Astrophysics & Cosmology

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Lecture - 30
Distances

Welcome. Let me remind you of what we were doing at the end of the last class lecture. At the end of the last lecture I had introduced the conformal time which is defined through the expression given here.

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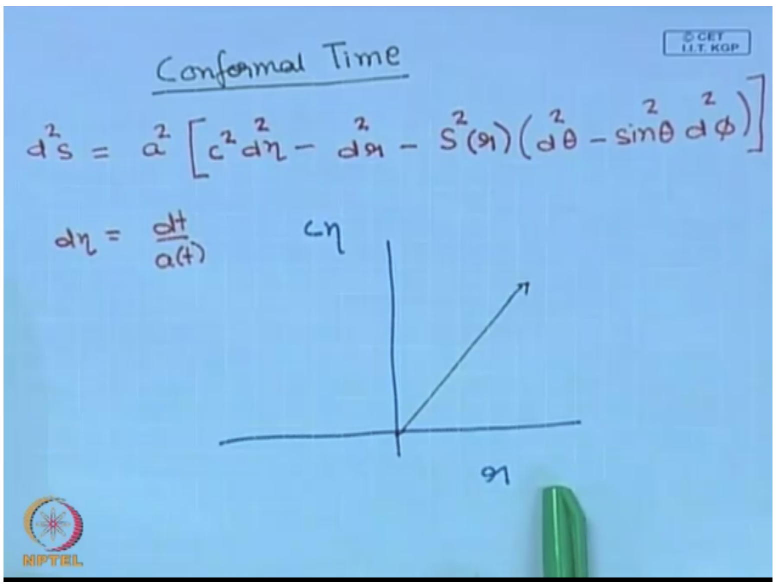

Conformal time is represented as eta and d eta is dt where t is the cosmic time divided by the scale factor. And the line element which allows us to calculate the interval between 2 events can be written in terms of the conformal time in this way. We have a factor of scale factor square outside which is a conformal factor and inside we have c square d eta square - dr square - s square r into the line element due to the angular displacement.

Now, I had also pointed out that the main advantage of conformal time is that it provides a very convenient way to look at the propagation of light in space time diagram. So, we are used to a space time diagram where we have c into the time ct over here and the displacement r over here. This is a space time diagram where we have the comoving distance r along the x-axis and we have c into the conformal time eta along the y-axis.

And the propagation of light in this space time diagram is along 45-degree lines. If I had used the usual cosmic time and the comoving coordinate. The light would not propagate along 45-degree lines because of the scale factor inside. But here the scale factor has been taken outside and we know that light propagates along null intervals so d square is 0. So, essentially d eta if light is propagating in the radial direction. cd eta will be = dr.

So, the propagating of light is along 45 degree line that is the very useful aspect of this conformal time. So, let us now use the conformal time and go back to the situation that we were discussing. The situation that we were discussing is as follows. There is an observer sitting here these are the x, y, z we are working comoving coordinates.

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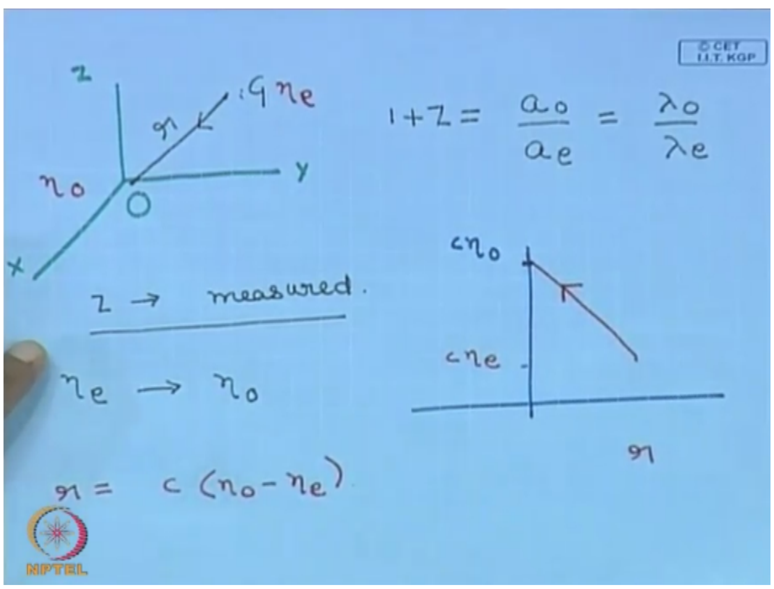

These are x, y, z comoving and there is a galaxy G some comoving distance r away and there is light being emitted by this galaxy which reaches this observer. That is the situation that we were considering. It's a very general situation and here we are considering the situation where the observer is at present but it need not be so. And we have already learned one aspect of this. We have learned that the radiation coming from this galaxy to this observer will get red shifted.

The wavelength will increase and we have calculated the red shift. The red shift we have seen is

1 + Z = the ratio of the scale factor when the light is observed to the scale factor when the light is emitted. This is the ratio of the observed wavelength to the emitted wavelength. This is something that we have already calculated. Now, an observer sitting here will not be able to measure the distance to the galaxy.

The quantity that he will measure is the red shift this is something I have already told you that the redshift is the quantity that one directly can measure easily sitting on earth. So, an observer on earth will essentially be able to measure the red shift of the galaxy that is the quantity that is going to be measured. So, the red shift in most situations is the quantity which is measured. For any cosmological astronomical sources at a cosmological distance.

The red shift is the quantity which is normally directly measured. "Professor - student conversation starts" Sir (()) (05:15) we are getting from any distant object. How can we predict, how much is the red shifted there? You cannot predict if you can identify some spectral line. This is something I have discussed if you can identify some spectral line in the spectrum in the radiation then you know where the spectral line will be emitted if the object is at rest.

So, you know lambda emitted. And now if you can observe the wavelength where you are receiving it you will know the red shift. So, you basically have to do spectroscopy. You have identified some spectral line in the radiation spectrum. So, this depends on spectroscopy. Once you can identify some spectral line you can then do spectroscopy and determine the red shift. This is the quantity that is measured.

Before we go into anything else let us first see what is the relation between the conformal time and the distance? That is for the propagation of light. What does it look like? So, we shall come to this red shift relation with the red shift later on. Let us first see what is the relation between the conformal time when the light is emitted let us say that the light is emitted when the conformal time is eta 0, eta emitted it is received at a conformal time eta observed.

It is quite straight forward that if you draw this in a space time diagram then the whole thing set of events will look like this. This is the observer let us say c eta observed this is the conformal time at the instant when the observation is done which may be present in most situations it will be present. And if you look at the propagation of light that arrives here it will have travelled along what is the called the backward light cone.

So, light if this is r the trajectory of the photon that is reaching us will look like this in this diagram. It will come from the past. So, these 2 events light being emitted from here at eta emitted, light being observed here the same photon being observed here at eta observed will in the space time diagram look like this. So, this is c eta emitted and this is the distance from which it is coming the comoving distance.

So, it is quite clear that the comoving distance r = c eta observed -eta emitted. Now, the interesting and useful thing if to relate this to the red shift. So, red shift is what you can measure. Suppose there is a galaxy which I am observing at a red shift of 1. I would like to determine what is the comoving distance? What is the value of the conformal time when it was emitted what is the value of conformal time?

Present, I know so these are things I would like to calculate. So, that is what of interest. How to determine these from the red shift? So, let me discuss this. So, the way you do this is as follows. We know that light propagates along a trajectory dr = cdeta and d eta we also know is dt/a the scale factor.

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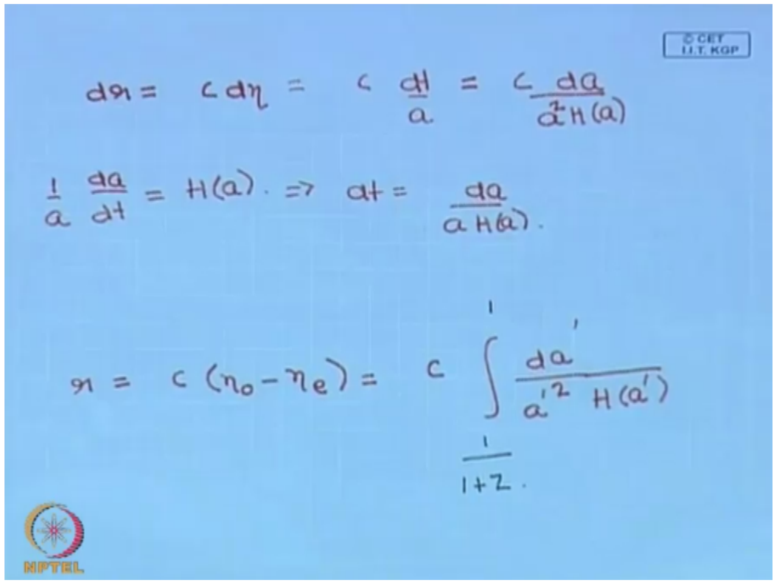

Let us look at the definition of the Hubble parameter. Hubble parameter is defined as follows da /dt, 1/a is the Hubble parameter H at any (()) (09:42) that is how it is defined. So, from this we can replace dt/da divided by a*h. So, this implies that dt = da, a times the Hubble parameter and we can put in here. So, what we have here is that this =cda/aHa, there will be a square here because there is 1/a coming from here and there is already 1/a there.

So, you see you can calculate, you can now integrate this. And this integral will give us r = ceta observed -ceta emitted =c. Let me keep the factor of c and I have to limit integrate this over the limits of the scale factor. The range being the integral being ba prime/a prime square H a prime and the limits are the value of the scale factor when the light was emitted and the value of the scale factor when the light is observed.

Now, we will assume that the observer is at present when the scale factor is 1. So, a0 is 1. So a emitted is 1/1 + z. So, the limits of this integral are 1 and 1/1 + z. This expression is very important expression it allows us to calculate the difference in the conformal time between the light which we receive from the red shift z and light being emitted from the red shift Z and received reaching us now.

You can in general use it to calculate the difference in conformal time between any 2 events. Here we are focusing on a particular event where the light being emitted at a red shift z and reaching us now. It also gives us the comoving distance corresponding to that source in terms of z. How do you determine H of a? That we have already worked out from the dynamics of the expanding universe if we use this expression to determine H of a.

So, this is determined H of a is determined by the dynamics of the universe. So, you have to use this in this to determine the comoving distance to any galaxy observed at a red shift z. So, let us now work out one particular case which is quite simple. So, the case that we are going to work out is where our universe is matter dominated.

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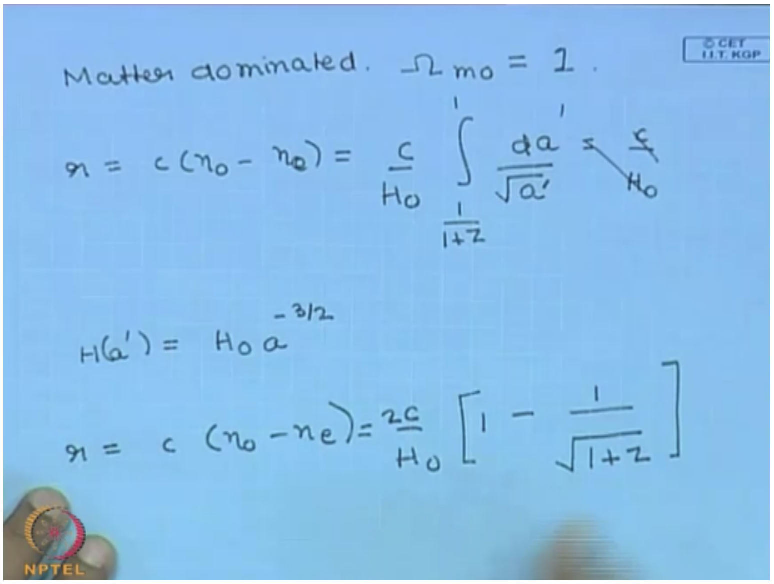

So, we will assume that omega matter 0 is 1 there is only matter in the universe nothing else dust pressureless matter like the galaxies that we have. So, we will assume that there is only pressureless matter in the Universe. There is no cosmological constant curvature whatever etcetera etcetera.

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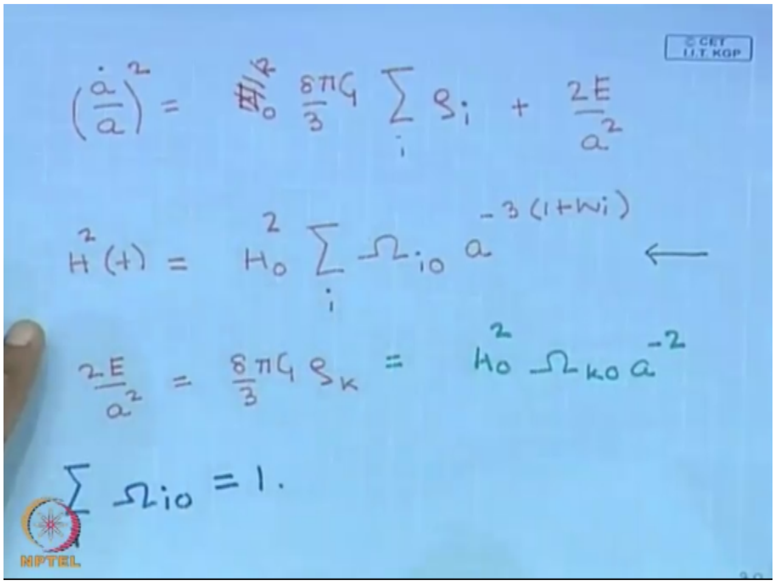

So, this is a very simple situation and it gives us considerable insight into what happens. So, let us use this so H square is H 0 omega matter is 1. So a to the power -3. So, we will put that in here. Let us put that in here. So, what we have then. Let me just write down r = c. So, we are going to put in the value of H a here and H a in this particular modal. Let me write down H a first so H a prime = H 0 omega matter is 1 omega matter 0 is 1.

And I have a to the power -3/ 2 because I have to take square root. So, a prime to the power -3/2 that is what we have. This I have to plug in to the expression over here. So, if I have a to the power -3/2 I have a square here. So, I have a to the power half. So, the integral that I have to do is essentially that this = c/ H0. I have taken the H 0 outside. Sorry, I could have done it here itself C by H 0 divided by square root of a, a to the power half.

That is the integral that we have to do. This will get multiplied by a square so I will be left with square root of a prime. And we know what this integral is so this integral is going to be there is no need for this we can straight away put in the value. So, what we have is r = c eta 0 - eta emitted. This = C/ H 0. The integral over here is 2 square root of a, 2 root a prime and the upper limit is 1.

So, I can put the 2 here the upper limit is 1 so I have 1 - the lower limit the lower limit of a is 1/1+ z so 1 - square root of 1 + z. This is the relation which tells me the interval of conformal time how that is related to the red shift of the source. It also tells me the comoving distance to the source in terms of the red shift. In a modal where omega matter =1 the only matter documented. Now, the first thing that you should be wondering we know that in the nearby Universe Hubble's law holds.

The Hubble's law is that the distance is proportional to red shift. So, let us see if this is consistent with that. Is the distance in the nearby Universe proportional to the red shift? Hubble worked in a regime where z is extremely small. This expression is valid everywhere for all values of z. So, let us just look at this in the lower red shift regime. In the low red shift regime we can do a tailor expansion of this.

So, if we consider z > 1. 1 + z to the power -1/2 is approximately =1 – 1/2z. So, what we have here is that r =2 c / H 0, 1 -1/2 z so we are led to the relation that r = C by H 0*z which is Hubble law.

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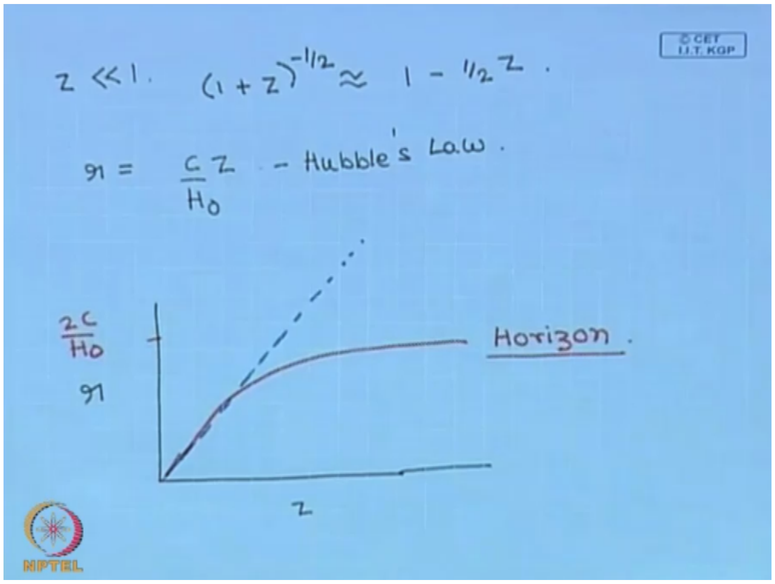

So, we see that in the low red shift limit it reproduces what we started off with. We started off the whole business from the Hubble's law which was observed at low red shifts. But if you go to higher red shifts you see the distance the relation between distances this is comoving distance remember and the red shifts changes. So, let us plot this now and try to see what happens at higher red shifts.

So, initially the graph between the comoving distance r and the red shift z is linear as predicted by Hubble's law. But as you approach red shift unity so this is when the z is much < 1. But now you see as you approach unit T so let us look large as you approach unit T there will be deviations from Hubble's law. The tailor series will no longer be valid and for very large z let us see what happens at very large z.

At very large z, the distance will reach a constant value which is 2c/H0 that is the furthest distance comoving distance from which a red shift of infinity corresponds to 2c/H0. So, this will look something like this. So, it will reach a constant value which is 2c/H0. So, it will look something like this. It will saturate you cannot reach have a value larger than that comoving distance.

This you see is very interesting, as you go higher and higher red shifts according to Hubble's law the distance should keep on increasingly linearly. But here you have the fact that the comoving distance the furthest comoving distance from which you can receive any radiation which corresponds to red shift infinity is a fixed value 2c/H0. What is the interpretation of this? Well, the interpretation for this to understand the interpretation of this let us go back to our space time diagram.

So, this is the observer. Observer is receiving light from this source and the source in this space time diagram is here. As the source is moved further and further back away you are going further and further back in time. But you see there is the big bang which occurred. Let us say the big bang we choose it so that it occurs at eta = 0. This is the big bang eta = 0. So, there can be no source which is big bang.

This line that is the event that is the big bang so at this value of eta which is 0 we had the big bang so eta emitted = 0 is the big bang that is the origin. So, the furthest distance that you can have a source light started travelling from which light could reach us is this. If you can receive a radiation from this distance that is the big bang the scale factor is 0. If the scale factor here is 0 you see that the red shift a emitted is 0, the red shift if infinite.

So, red shift infinity essentially corresponds to the big bang and this is the furthest distance. The distance that photon emitted at the big bang which can travel till now that is the distance 2c/H0 this distance is called the horizon. That is my horizon now. My horizon now in this cosmological modal is 2c/H 0. This is the furthest comoving distance that I can see I am the observer over here.

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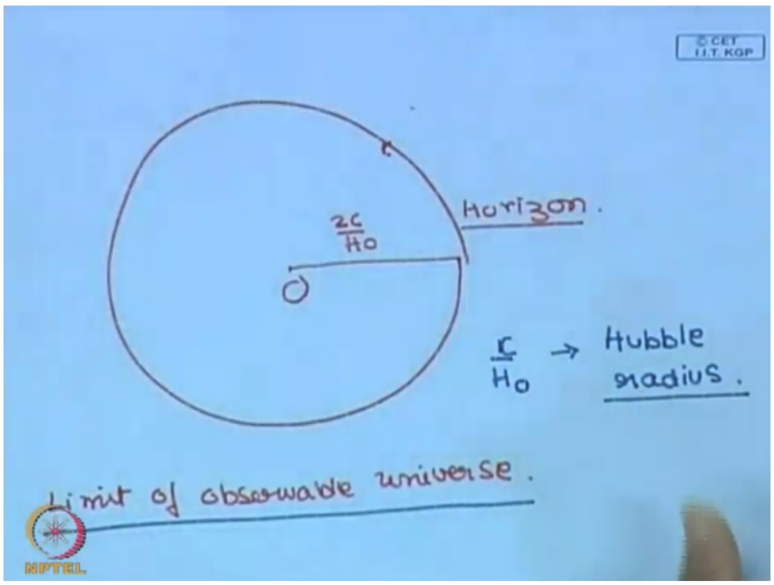

Photons emitted here to reach me now photons should have been emitted at the big bang. If photon has to come from here to me it should have been emitted before the big bang which is not possible. So, given the fact that the Universe has been in existence for a finite time there is only a finite distance which I can see up to and that distance is the horizon. That corresponds to the big bang. I cannot see anything in principle or before earlier to that.

Because the Universe was not existence. The photons started travelling a finite time ago a finite time period ago and the comoving distance it could cover in this cosmological model is 2c/H0. "Professor - student conversation starts" There is no boundary but this is my observable boundary. This is the boundary of my observable Universe. Not that the Universe does not exist beyond that. If I rate one day the distance I would be seeing is little bigger.

So, it keeps on increasing slowly. Because the age, the time that has elapsed since the big bang has increased so I would be looking a little further. "Professor - student conversation ends".So, this is the limit of the observable universe. I cannot at present see any part of the Universe bigger than this. In the past, that would have been smaller. Now, you see 1/H0 defines the time scale called the Hubble time. So, C/H0 is called the Hubble length scale, Hubble radius.

This gives an order of magnitude estimate of the distance from which furthest distance which you can see and the exact value in this particular cosmological modal that we have been considering, we have been considering a particular cosmological modal if you remember. And the particular cosmological modal that we have been considering is omega matter = 1. In that particular model, the horizon is at a distance 2c/H0 that is the furthest distance you can see.

So, that is the furthest region which can be causally connected to us through cause and effect. Anything beyond that cannot be is not causally connected to us. It cannot be the cause for any effect now. Because no signal can propagate faster than the speed of light. Now, you should realize that the statements that we have made are all depend on the cosmological modal. Suppose, I was in a different cosmological modal by that we mean if the density parameter had different set of values.

Then my Hubble H square a would be a different function and these results would be changed. So, this entire relation, exercise that we have worked out is this relation is specific to a matter dominated universe. Till it get modified the relation between comoving distance and red shift. The difference in conformal time and red shift everything will get modified if I change the constituent of the Universe.

Let me move on our interest here in distances and we have seen that in a cosmological modal there is a furthest distance that we can see and the way that distance from which the radiation is coming the comoving distance changes with red shift has to be calculated using the formula which we have just derived and we have worked out a particular example where omega matter is 1. But the comoving distance is not a distance which is of physical interest.

After all what does the comoving distance imply? It has for most purposes it has no physical direct physical implication. The actual distance at any instance of time that is called the proper distance.

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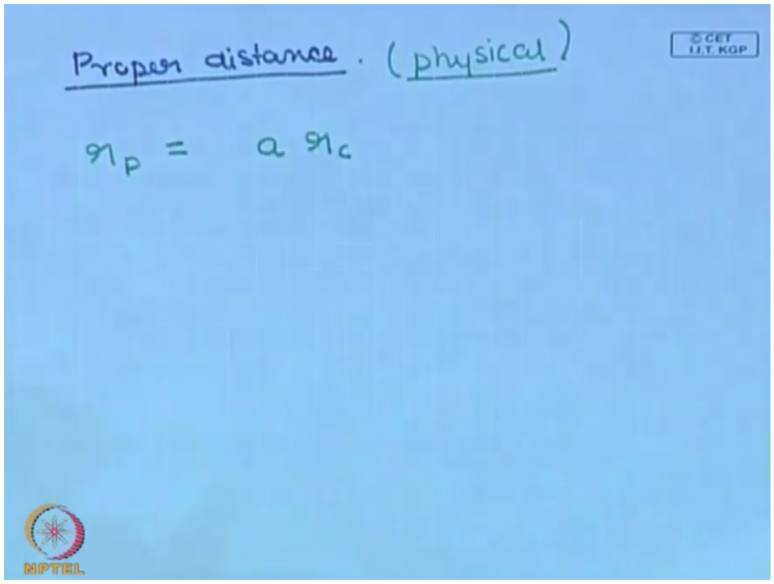

So, this is the actual physical distance. This is of quantity which is physical if you add a meter stick you would be actually measuring this. The proper distance and the proper distance r proper = the scale factor into the r comoving. So, if I call this r comoving then the proper distance is the scale factor into the comoving distance and the scale factor is the function of time. So, given a fixed comoving distance the physical proper distance keeps on changing with time.

So, obviously you cannot use light the propagation of light to measure the comoving distance. So if I told you that the comoving distance to the particular observer here is whatever 100 mega power 6. How would you determine the physical distance? You cannot send light from here to here because the Universe expands as the light propagates. So, one in hypothetically what you could do you could have observers all along.

Who could simultaneously measure the distances using meter rules, meter sticks and then finally you would have to add them up later on. But that is all just hypothetical, it's a thought exercise you cannot really carry it out. So, let us now move on to distances which are of physical importance, measurable quantities. So, the first such thing is the angular diameter distance. So, let me introduce this the angular diameter distance.

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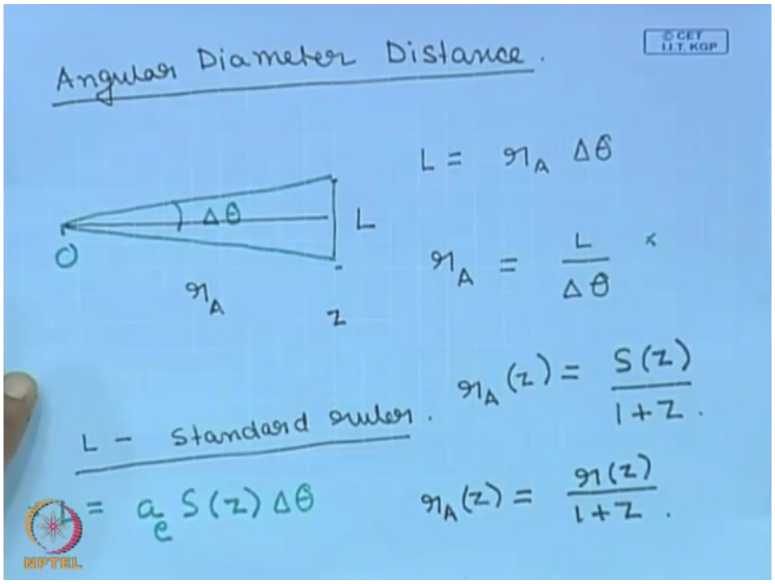

This is the distance. This is one of the distances which come in important in observations. How is the angular diameter distance defined? The angular diameter distance is defined as follows. Suppose, I have some object whose actual physical length is known. Let us say it is L, its proper length is known this is an object. Such an object is called a standard ruler we have reasons to believe that its length is unknown.

Obviously for example let us take a galaxy suppose we have reason to believe that galaxies have physical size 10 kilo parsec. I am just giving you an example which is not realistic but just to make the idea clear. Suppose, we have reason to believe that all galaxies have physical size 10 kilo per se then I know that if I see a galaxy somewhere that its size is 10 kilo parsec. Now, we can define the angular distance as follows.

I am an observer sitting here if I see the galaxy it will then subtends and angle delta theta on the sky. These angles are extremely small so we know that the length L will be rA, the angular diameter distance into delta theta. So, if you know the size of an object of the galaxy the physical size the proper size and from earth if you can measure the angle it subtends on the sky you can then determine the angular diameter distance to that object L/delta theta.

So, if you have some prior knowledge of what is the size of that object suppose the object, what is called the standard ruler we know its size is fixed irrespective of where you place it and if you can measure the angle from the earth from here then you can determine the distance to that object and that distance is called the angular diameter distance. Now, here you see it is essential to distinguish between different distances.

On earth in the standard Eulerian geometry which we deal with these different definitions of distance all coincide. But in cosmology they will not coincide as you shall see. So, you have to be careful which distance you are talking about. So, this is one definition of distance it is called the angular diameter distance. Now, let us ask the question suppose I see a galaxy at a red shift some standard ruler at the red shift z so I know its proper size L and I can measure the angle it subtends in the sky.

So, I know L, I know delta theta let us calculate the angular diameter distance as a function of Z what is the predicted angular diameter distance as a function of z. So, to do that let us go back to our line element this is our line element. So, you see the object is at comoving distance r let us assume that the object is a comoving distance r and it subtends an angle delta theta. So, the length corresponding to that you can determine from here.

The length corresponding to that L what is the length it is a*s*delta theta. You see I have not yet talked about let me just digress a little bit I have not talked about the difference between r and S of r. There are 2 different functions over here. Here you have r, here you have S of r. In a specially flat geometry r and S of r are same. Where as in a curve space this becomes sin if it is positively curved it become sin hyperbolic if it is negatively curve.

What is the significance of this? Let me briefly just mention what is the significance of this. Suppose I have a sphere of radius r forget about the scale factor. I have a sphere of radius r then the distance from the origin is r comoving distance from the origin is r. So, this is my observer.

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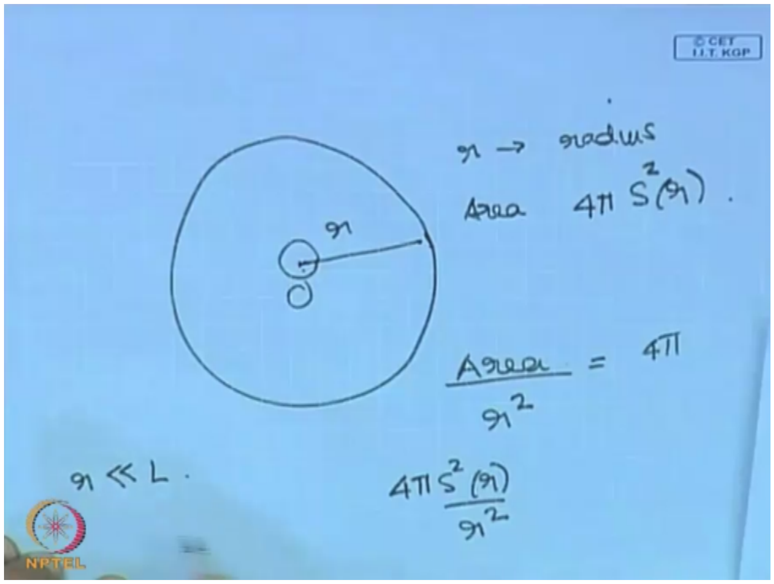

This is a sphere of comoving radius r. So, this is the distance from surface of the sphere from the observer, comoving distance is r. Now, let us ask the question what is the comoving surface area? To calculate the surface area you will not vary r what you will integrate over is this angular part. You will do the solid angle integral that will give you 4 pi. A solid angle part has not been changed so the area of that sphere is going to be S square*4 pi.

So, you see in this curve geometry the radius is r of a sphere. But the area is not 4 pi/r square it is 4 pi S square r. This is the main difference because of the curve space. The ratio of the area of a sphere to the square of the radius. This ratio in flat space is a fixed number which is 4 pi where as in curve space this ratio becomes S square r/r square which is not 4 pi sorry this is a 4 pi times this.

Which is not the same as 4 pi this ratio becomes dependent on r that is the crucial difference. For small spheres if you make the radius very small how small should it be? If r < radius of curvature. So, let me show you the thing that we had written yesterday and everything should be much more clearer here.

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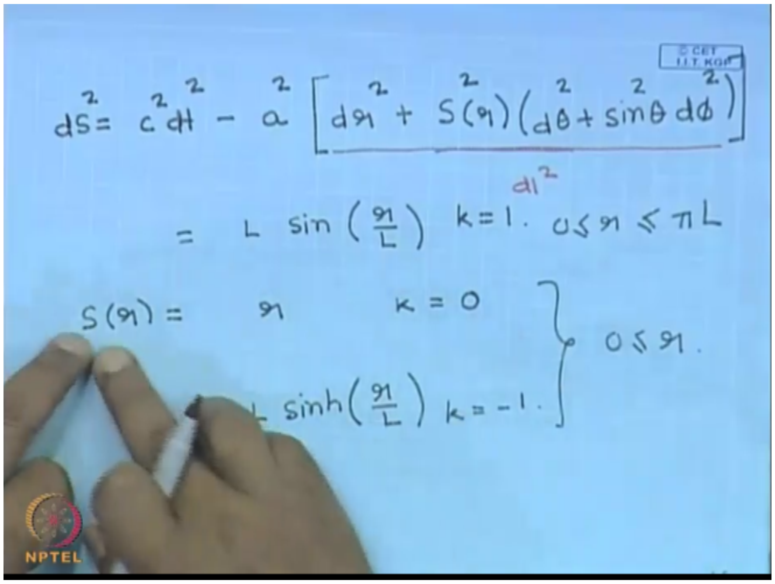

So, the function S of r that occurs over here is r if it is specially flat. This same function S or r that occurs over here is L sin r/L if it is passively curved. For small r let us take the limit of small r for small r sin r/L is r/L. It goes over to r. So, you see on length scale much smaller than the curvature the curvature has no effect. It is just like the flat space. If I consider a sphere which is very small this ratio tends to 4 pi it has no effect, the curvature has no effect.

It is only on length scale comparable to the curvature that the curvature becomes important which we know from our familiar experience on earth if I look at a small part of the earth. I will not know that the earth is a sphere only if I travel large distances does the fact that the earth is a sphere becomes important, same thing here. The ratio of the area of the sphere to the radius square is different from 4 pi only if I go to length scales comparable to the curvature.

So, with digression I have told you about the significance of this S of r. Let us now go back to the problem that we were discussing. So, for the problem that we were discussing or I can draw the picture again. So, there is an object here whose proper length is L and it subtends an angle delta theta. So, we know we want to calculate the angular diameter distance which is defined as L by delta theta.

Now, L the physical distance the proper distance corresponding to an angular separation delta theta is a S delta theta. So, this L is a S of r which I can write as S of z because I can calculate r if

I know z and if I know r I can also calculate s. So, I can write this S of z*delta theta and the value of the scale factor at the instant when the light was emitted because this we have to evaluate –we know that delta theta*S*a = the physical length of that object at the time when the light was emitted.

So, this should be a emitted so all you have to do is that you have to compare this with this and you see that the angular diameter distance = S z divided into a emitted and a emitted we know is 1 + z. So, this is the angular diameter distance how that changes with red shift. Let us now look at the cosmological modal that we are discussing. We are discussing a specially flat cosmological modal.

And in this specially flat cosmological modal r is related to red shift in this way. And in the specially flat cosmological modal S of r is exactly = r. So, let us in this modal look at the behavior of the angular diameter distance as a function of red shift. So, we would like to look at the behavior of the angular diameter distance as a function of red shift. So, the angular diameter distance as a function of red shift in this particular modal essentially follows rAz = rz / 1+z.

That the way the angular diameter distance behaves. So, let me draw a picture of this. Let me draw a graph for this.

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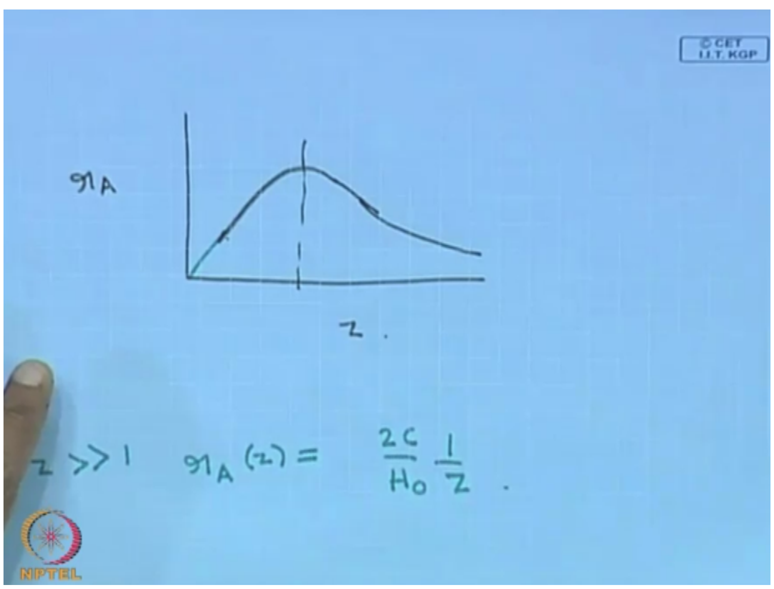

Again you see for small z that angular diameter distance is the same as the comoving distance which we have seen follows the Hubble's law. So, for small z it is going to be a linear behavior which is exactly the same as Hubble's law. Now, let us look at what happens at large z. At large values of z this term vanishes from r but the angular diameter distance k is r/ 1+z at large z, 1 + z is approximately z. So, for very large z the angular diameter distance = 2c/H0 1/z.

So, you see initially it increases with the slop which is z by H 0 and for large z it falls with the slope 2c/H0*1/z. So, it falls off here as 1/z and in between it will have a peak value. So it will look something like this. The angular diameter distance as a function of H. Whereas the comoving distance to any object keeps on increasing with red shift and then it becomes constant. You see the angular diameter distance here has a very strange behavior, why is this change?

Let us try to appreciate that. It is strange because of the following reason. From our familiar experience we know if there is an object of a fixed size and if you move further and further away from the object it will subtend a progressively smaller angle. But in cosmology what happens is that if you have an object of a fixed size if you place it at progressively further and further distance from you the angle will decrease still a certain red shift.

After that the angle will increase again because the angle which the object subtends is L, r*delta theta. r will initially increase see L is fixed so angle is basically L/r. r is initially increasing so delta theta is increasing. But after a certain red shift the r will again decrease. And finally if I have an object at the big bang it will subtend and what angle will an object at the big band subtend?

It will subtend 4 pi it will never go to infinity. It will subtend it basically fill the whole sky. Well, why is this happening? It is happening because the Universe is actually, if I go to past the Universe is smaller. So, if you look at the big bang all the light that reaches you from the big bang were emitted from the same point. So, 0 angle there will subtend fill whole sky. So, as he approached the big bang.

The physical distance between 2 point from where the light the was emitted will actually keep on decreasing. So, if you look at the fixed solid angle the corresponding physical distance keeps on decreasing if you look at a fixed angle. It is a combination of 2 things basically. One is that the distance increases with red shift other is the fact that the Universe is itself, the scale factor itself is a decreasing function of red shift.

Combination of 2 factors which give rise to this behavior the angular diameter distance. So, let me remind you again the angular diameter distance is important if you can measure the angle and the red shift and you know the size of the object. Let me now introduce another kind of distance which is of relevance in cosmology this is called the luminosity distance.

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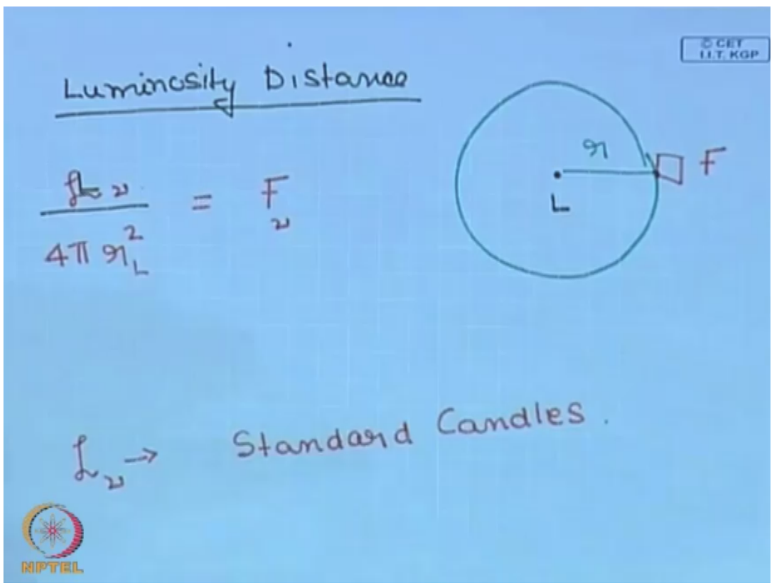

So, what is the luminosity distance? The luminosity distance is defined as follows suppose I have a 100 watt bulb fixed luminosity and I observe it from a distance. So, this is my source which has luminosity L and I observe it from a distance r from where I can measure the flux so sitting here I cannot measure obviously the luminosity of the source. What I can measure is the flux F which is the amount of energy. So, luminosity is the energy that is emitted per unit second.

This is the energy that I received per unit area per unit time and we know that these 2 are related as 4 pi r square this = flux. So, the r that occurs over here is the luminosity distance it is called rL. So, essentially what we do here is if you know luminosity so there are certain objects let us put a different sign here otherwise I will be confused with the length scale that we have introduced.

There are certain objects whose luminosity is known these are called standard candles like a 100 watt bulb. If I believe if I have some prior information that my source is a 100 watt bulb and if I can measure the flux I can get the distance through the source. In cosmology, there are astronomical sources for which we believe that the luminosity is known. We have reason to believe that we know the luminosity.

So, if you can measure the flux from such a source you know the luminosity you can measure the distance. Like we have already talked about one such thing that is the cepheid variables If you can measure the period you know the luminosity. So cepheid variables act like standard candles. And other things which I should mention now is that here we shall work with the luminosity in a certain specific frequency range.

So, this is the luminosity in a frequency range delta nu. I have to bring today's class to an end over here because we have run out of time. So, I shall do the calculation of this luminosity distance in the next class. Let me briefly recapitulate what we have learned in today's class. In today's class, we first learned about the conformal time conformal time and we saw that it is very useful thing to look at the propagation of light.

And we learnt then how to calculate the comoving distance to a source and the conformal time when the light to calculate the conformal time when the light was emitted from the source in terms of the red shift. The measured red shift we learned how to do this. And we show that this relation we show there is a furthest distance to which you can see. So, if you have a source which has a red shift infinity that is the furthest red shift that you can have.

And there is a finite comoving distance corresponding to red shift infinity. This we learnt I told you is the horizon. So that is the furthest distance of photon emitted at the big bang can propagate. Next I told you about the proper distance which is the actual physical distance between 2 objects. Unfortunately, we cannot measure it directly. Finally, we moved on to things which are of physical significance in measurements.

The first thing that we talked about was the angular diameter distance and we show that the angular diameter distance is the relation between the angular size of the object and the proper size of the object. So, it relates the angular diameter measured angle subtended by that object to the proper size of that object. And then finally we started discussing the luminosity distance which relates the luminosity to the flux.

So, let me end todays lecture here we shall continue with the luminosity distance in the next class.