scale so a small increase this is an increase of 10 times. So, this is more than double the difference between this and this is more than double. Similarly, the difference between this and this is roughly double. So, the point I am trying to make here is that both are an S have depend for the red shift.

Dependence of both of these things is different in different cosmological models.

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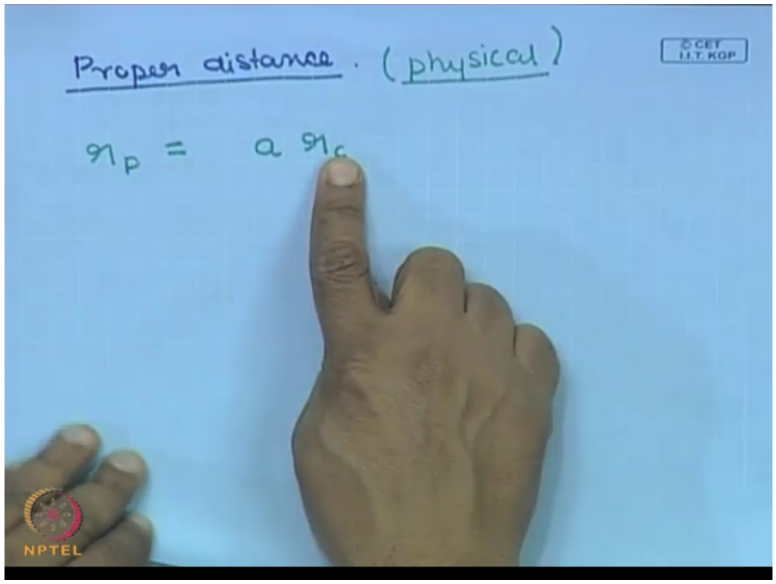

But then I told you that comoving distance is no a physical quantity the first physical quantity that you can define is the proper distance which is actual physical distance between 2 points. But again you cannot measure this. So, for the observational point of view and first important thing that we discussed is the angular diameter distance and let us recapitulate how the angular diameter distance is measured.

The angular diameter distance is defined so that if I have an object whose proper size is known then the angle it subtends is the angular diameter, the ratio of the proper size to the angular diameter distance or what is more useful is that if I know the angular diameter distance and if I can measure the angular subtends I can get the proper size. And usually there are certain things called standard rulers.

For which we have some idea of what the proper size is and the angle can be measure. This is where the angular diameter distance is particularly relevant. And the angular diameter distance is defined as the ratio of Sz/1+z. And if its specially flat then it is r/1+z. So, this picture shows you the angular diameter distance for different cosmological modals. The picture on the screen shows you the angular diameter distance how it varies with red shift for different cosmological modals.

So, you see that for all the modals it has a maxima somewhere around between 1 and 2 and then it falls off. And again you see that the angular diameter distance in this red shift range between 1 to 0 to 6 Lcdm is much larger then all of them. Lcdm and ocdm are actually quite close nearby whereas the ones which have significant amounts of matter. Omega matter is close to 1 the distance are much smaller considerably smaller.

And if you look at the behavior over a large red shift range this is what it looks like. And at large red shifts the open cdm modal has a larger angular diameter distance compared to Lcdm. An Lcdm has larger than the standard cdm or curve one with the positive curvature. So, this is the angular diameter distance. It basically refers to angles. The next thing that we were discussing in the last class is the luminosity distance.

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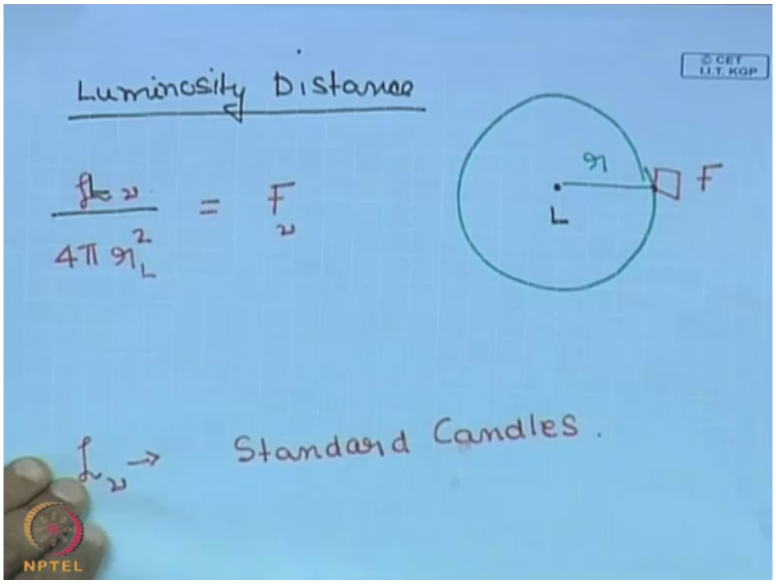

And I had told you what the luminosity distance is. It is based on the relation between luminosity and the flux. So, now let us derive the luminosity distance r L. So, let us consider a source whose luminosity the amount of radiation it emits, in the frequency range L nu emitted, delta nu emitted.

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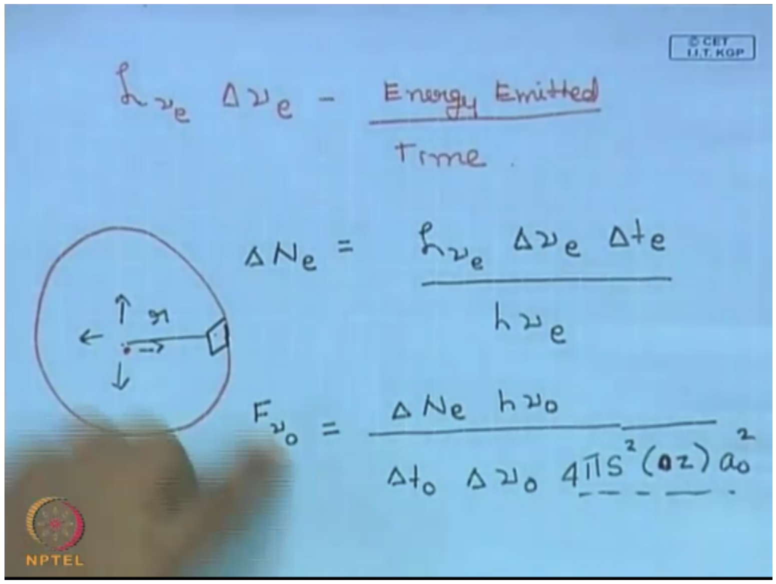

This is the energy emitted in the frequency intervals. So, energy emitted in the time interval per time. So, L nu is the energy emitted in the frequency interval delta nu e basically. So, there is a source over here which is emitting radiation and this is the luminosity in a particular frequency interval. So if I multiply with frequency interval I get the amount of energy it radiates per unit time.

And we want to find what will be the flux that will be measured by an observer over here. How is the derivation done? The derivation is done that you consider the photons that come out of here they will be distributed over a sphere like this and you divide that by the area of the sphere you will get the flux. The time over which it is emitted and the time over which it is received is the same usually not in the usual context.

So, you have a 100-watt bulb and you want to do it. But here you see the time interval also gets changed because of the expansion of the Universe. So, let us put in the time interval and the frequency also get changed because of red shift. So, let us now calculate the correct relation between the luminosity and the flux taking into account all of these. So, to do this let us calculate first the number of photons that were emitted. The number of photos will not get changed.

So the number of photons that were emitted delta N emitted = --in a time interval. So, this is the number of photons that were emitted in a time interval delta T e in the frequency interval delta nu e is given by Lnu e delta nu e delta t e/ h nu e. This is the number of photons that were emitted in the frequency interval delta nu emitted in the time interval delta t emitted. This is the energy that was emitted in this frequency interval.

And time interval divided by the energy of each photon. I will get the number of photos. Now, the same number of photons that were emitted will propagate out and they will pass through a surface of some radius at some later time. So, the same number of photons will be received over here. And to determine the flux at the observer what I have to do is I have to divided delta nu delta N emitted by I have to multiply this with the frequency of each photons.

So, the frequency of the photons would have changed by the time it comes from here to here. It will be h*nu observed. This is the total energy that is crossing this surface and it will be received by an observer over here over a time interval delta t observed over a frequency interval delta nu observed. This will be the flux in the frequency interval observed over here. Now the question what is the ratio of these emitted frequency and observed frequency?

We have already worked this out. Sorry, divided by the area of the sphere. And the area of the sphere is going to be 4 pi*S. The area of the sphere is going to 4 pi*s square, that is the area. It is not going to be C delta T, it is going to 4 pi S square. That is how you calculate area in cosmology. The distance is r the distance from here to here is r. So, the area is 4 pi/s square, not 4 pi/r square. So, this is the flux. And we know how to calculate S.

So, the situation that we are considering the observer is at the red shift 0. The source is at the red shift z. So, we know what this r is we know what s is. S is a known function and we want the physical area here this is the comoving area you have to divide it by the physical area. The physical area the proper area this will have to be multiplied with the scale factor square. This will give me the comoving area but the scale factor at present has a value unity.

So actually there should be a a0 square here also. Like this is the area of the sphere at the instance when the observation is done. But a0 we have assumed to be 1 because the observer is at present. So, this is 1 this will give us the flux in the interval delta nu. So, now you see we have to determine what is the ratio observed frequency to emitted frequency. Let me put this here. So, what we will have is that the flux = the luminosity.

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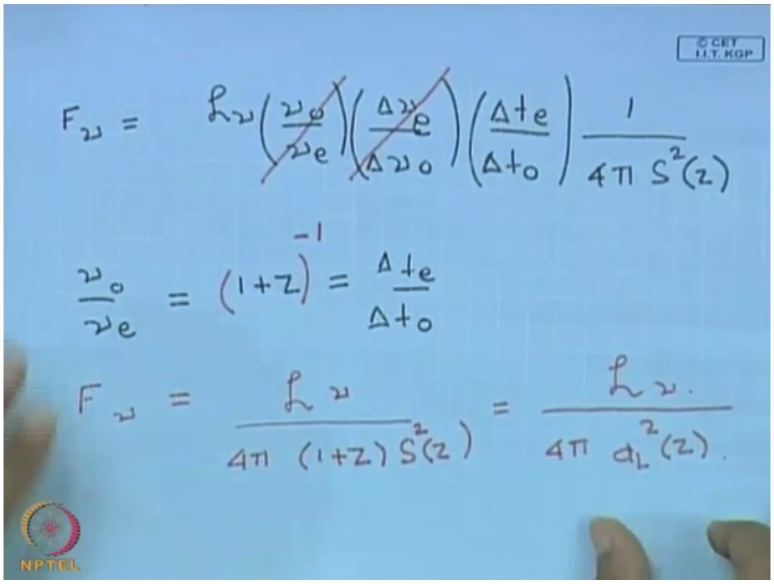

And then I will have here the nu observed by nu emitted coming from this and this I will have delta nu observed by delta nu sorry delta nu emitted/ delta nu observed. Delta t emitted/ delta t observed into 1pi/ 4 pi S square. That is what we have. So, I have just put this delta N over here. So, I will have the ratio delta t emitted/ delta t observed, nu observed/ nu emitted and delta t and delta nu ratio is okay. So, this is what I will get fine.

Now, we also know that nu observed by nu emitted = 1+z which is also = t emitted/ t observed. We have worked this out. How delta t emitted? How time intervals change because it is light that is propagating so we have worked this out right in the beginning. So, you see that this ratio will cancel out with this ratio and this ratio will give me a factor of 1+z. So, delta t emitted/ delta t observed, this will be the inverse of this so delta t emitted/delta t observed.

So, what we have here is that the flux, no this will be other way round, sorry. "Professor - student conversation starts here" The frequency of light as it propagates what happens? Does it increase or decrease? How much distance source, will the frequency be higher or smaller? Smaller. It will be smaller, right. "Professor - student conversation ends here". So, the observed frequency will be less than the emitted frequency, observed wavelength is more.

So, it will be 1/1+z and the time interval is opposite is the frequency. So, there will be a factor of 1+z-1 here. So, what I will get is 1+z Sz square. Compare this with the definition of the luminosity distance this is 4 pi dL square z. So, what we get from here is that the luminosity distance dl = square root of 1+z*Sz.

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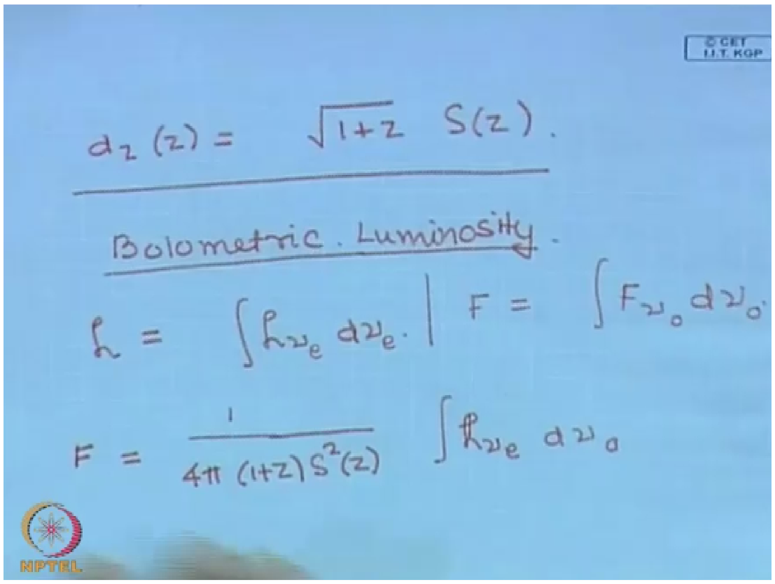

This is if you are dealing with luminosity and the flux per frequency interval. Now, quite often you do not deal with this. What you deal with is the bolometric luminosity. So, what is the bolometric luminosity? The bolometric luminosity is the total luminosity integrated over all frequencies. So, this is the bolometric luminosity which is L nu integrated over d nu. Similarly, the bolometric flux is going to be F nu integrated over d nu.

Now, the point to remember is that here the integral will be over the emitted frequency. And here the integral will be over the observed frequency after all this is what you observe. You see the light that you observe here. This is the observed frequency. This is emitted frequency they are not the same. You are actually what this gives you is the relation between the luminosity at a certain emitted frequency and the flux at another observed frequency.

Some observed frequency which is different and these 2 are related. The frequencies are related through the red shift that red shift formula, here this formula. So, if you take this expression and integrate it let us say we integrate it over the observed frequency. So, we integrate this over the observed frequency then what we get is that the flux, the bolometric flux = --so I will have 1/ 4 pi 1+z S square.

That is what I will have here and then I have the integral the luminosity d nu observed. Because I have integrated this over the observed frequency and this I have to write in terms of the emitted frequency. Because I know the luminosity, the bolometric luminosity is the integral of this over all emitted frequencies. So, I have to then multiply rather divide this, no what I have to do. So the flux can be written as 1/4pi 1+z square S square z Lnu emitted dnu emitted.

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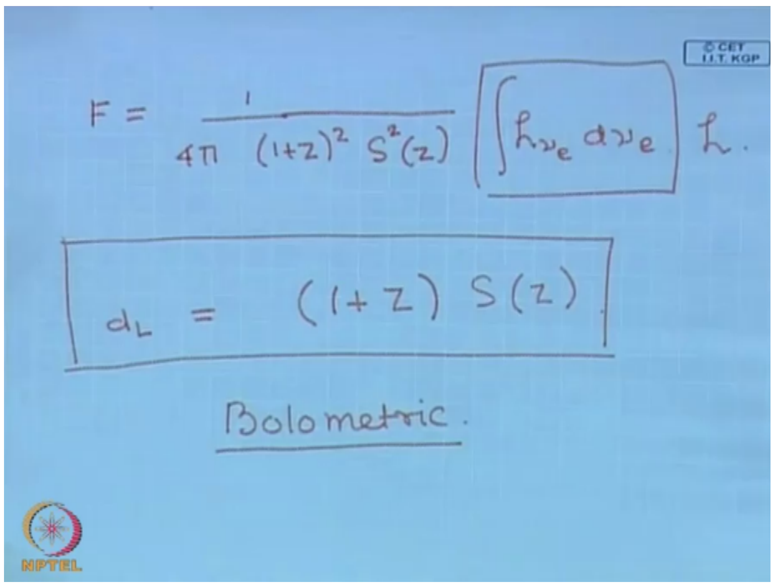

Because I have to multiply this with the factor of 1+z this becomes the d nu emitted and this thing is the bolometric luminosity L. So, we see that if you use the bolometric flux and the bolometric luminosity then the luminosity distance = 1+z into the --and throughout our discussion we shall we using this definition.

So, this is if you use the Bolometric luminosity and the bolometric flux. So, let me show you how this varies with red shift. So, this picture it is graph over here. The graph on the computer screen shows you how the luminosity distance varies with red shift for the different cosmological models that we were discussing. So, you see again the ones which have large amounts of matter dominated both these models.

The behavior is quite different from the modal where omega matter is .3. And you see again here there are differences between the Open cdm and Lcdm model. This shows you the same thing how the luminosity distance varies with red shift over a large red shift range. So, we see that the distances which are important in observational cosmology are essentially the luminosity distance and the angular dimeter distance these are the 2 things which are important in observational cosmology.

Instances that we have been discussing till now the luminosity distance the angular diameter distance comoving distance etcetera all of them have the property that at low red shifts we recovered the Hubble's law. So, this is a very important thing so all the laws at low red shifts recovered Hubble's law and you can all the distances you can check that from the formula. And these if you have observations.

That low red shift then these observations do not depend on the cosmological modal. They only depend on the value of H0 which sets the cH0 overall length scale. So, the basic idea that we are going to discuss now is how to determine H0 and observation that low red shift can be used to do this. So, if you can measure the distances to different galaxies independently and the red shift independently.

If you can measure red shift and distances independently you can then determine the value of the Hubble parameter. I have also told you several classes ago that determining distances is the most difficult thing in cosmology you can infer the distance from the red shift that is easy. But independently determining the distances is extremely difficult.

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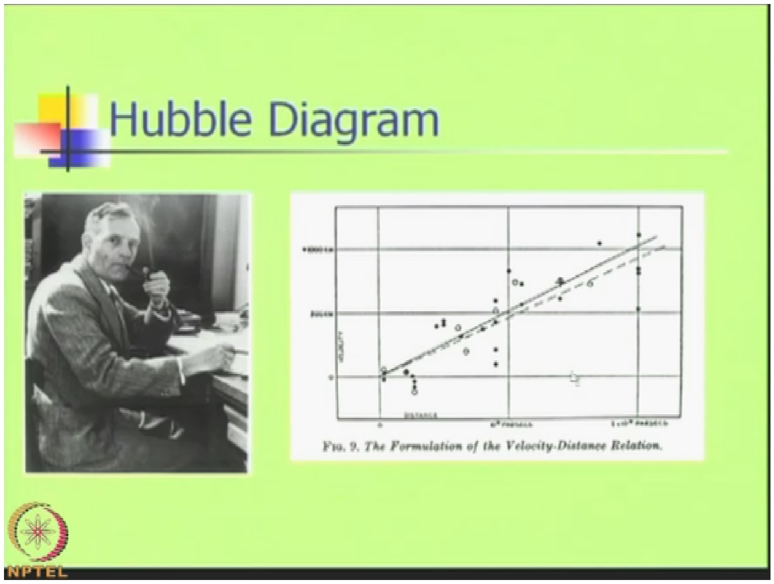

So, let us now take a look at this so this is what Hubble has done to start with. Hubble has measured distances using Cepheid variables. Distances to galaxies and he has measured the red shifts on the spectra and this is a picture of Edwin Hubble and he has measured these 2 quantities and he found that a straight line that they have a linear dependence. Hubble's observation unfortunately were based on very low red shifts.

So, he had only a few galaxies located very close to us and the slope that he got was far too high. He got a slope of around 400 which brings down the entire distance scale of the Universe.

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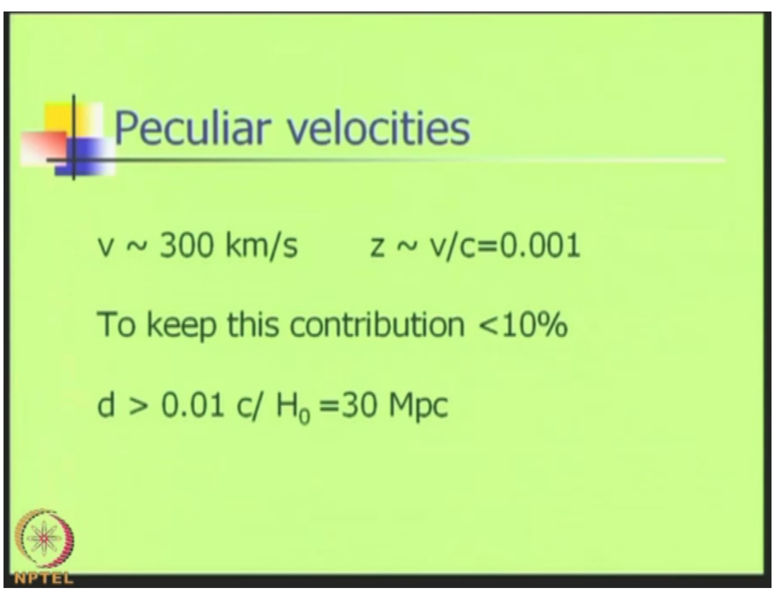

So let us now take a closer look at the future development, the subsequent developments since

Hubble. The basic problem in determining the Hubble parameter likes in the fact that you have peculiar velocities. And the motion of a peculiar velocity something new so let me send a little time discussing this. So, in the picture that we have learnt the observer here.

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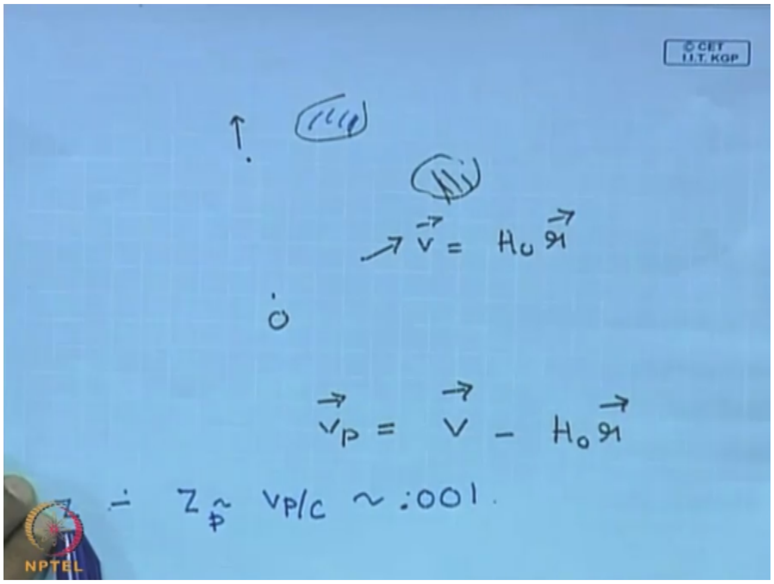

So, in the picture that we have learnt the observer here and this observer will see that all the galaxies are moving away with the velocity which is proportional to the distance and this velocity in the new by Universe and this velocity can be interpreted as the red shift that is in the new by Universe. If you look at the entire picture, then it is better to interpret it in terms of the scale factor. But in the nearby Universe this is fine.

Now, this is the component of velocity due to Hubble expansion, you can think of it like that. Now in reality in addition to this expansion the velocity due to expansion of the Universe the galaxies have other components of velocities also. And this other component is called the peculiar velocity. So, peculiar velocity is by definition the actual velocity of a galaxy minus what you expect from the Hubble law.

This definition is in the nearby Universe. If further away it gets slightly modified but it is essentially is this. It is the deviation from the Hubble expansion. Why are there deviations from Hubble expansion? From the motion predicted by the expansion of the Universe these deviations arise because the Universe is not exactly homogenous and isotropic there are deviations. And some parts of the Universe have more matter.

So, this is more matter. Some parts have less matter so I will put a different color. And the places which have more matter exert an extra force on this galaxy for example or a place where there is less matter will exert an extra repulsive force. So, the analysis till now has assumed that the Universe is homogeneous and isotropic. But if there are deviations these will cause deviation in the velocities also. And these are what are called peculiar velocities.

For example, Andromeda galaxy is not moving away from us. It is moving towards us. Why? Because in the neighbor between our galaxy and Andromeda galaxy in this region there is excess matter compared to the average density of the Universe which is causing the Andromeda and our galaxy to come together. So, there are peculiar velocities in the Universe this is an observed fact and the peculiar velocities typically have speeds of the order of few 100 kilometer per second.

So, you could take a typical speed to be 300 kilometers per second. And if you convert this to red shift using $z = v/c$ on the order of v/c then you get that this corresponds to red shift of 0.001. I have chosen 300 so that this comes out to be the division as easy. So, peculiar velocities will contribute to the red shift and the contribution will be of the order of 10 to the power -3. So, the red shifts that you see have a contribution due to the Hubble expansion.

In addition they have a contribution due to peculiar velocities, let us call it the peculiar red shift peculiar velocity/c. And this is of the order of .001 that is what we saw. This introduces errors in the red shift. The red shift that you measure will not be just due to expansion it will have a contribution due to peculiar velocities. So, this will be reflected in an uncertainty in the Hubble parameter.

Because what you will do is you measure the distance, we will measure the red shift and look at the ratio. If red shift has some other contribution not due to the expansion of Universe that will be reflected in an uncertainty to the Hubble parameter and we would like the uncertainty in the Hubble parameter in this. We would like the error due to this peculiar velocity to be less than 10 percent. Let us put an restriction like this.

If you want this condition to be satisfied then you have to look at a red shift of 0.01. If you consider a galaxy at a red shift of 0.01 then the effect on the peculiar velocity we know is going to be within 10 percent. And red shift of 0.01, you have to look at galaxies at a distance which is more than 0.01*c/H0 which comes out to be around 30 megaparsec. So, for a reliable determination of the Hubble parameter.

You have to look at this relation at distances beyond 30 megaparsec only then will the contribution of the uncertainty due to the peculiar velocity will be less than 10 percent. So, you have to look pretty far away you cannot look just a nearby galaxies to determine the Hubble parameter that is the first thing that we have to realize. Now once you look at galaxies at 30 megaparsec you can no longer identify Cepheid variables at that distance.

You can no longer identify the starts. So, a galaxy at that distance you cannot identify individual starts. It looks like a fuzzy thing in the sky that is all. You will not be able to make out the stars. So, one has to look at other things, other quantities to determine the distance.

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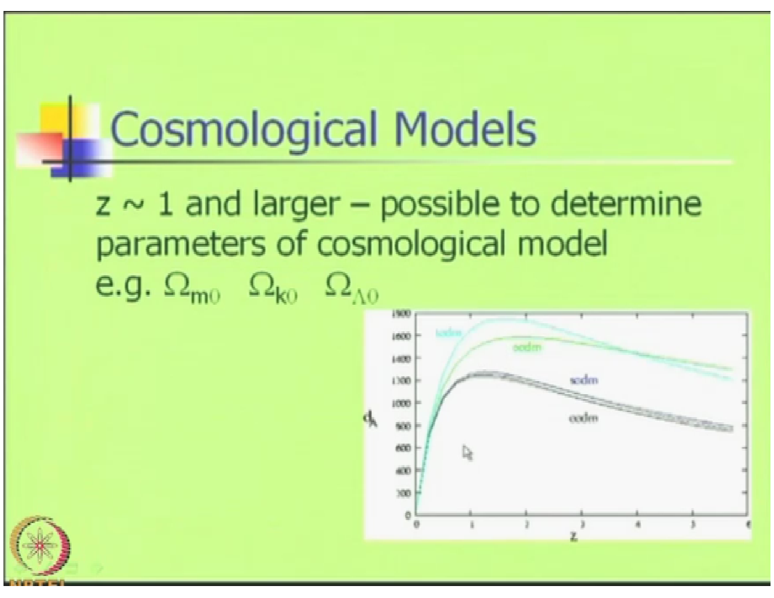

Other important thing is that if you can push this exercise to larger red shifts, red shifts of order 1. At red shifts of order 1 we have seen that the different distances for example the angular diameter distance this the way it varies with red shift depends on the cosmological modal. So, if you can measure the distance as a function of red shift you can determine which cosmological model we are in. So, you can essentially determine the values of these omega density parameters.

Because the distance will be different for different density parameter, you will get one value so you know which one we live in. That is the basic idea.

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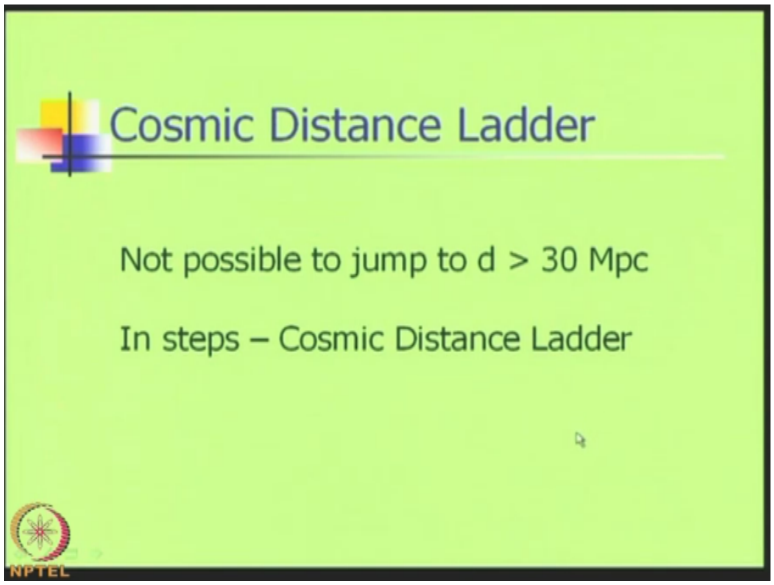

So, this brings us back to the issue of the cosmological distance ladder and we have already discussed the cosmological distance ladder to some extent we have learnt that you can use Cepheid variables to determine distances out to galaxies which are nearer. So, the globular cluster and nearby galaxies were you can make out stars you can use Cepheid variables to determine distances. Now, let us look at the cosmological ladder again.

So, we have to now determine distances beyond 30 megaparsec that is basic problem. And it is not possible to directly jump to 30 megaparsec because you cannot see features there. So you do not know how to do it. So, you have to again extend your ladder put various steps in between which overlap. So, let us go through this. So, we will now start the discussion from the largest scales other way round.

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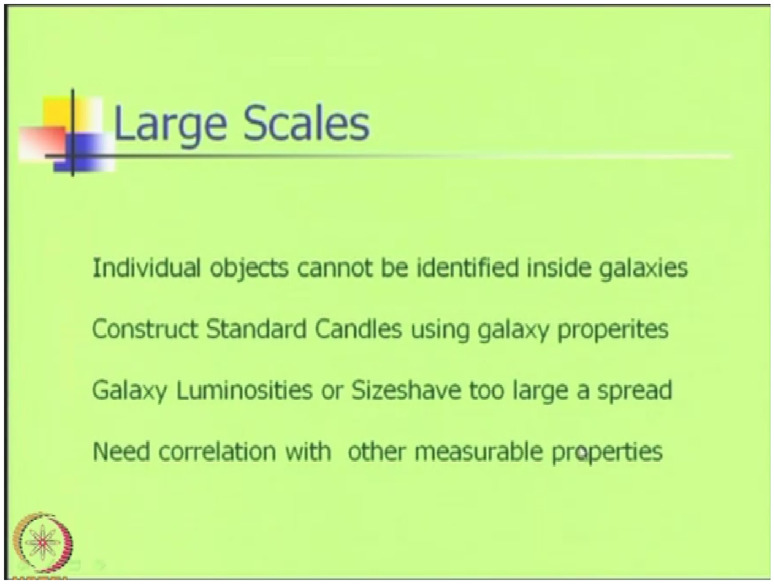

And work back to the place where he can use Cepheid variables. So, the basic idea is that at large distances like 30 megaparsec you can use properties of galaxies that is the only thing that you can use by and large, properties of galaxies. Now, question is that if you just take the luminosity of a galaxy. If you assume that all the galaxies have the same luminosity and use them as standard candles.

So, let us suppose we assume that I know the luminosity of all the galaxies they are same. And then if I take the flux and interpret that due to the variation in the distance then I can get the luminosity distance if I can measure the flux. Then I can get the luminosity distance as a function of red shift. This will not yield correct results because the galaxy luminosities are large scattered. Galaxies do not have a fixed luminosity.

The galaxy luminosity have a large scatter they come in a large range. So, our galaxy for example has a luminosity of around 10 to the power 11 times the luminosity of the sun that is the number of stars in our galaxy. There are galaxies which are considerably fainter and considerably brighter. So, the variation in the flux cannot be related to the distance. So, one has to look for other means of determining the luminosity of the galaxy.

And the basic idea is that one has to look at the correlation between the luminosity and the property, other measurable properties of the galaxy. So, you cannot measure the luminosity but there are other quantities which you can directly measure or you cannot measure the length of a galaxy. But there are other quantities which you can measure and if you can correlated the length to those then you can use this to determine the get standard candles and rulers.

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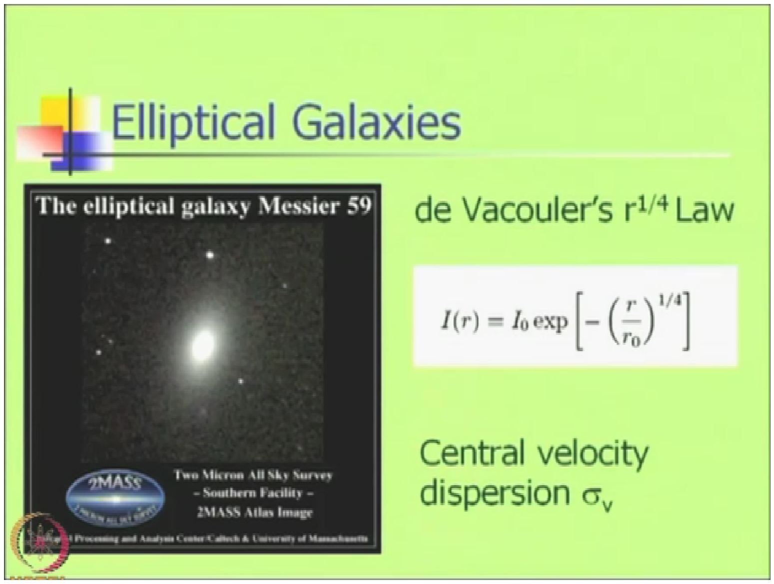

So, let us first start off with elliptical galaxies. So, we have already learnt that the distribution of specific intensity in an elliptical galaxy looks like an ellipse and these are believed to be ellipsoids, 3 dimensional ellipsoids. Now, the specific intensity variation inside the galaxy away from the center so specific intensity obviously fall off as you go away from the center. This fall of the specific intensity away from the center is well described by what is called de Vacouler's r to the power one 4th law.

So, the way the specific intensity surface brightness falls off as you go away from the center is well described by this law. I0 exponential –r /r0 to the power one 4th. This is called de Vacouler's r /r0 to the power one 4th law at the specific intensity is something that can be measured because the specific intensity does not changes, the light propagates. Well that is not precisely true in cosmology.

In cosmology, the specific intensity will change because of the red shift but that we know the red shift. You have measured the red shift of the galaxy so you can predict what the specific intensity is when the light is emitted. So, we can measure the central specific intensity for example of a galaxy that is not difficult to do. Another quantity that you can measure is the central velocity dispersion sigma v. What do we mean by velocity dispersion?

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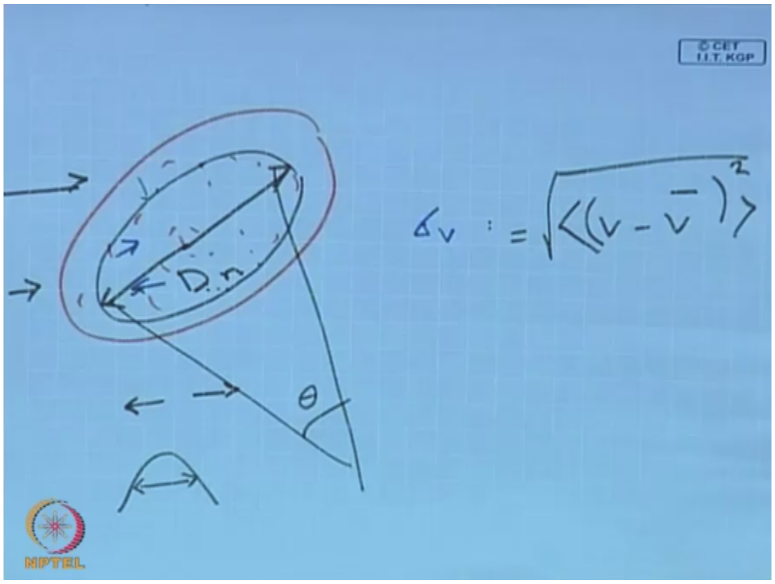

So, this galaxy is an ellipsoid like this. Let us say and it is a collection is stars. In an elliptical galaxy the stars are moving randomly in different directions like a gas at some temperature and this random motion that prevents the galaxy from collapsing. Otherwise the galaxy would have collapsed under its own weight. So, there are random motions which prevent it from collapsing. And the sigma v, what is sigma v?

So, we are observing from somewhere over here large distance away and we can measure the line of sight component of the velocity. So, we will be able to measure only the component of the velocity along our line of sight due to adopt from Doppler shifts. So, if you can look at the spectrum of this galaxy in some line then the line will be some broad thing like this where the width is determine by the velocity dispersion.

You will not be able to make out individual stars typically. But you will be able to make out you will get a broad line due to the random motions of the galaxies. The width of the line will reflect the random motion of the galaxies because each spectral line will not be at the same place it will be shifted due to Doppler shifts. And the RMS variants, standard deviations of these velocities can be estimated from the line width and that is what is sigma v the velocity dispersion.

So, it is v-v mean, the mean value of this square. That is how you calculate the mean square RMS value, deviation from mean value. So, you can measure the central velocity dispersion. So, these things can be measured I0 and sigma v.

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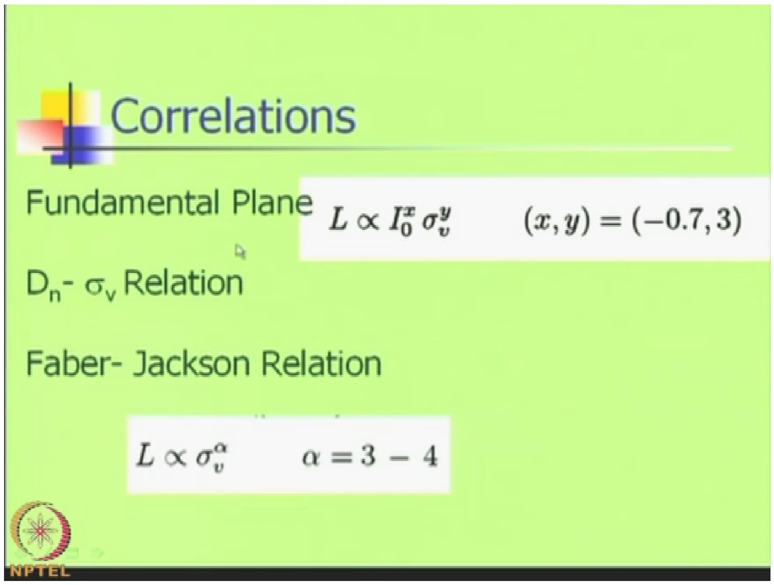

And it has been observed that there are some relations which are referred to as the fundamental plain. So, there is a relation which is referred to as the fundamental plain. So, what is this fundamental plain? These are essentially co-relations between different galaxy properties. Here the 3 properties that we are looking at are the total luminosity of the galaxy. The specific intensity at the center, surface brightness at the center of the galaxy and sigma v and it is found purely empirically.

That the luminosity is proportional to the central specific intensity to the power x and sigma v to the power y. So, this is a plain in a 3 dimensional space which has a luminosity, central specific intensity and sigma v as the axis. It is called the fundamental plain basically. So, galaxies lie in a plain in this space of luminosity, central specific intensity and sigma v. So, if you can in principle if you can measure the value of this central specific intensity.

And the velocity dispersion at the center you can determine the luminosity. Because galaxies all lie on a particular plain in the space of these 3 parameters. So, this is one possible thing that you can use. Another possibility is what is called Dn sigma relation. Dn is the radius of the or radius diameter of the galaxy. And it is defined in such a way so that the average specific intensity inside this radius has a particular value.

So, you have an image of a galaxy you find the diameter inside which the mean specific intensity has a particular value. It is found that is related to the velocity dispersion. So, if you can measure the velocity dispersion of the galaxy then you can predict what the length scale corresponding to this D. You can predict Dn and from the image you can measure the angle that this subtends. So, you know this length you know the angle.

So you can determine the angular diameter distance. Well at low shift they do not differ so we can determine the distance. So, essentially if you can measure the sigma v you can predict the diameter of the galaxy. And you can measure the angle from the image then you can use this to determine the distance. Further, there is another possibility which is called the Faber Jackson relation. This relates the luminosity to the central velocity dispersion.

So, these are all empirical things, findings. Some of these can be justified there is a physical bases for some of these which depends on the –you have physical arguments for some of these relations based on the Virial theorem. Because we know that the galaxy is in equilibrium so the kinetic energy should and gravitational potential energy are related one is half of the other. From there you can find that such relation should exist and these are observationally found.

So, they allow us to determine either the luminosity or the diameter of the galaxy from measured quantities.

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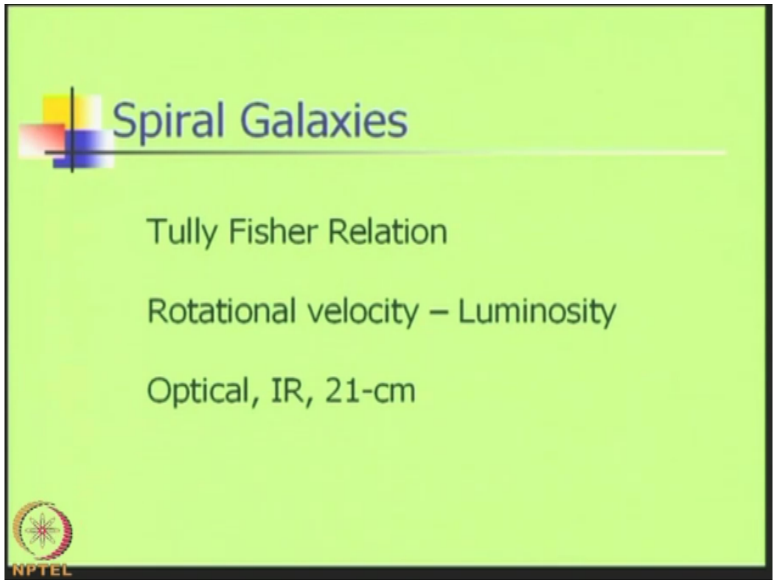

For spiral galaxies so we have also learnt that there is another type of galaxy called a spiral galaxy. In spiral galaxy we have a disk and the disk is rotating. Spiral galaxy it is found that again the rotational velocity is related to the luminosity and this is called the Tully Fisher relation. So, this can be done either in optical or in infrared or 21 centimeter neutral hydrogen which we have discussed.

So, here again if you can measure the rotational velocity which is easy from Doppler shift you can predict the luminosity of the galaxy. So, let me briefly summarize we have what we have learnt in the later part of today's talk. We have learnt about how we can use galaxies as either standard rulers or standard candles. So, let me bring today's lectures to a close over here. We shall resume our discussion from here in the next lecture.