Astrophysics & Cosmology

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Lecture - 29
The Cosmological Space – Time

Welcome, let me remind about you that in the last class we had embarked upon trying to understand the nature of the space time in cosmology.

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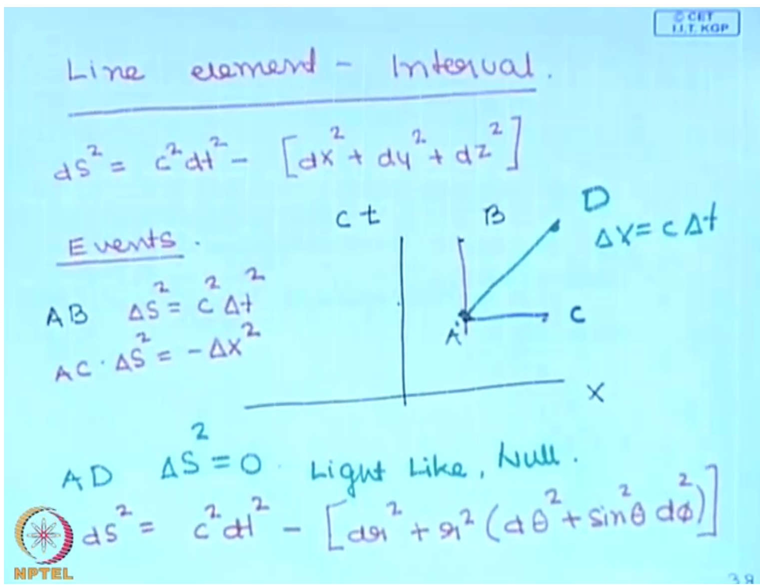

And I had started off by first briefly recapitulating the nature of space-time without the effect of gravity and the space-time properties are can be characterized by this quantity dS square which we refer to as the line element or the interval and this formula tells us how to calculate the interval between 2 events, events can be represented in the space time diagram like this, so each event has a corresponding position and time when it occurred.

So you can represent events on a space-time diagram and if A and B are 2 events you can calculate the interval, if these 2 events correspond to the same point, same co-ordinate at different time, so imagine a person sitting at one place then you will get positive value for the interval and these 2 events are said to be time like, what was very important I had pointed out is if you look at the propagation of light, light will propagate along a curve.

So if this is the curve which along which the light is propagating we are in considering only the x axis, let us assume that the y and z direction remain fixed, then x is delta x=c delta t for light, so delta x=c delta t and if you put that in here and calculate the interval you will find that the interval turns out to be 0, so for 2 events which are along the trajectory of a photon the interval is 0.

The expression for the interval or the line element can also be written like this where we have written this spatial distance separation in spherical polar coordinates, now in Einstein's theory of relativity the gravitational field manifest itself as the curvature of space-time.

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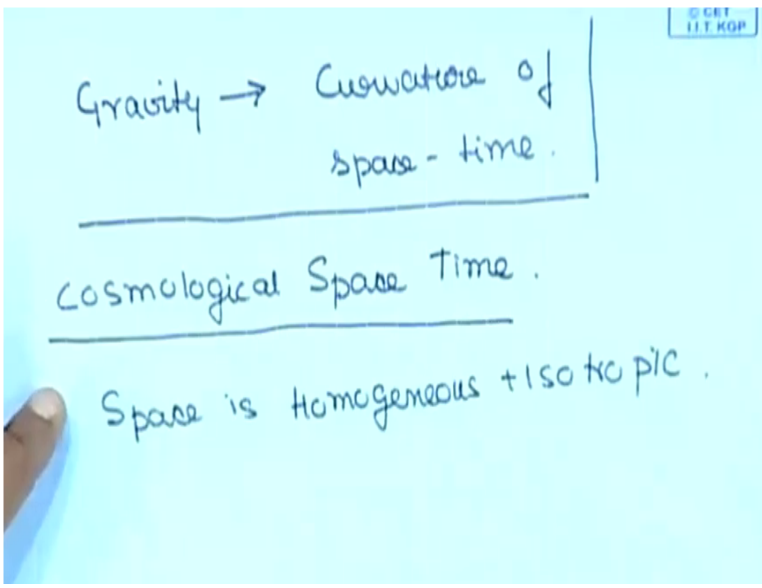

And here in cosmology, the cosmological space-time has the property that it has to be space rather the cosmological space time has the property that the space has to be homogeneous and isotropic.

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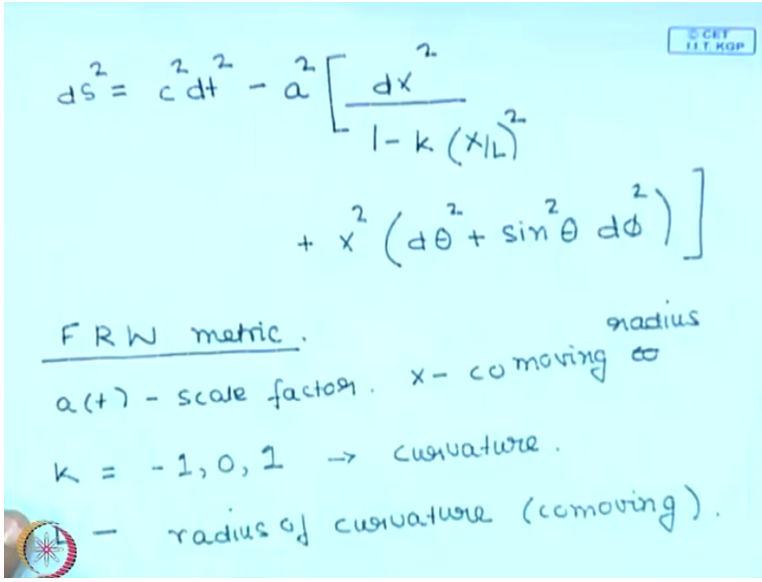

So the line element or the metric corresponding the metric allows us to calculate distance between points the line element, the cosmological line element where this space is homogeneous and isotropic has been written over here this is called the Friedman, Robertson Walker metric. It was independently written down by Friedman, Robertson and Walker and possibly by other scientist also at the same time.

And I was trying to explain to you what the various terms in this mean, now time here is the cosmic time, c is the speed of light, a is the scale factor which is already familiar to us it is the it quantifies the expansion of the universe, the x is the comoving radius we are working in spherical polar coordinates, so x is the comoving radius we are working in comoving coordinates because you can see that there is the scale factor over here.

So x is the comoving radius and you have this extra factor if this factor K were 0, if the factor K were 0 then this line element would be exactly the same as this line element except for a scale factor squared over here, but we have this extra term K*x square/L square also appearing and K this presence of this non-0 K indicates that there is spatial curvature my spaces is curved, my space time is going to be curved because of this scale factor okay.

But the presence of this K indicates that my space is curved and L is the radius of the curvature in comoving, so this is the comoving radius of curvature of that space.

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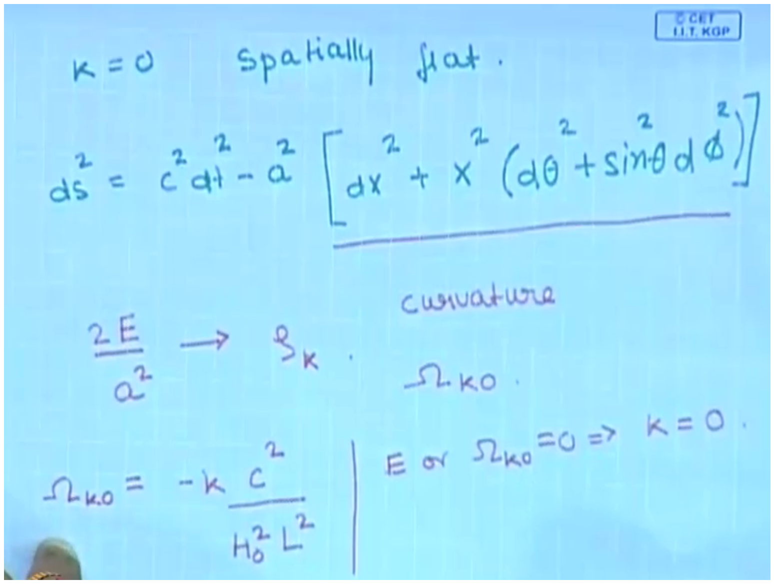

So K=0, if I have K=0 this is spatially flat and here the line element is dS square is=c square dt square-a square [dx square+x square (d theta square+sin square theta d phi square)], so this is how you calculate the length in the usual flat curved space polar coordinates and the lengths these are comoving lengths get multiplied by a square, lengths square gets multiplied by a square because universe is expanding and the line element is just given by this.

So I would think that this should be quiet comprehensible to all of you, comprehendible to all of you, you should be able to understand this quite easily okay, now the question is what happens when there is a K and L which are non 0, L there is no the radius of curvature, there is no radius of curvature infinite it is infinite in this when K is 0 okay.

Now I have told you that we had introduced when we solved the Einstein, when we solve the equation for the expansion of universe we had encountered a constant of integration 2E and this appeared in my equation for the Hubble parameter in this combination if you remember and this we had replaced we had parametrized this by a fictitious density omega K which whose contribution for the a square as universe expands.

And the contribution from this fictitious thing which I have told you corresponds to curvature was quantified using omega K0, so if you now take you see in Einstein's theory of gravity you do not use the laws of gravity or modified they are given by Einstein equation and if you write down these equations you have something some relations between the metric, so they relate the curvature of space time to the matter which produces gravity.

And they give you relations between the scale factor and K and L so these things are related to matter density and these equations turn out to be exactly the same equation that we derived from Newtonian considerations, with the modification that rho gets replaced by rho+3P/c square okay, so they turn out to be exactly the same, so if you now workout Einstein's equation in those equations you will have K and L appearing.

If you relate those if you identify that equation with the equation that we have derived you will find that you can relate omega curvature or 2E/a square to this K and L which I have introduced here it follow some Einstein equation, and the identification is let me just write it down I will not work it out here, the identification is that omega curvature 0=-K c square/H0 square*L square okay.

Let me remind you again this is something that we have already encountered we have introduced to this to essentially parameterized contribution from this constant of integration, this is the value of K and L are related to that constant of integration which we had encountered that relation comes if you look at the Einstein equation, I will not do the derivation here let me just place it before you that relation comes out to be this okay.

So you are led to this relation if you really work through the Einstein equation, so it follows that if the constant of integration is 0, then so if E or omega K0=0 it implies that K=0, the space is specially flat, this part of the line element refers to the space it is spatially flat, it the usual flat space okay, now the question is what happens when this constant of integration is not 0 what is the curvature? What is the value of K?

So it is quite obvious that if omega curvature 0 if the constant of integration is positive these are the models if I have only matter and this constant of integration these are the models that will keep on expanding forever, this is the critical model.

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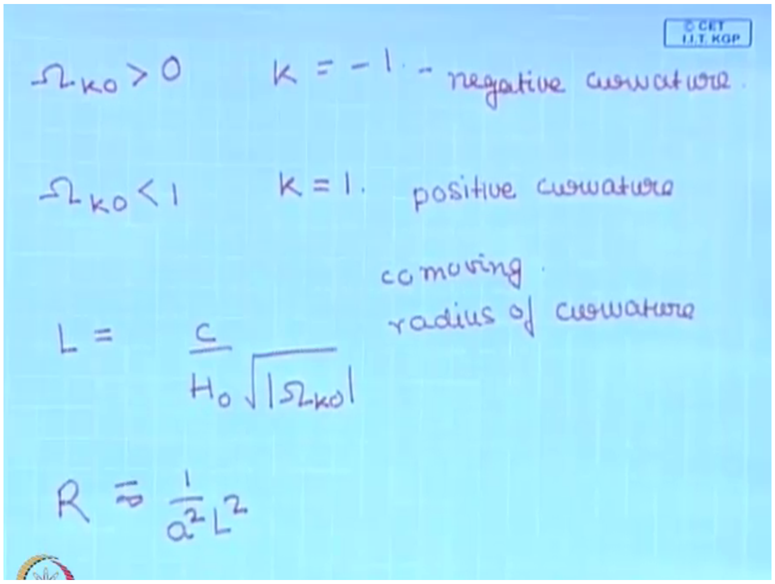

If I have omega curvature 0 > 0 then K=-1, K is -1 and if omega curvature is <0 then k is 1 and this is called, so the previous one was specially flat this K=0 is specially flat, this has the positive this is space of positive constant positive curvature and this is the space of constant negative curvature okay, and the radius of curvature L is, this is the radius of curvature comoving radius of curvature and the curvature is 1/the radius of curvature squared.

So if you want the physical radius of curvature then it is a*L and the curvature which is denoted by R is the order of is actually=1/a square L square the curvature of space not of space time or space okay, so this is a brief gist of the properties of space time, let me know explain to you try to explain to you what we mean by a space of constant let us say positive curvature okay so let us look at this case a little closer take a closer look at this case where K=+1.

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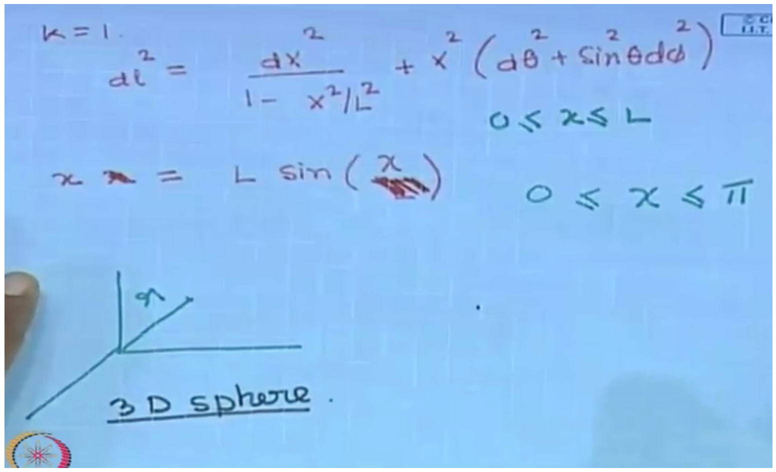

So the situation where K is =+1 and we shall only deal with the spatial part of this which allows us to calculate the comoving distance, so let us try to interpret what this space looks like, so the line the distance between the 2 points in that space is given by dl square=dx square 1-x square/L square+x square (d theta square+sin square theta d phi square), let us try to get the picture of what this space looks like, what is this space look like?

For this it is convenient to introduce new coordinates x=L sin sorry not x, chi or sorry x=sin chi/L so this is chi/L, x=L sin (chi/L) before we embark up on this let me point out that the range of x you see this space the radius x is the distance from the origin, x is the radius and in the usual flat space the radius can go from 0 to infinity there is no restriction, you see in the usual flats space look at this there is no restriction on the radius it can go from 0 to infinity.

But in the space there is a limit the value of x cannot exceed L so x=the range x has to vary in the range beyond which this thing will become negative obviously distance is cannot be negative, this allows us to calculate the distances between different points and the square of the distance that obviously cannot be negative, so L has to be in the range 0 to L, x has to be in the range 0 to L or equivalently chi has to be in the range 0 to pi.

The variable chi that we had been introduced has to be in the range 0 to pi okay, x is the distance from the origin and x is restricted 0 to L, so you cannot go distance further than L away from the origin it is a different kind of space all together from the usual space that we are normally used to.

“Professor-student conversation starts”. Yes, is there a question? Sir why cannot x lead from – L to L? Well x is the distance right it is a radial distance how can the radial distance be-, so the x is the radial distance from the origin where in the polar coordinates system, so x goes from 0 to L not from –L to L okay fine. “Professor-student conversation ends”.

So let us now replace the variable x with chi, so to do that all that you have to do is you have to replace it here and this becomes sin square sorry let me remove this chi/L here, let us make chi is dimensionless okay sorry otherwise I have to put extra factor of pi L, so X=L sin chi and the range of chi is 0 to pi okay, so with this, this term it becomes d chi square because the numerator becomes L square cos square chi d chi square chi the denominator also cos square.

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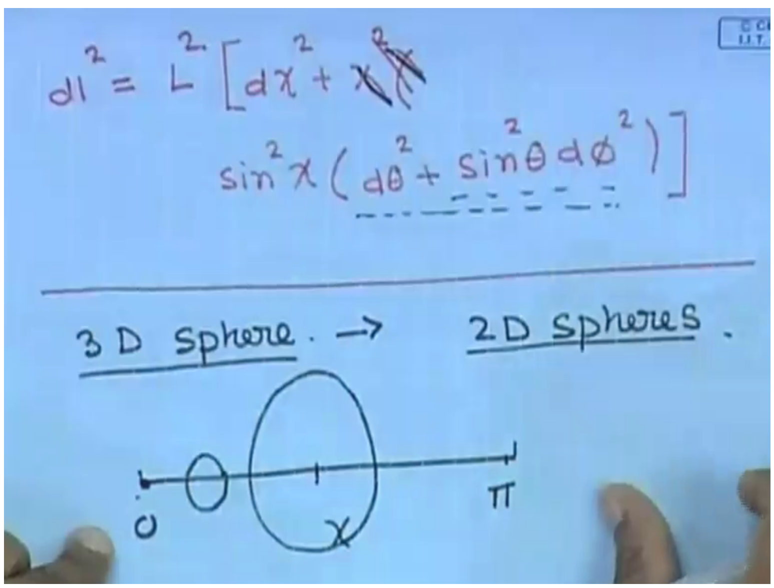

So the line element over here if I write it in terms of this variable chi, it takes on the form dl square=L square d chi square+chi square sorry it will not be +, sin square chi d theta square+sin square theta d phi square, this is how we calculate length all that I have done is I just changed variables and gone from x to chi where chi is sin, well x is L*sin chi.

“Professor-student conversation starts”. Sir where do you get this x=L*sin chi? I am making a substitution that is all, I am given this formula for calculating the distance between points every I am replacing x with a new variable chi, I can always change my coordinates right so that is all I am doing for my convenience, the convenience will be cleared just now. "Professor-student conversation ends".

So you see the way you calculate the distance in this co-ordinate system, it is a d chi square sin square chi d theta square+sin square theta d phi square okay, so this is like a spherical polar co-ordinate system except that you have this extra coordinate chi appearing okay, so this is the thing that we would try to interpret, the interpretation of this is quite easy, to interpret this let us fix the value of chi, some let us look at some fixed value chi okay.

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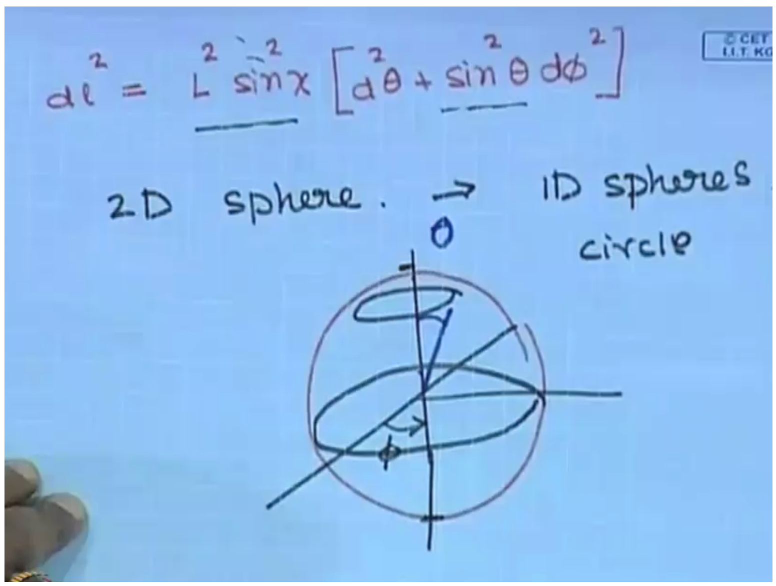

So if a look at a fixed value of chi then the if I look at a fixed value of chi let me write this down here, if I look at a fixed value of chi then the distance between 2 points the chi does not change becomes L square sin squared chi [d square theta+sin square theta d phi square], so I am looking at a fixed value of chi, let me now look at a fixed value of theta phi is the only variable.

So for a fixed value of theta phi is the only variable so now you see that you can think of this as being a it is basically a circle if phi is the only variable we have a circle right phi is the only variable so we have a circle and the distance between 2 points and that circle is d phi*L sin chi sin theta let us just ignore this part.

So this is the circle and that radius of the circle just increases with theta, it is 0 when theta=0, it is 0 when theta =phi and the radius of the circle this is the radius of the this whole thing is the radius of the circle, we will just focus only on this part, so the radius of the circle keeps on increasing with the theta, so let me draw it here, this is the this you can see is the surface in 2 dimension the 2 dimensional surface 2-D surface, it is basically 2D sphere right nothing more than that.

The radius of the sphere is L square sin square chi sorry the radius is L*sign chi and let me draw it, so this is my x, y, z and this is my sphere this is the angle theta, for a fixed theta this is my angle phi various in this direction in the x y plane, this is my circle so essentially what this tells us only this part, what it tells us is that I can think of a circle as a collection of as a sphere collection of circles.

The radius of the circle is 0 at the north pole, the radius of the circle is also 0 at the south pole where theta is 0 and phi respectively, the radius of the circle increases as it comes near the equator and it is maximum at the equator and again it become 0 at the 2 poles okay, so 2D sphere can we thought as the collection of 1D spheres, a circle is a 1D sphere okay circle basically circle, and they act up like this okay.

I hope it is clear, so 2 dimensional sphere this is a 2 dimensional sphere this surface is 2 dimensional can be thought of as a collection of 1 dimensional sphere at different values of theta as the radius of the sphere varies from 0 to the maximum value which is given here that is the maximum value into this that is the maximum value and then again it goes to 0 at the other pole okay so this is the 2D sphere.

Now look at this, this you can see is a collection again so this is the 1D sphere, 1 dimensional sphere collection of which gives me a 2 dimensional sphere this is a collection of 2 dimensional spheres along the variable chi, the radius of the sphere is L sin chi the range of chi is 0 to pi, this is chi, so for chi=0 I have a sphere of radius 0 okay it is a point, and then for chi=pi this sphere has the maximum size and again it become 0 here.

So you see this is a collection of 2D spheres, this is what is called whose radius keeps on varying as I go towards the equator, the equator is at chi=pi, pi/2 and then again it decreases as I go to chi=pi, this is what is called 3D sphere it is a 3-dimensional volume which is a sphere, it is a generalization of a 2 dimensional sphere, it is a 3 dimensional sphere okay.

And this is a collection of I can think of it as a collection of 2D spheres which I have been all stuck together okay or I can generalize this I can have 4 dimensional sphere, 5 dimensional sphere, 6 dimensional sphere, all I have to do is more variables theta sin square theta etc. okay, so the radius of this 3 dimensional sphere is L just like the radius of a 2 dimensional sphere here is L square forget about this is L right this represents the 2D sphere with this=1.

This represents the 3 dimensional sphere with radius L okay, so K=+1 is this space is essentially 3D sphere, so 3 dimensional sphere the surface of the earth for example is a 2 dimensional sphere okay, this is a 3 dimensional sphere and the way you can imagine this is that there is a space in that space 2 dimensional sphere for example on the surface of the earth if I go forward on the surface of the earth after sometime I will circum navigate the earth and come back from the back.

If I go this way, I will after sometime go around the entire earth and come back from this side. Similarly, in a 3 dimensional sphere if I go up given adequate time I will go around the entire sphere and come back from the bottom it is a 2 dimensional sphere it is a finite volume it has no boundaries though okay there are no boundaries but the volume is finite just like the surface of the earth it is a finite area but the boundary there are no boundaries okay.

So this is what we have when K=+1, it is a space of constant curvature just like 2 dimensional sphere and we have some idea now what it looks like. Similarly, K=-1 is a space of negative curvature okay, K=+1 is a space of positive curvature and K=constant positive curvature, K=-1 is a space of constant negative curvature okay.

And it is convenient to rewrite this cosmological metric sorry not this it is convenient no it is convenient to rewrite the cosmological metric in the following form, let me just write it down for you, so it is convenient to rewrite the cosmological metric in the following form.

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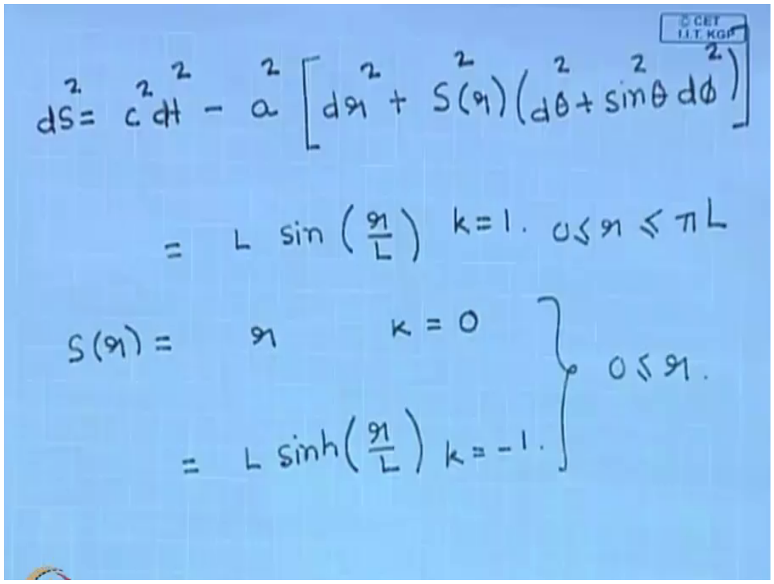

We will write as dS square=c square dt square-a square and then we have [dr square+S (r) square d theta square+sin square theta d phi square] okay, so I have written the same line element which I have written down right at the start of today's class, it is exactly the same line element where we have now introduce so this is the line element I was talking about, I have just written it in terms of new variable r, r is also a comoving distance, I written it like this.

Where remember that we had x square here we now have a function of r over here, this function of r S (r)=r if K=0 that is very familiar flat space okay, d square r+r square d theta square+sin square theta etc. okay, now this function S (r) is L sin (r/L) if K =+1 we just saw this except that we use the variable chi which was dimensionless.

Now we will use the variable r/L, so the range of r is from 0 all the way to pi*L okay and this is if K =+1 positive curvature, if K is =-1 then we will represent this a sine hyperbolic that function S becomes sine hyperbolic we have to use the exactly same thing as you did for K=+1, you will then find that the function S is of the form sine hyperbolic L*(r/L) if K=-1 and there is no restrictions for both these cases r can take any value with more than 0 okay.

This space r infinite these 2 in these 2 cases negative curvature and no curvature, but if it is positively curved then the space is finite okay, so this is the general cosmological metric that line element that we shall we using, so the cosmological space time in general can be represented like this, the form of this function S (r) depends on the value of the omega curvature 0, how much curvature basically is parameterized by that okay.

So this is the form of the cosmological space time, now let us use this to calculate certain interesting things right, after all the space time the form of a space-time is important only if it is useful, so the question is what use is this in cosmology, we have already seen that we have determined the evolution of the in principle one would take this and put it in a Einstein's equation determine the equations governing a.

But we have already done this from Newtonian considerations just extending it a little bit okay, so now we will see what other utility this thing has and the place where you cannot do without relativity is essentially if you look at a propagation of light, propagation of massive particles can be very well studied without the aid of relativity in the most situations if they move slowly, but things are moved at the speed of light you cannot study without relativity.

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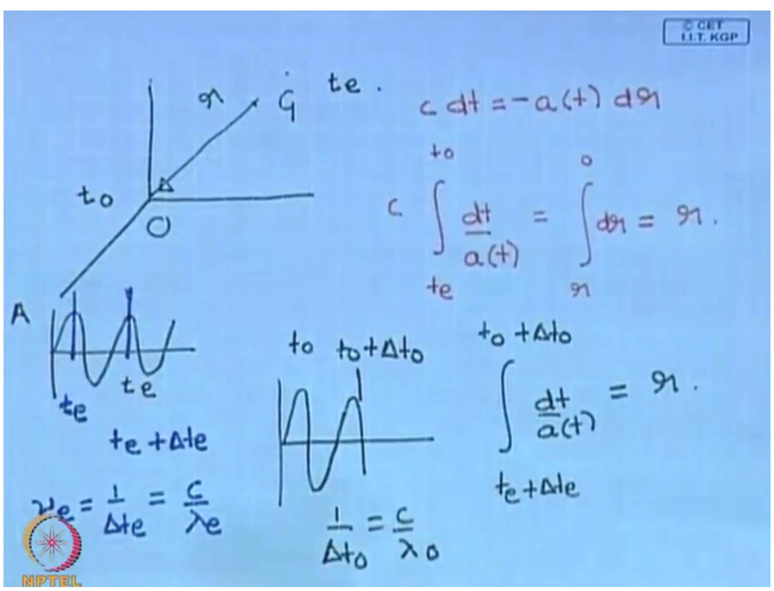

Because the Galileo's laws of transformation do not hold okay, so let us look at the propagation of light in this space time, so the situation that we are going to consider is as follows we have an observer sitting over here and there is a galaxy over here and the observer receives some light coming from the galaxy that is what we normally have in Astrophysics right, if you have observation observer is receiving light coming from a galaxy.

And let us say that the galaxy is at comoving distance r with respect to the observer, now and later say that the light was emitted here at a time t emitted and it was received here at a time t observed this is t observed cannot be necessarily t present t o and t e, so we would like to see the propagation study the propagation of this light, I have already told you that the most important thing about this interval is that it is 0 along the trajectory of a photon along the trajectory of light.

So the propagation of light from this galaxy to this observer is along trajectory which satisfies c dt=a (t) dr the light we are assuming propagates entirely along the radial direction, so these terms are not going to be there d theta and d phi and this is nothing very special because we can always choose the coordinates system, so that one of the point is observer is at the origin and galaxy is somewhere else okay.

So this is the situation that we are dealing with light propagates along a curve which satisfies this, so we would like to now to determine the trajectory you have to now integrate from t emitted to t observed right, so what we can say from this is that see the integral t emitted and there will be a-sign here, because as time increases the distance r decreases well you would have taken a positive sign if the lights were going outwards okay.

So we have to integrate this and we have to integrated from t emitted to t observed and we have to integrate dt/a (t) and we have to integrate the right hand side from r to 0 this will give us r, now light we know is a wave okay, so there is a wave actually which is coming from here and there is a wave which is being received over here, we would like to now calculate the redshift of the light.

We address the question does the frequency of the light change when it propagates from the galaxy to the observer that is the question that we would like to address, so we know that light is wave so let us just draw a wave which is emitted, so the wave which is emitted at the emitter will look something like this, this is t emitted and this is the oscillation that is being transmitted that is being emitted, so this is t emitted.

This is the amplitude of light let us call this some capital A amplitude of the wave that is being transmitted and let us say that we are this is the time instant t e when 1 crust is emitted this is the time instant t e+delta t e when the next crust was emitted, so there are 2 crusts in this wave which are emitted at the time interval of delta t e and we know that the frequency of the light mu emitted=1/delta t e and this is also=c/lambda emitted okay.

So basically delta t e is proportional to lambda emitted, similarly, this is the emitter what the emitter emits, the observer will also receive some thing is exactly identical except that maybe the time period will be different, so the observer will receive the first crust at a time t observed and he will receive the second crust at the time t observed +delta t observed and the observed frequency are 1/delta t observed=c/lambda observed which is okay.

Now so 1 there is 1 crust which is emitted at t emitted and received here t observed and that will follow this trajectory it will travel at the speed of light, so we have written down the equation that governs it, let us now write down the equation that governs the propagation of the second crust, the second crust will follow exactly you see in the comoving co-ordinate system the galaxy remains at the fixed coordinate, the observer will also remains at the fixed co-ordinate.

So this right hand side will not change, the right hand side is the comoving separation from the galaxy to the observer which remains fixed okay, the limits of the right hand side integral the left hand side integral will be different, so for the next crust for this crust and this one, the limits of the integral will be t emitted +delta t emitted that is when it was emitted and the time when it reaches here will be t observed +delta t observed, this dt/a (t) is also=r right.

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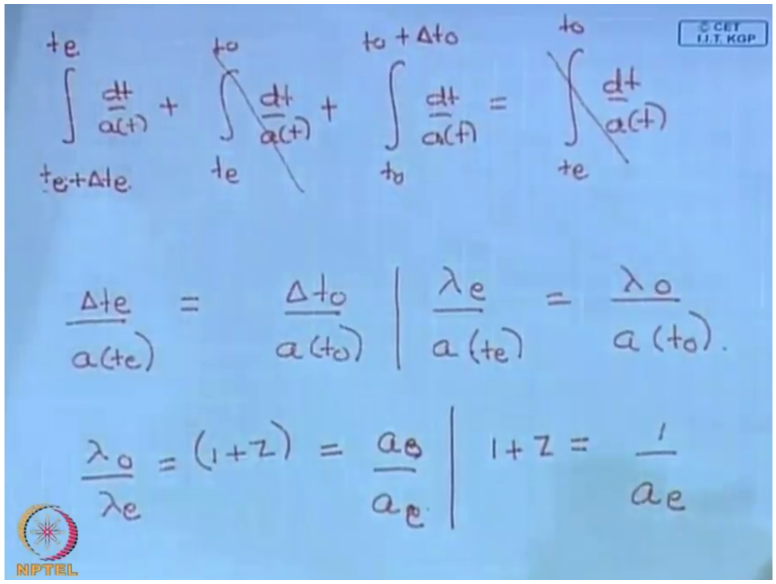

The second crust has to follow exactly the same equation, so now we can equate these 2, so let me equate these 2 and I can expand out this integral, so let me expand out the integral first, if I expand out the integral what I have is t emitted +delta t emitted to t emitted, so this is what I get if I expand out the second integral, I have just taken the thing the integral limits of the limits are from t emitted +delta t emitted to t observed +delta t observed.

I have written it out broken it up into these 3 parts and you can see that this part this exactly cancels with this, so we are led to the relation that this integral should be equal to this, now you see this integral delta t emitted is extremely small, so we can replace the integral by just delta t emitted by the value a (t) emitted, we may assume that the function a (t) does not change in this interval okay. Similarly, we may assume that the function a (t) does not change in this interval.

So we are led to the relation that this=delta t observed/a (t) observed okay and since the time interval and the wavelength are exactly proportional, we see that the wavelength satisfied this relation, so the wavelength actually changed because the time intervals change okay, so the wavelength of the light which is emitted is different from the wavelength of the light which is received okay and this is occurring because essentially because of the expansion of the universe.

The first peak over here the first crust has to travel a smaller physical distance a (t) is an increasing function of time, so the physical distance at the first crust has to travel is less compared to physical distance the second crust has to travel, so the time the first crust takes to cover the separation is less the time that the second one takes is more, as a consequence delta t observed is more than delta t emitted okay.

And this implies that the ratio of the wavelength also sales according to the scale factor okay, so essentially you can think of it as follows that the wavelength gets stretched with the expansion of the universe because the ratio of the wavelength to the scale factor remains a constant, as the universe expense if I have a wavelength light of wavelength of 1 meter, the size of the universe is becomes doubled the wavelength also becomes 2 meters okay.

And then we can now calculate the redshift z remember that if I look at the ratio of the observed wavelength to the emitted wavelength this ratio is what I call 1+the redshift, so light is emitted at the wavelength lambda emitted, it is received at the wavelength lambda observed this I called 1+the redshift okay and this here is then a emitted/a observed okay.

The ratio of the scale factors and if I assume that the observer is sitting at present then we have the 1+the redshift is this should be a observed by a emitted because they are proportional so this is observer in the top, emitter in the denominator this=1/a emitted, if I assume that the observer is sitting at present okay, in general it is the ratio of the scale factors, the observer is sitting at present.

So let us assume that the observer at present see is light coming from a galaxy faraway, the value of the scale factor we have assume to be 1 at present, so I can put a0=1, 1+the redshift is essentially 1/the scale factor when the light was emitted okay, so this now allows us to interpret the redshift straight away in terms of the scale factor of the universe when the light was emitted, so let me ask you a simple question suppose I observe a galaxy at redshift 1.

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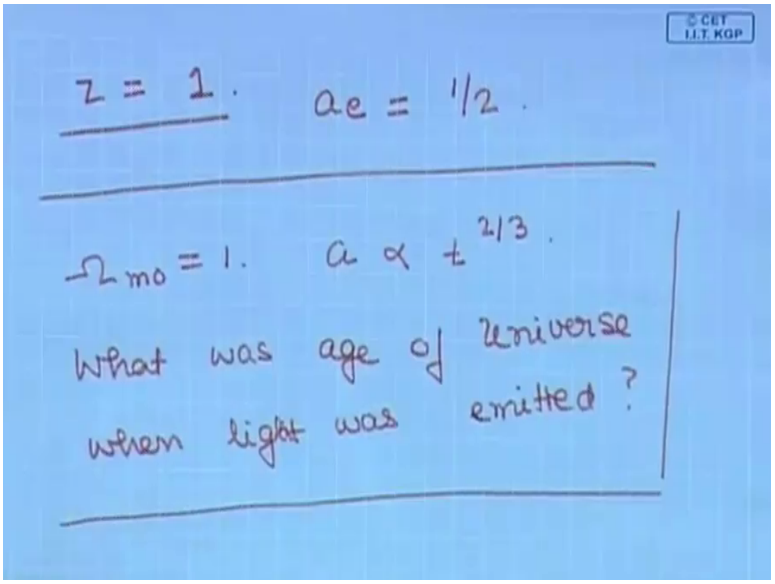

If I observe a galaxy at a redshift 1 I know then that the scale factor when the light was emitted the scale factor had a value which is how much is it, it is half, the universe was half the present size and suppose we also assume that the universe is matter dominated, completely matter dominated then we know that the scale factor is proportional to t to the power 2/3 this is something very important that you should remember.

As a universe is matter dominated the scale factor is proportional to t to the power 2/3 okay, if this is very important because this gives an with approximation to the real universe for most of its evolution except when it was radiation dominated in the past, but we do not see objects in that epoch okay and if it is if there is a cosmological constant then it will be different in the future may be somewhere near the present but most of the evolution it is like this.

We can use it for most purposes we can use it to make estimates, so if I know this then I can ask you the question, what was the age of universe compared to the present age when the light was emitted, so what was the age of the universe please work out this problem it is a very simple exercise, what was age of how far back in the universe are we seeing, when the light was emitted, so how far back in the universe are we seeing, when we see a galaxy at redshift 1.

And you can determine this from the fact that the scale factor is proportional to t to the power 2/3, so we know the value of this scale factor now is 1, it is half at the time when the light was emitted, so I determine the age of the, how far determine the age of the universe at the instant when the light was emitted or how far back in the time compared to the present epoch is the light coming from okay right.

So we have learnt how the redshift of a source we have learnt that due to the expansion of the universe the light from distant sources, light from sources which we see from the past of the universe get redshifted light from distant sources get redshifted and the next thing that we would like to calculate is that we would like to okay.

Before we go into anymore calculations let me now introduce something called the conformal time which is very useful if we are studying the propagation of light in an expanding universe, so the conformal time is introduced as follows, the same cosmological metric can also be written in the following way so let me introduce something called conformal time.

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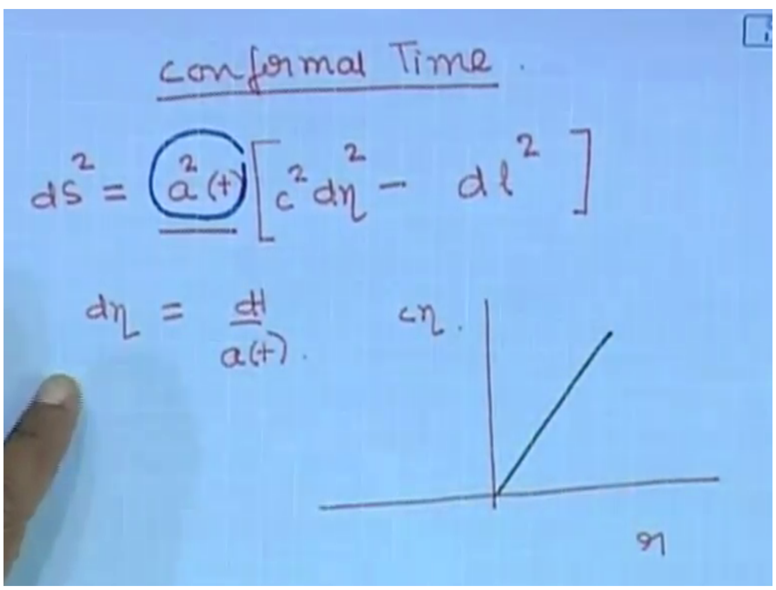

So I have not written out the length element in full.

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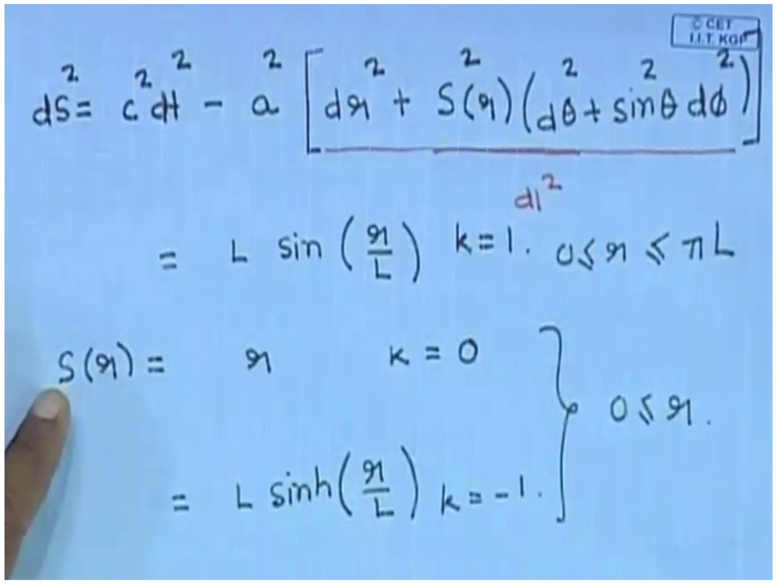

I have just represented this entire thing by dl square it is essentially the length distance between 2 points okay, so the same interval expression for the interval can also be written in this way, where we have introduced a conformal time d eta which is dt/a (t), you see all that we have done is that we have taken the factor a (t) which occurs over here common outside okay, so to do that I will get dt/a (t) here, so I have call that d eta the conformal time.

And line element now looks like this okay, this is called conformal time because this is very much like the metric and the absence of gravity and the absence of gravity that is correct right in the absence of expansion and the expansion of the universe as gone in as conformal factor outside, so if I have an element line element and I multiplied with the function this is called a conformal factor okay, so that is the reason why this is called conformal time.

Now if you look at this the advantage of this using the conformal time is as follows, let me look at the propagation of light again in a space-time diagram, in my space time diagram if I plot r and here if a plot c eta then the propagation of light is along 45 degree lines, because light will propagate along curves where dS is 0, so c d eta=dl I am looking at only the radial direction, so r will be=c eta, so the light propagates along 45 degree lines in this space time diagram.

If you go back to the original cosmological metric if I were to draw the space-time diagram in terms of t and r the comoving coordinate the light would not propagate along 45 degree lines because of this factor a (t) here okay, so the conformal time is very useful if we are looking at the propagation of light, the light covers equal comoving distance in equal intervals of the conformal time that is the important property okay.

So let me time is nearly over, so let me bring today's lecture to end over here, let me recapitulate before what we have learnt today, we learned first how to interpret the cosmological space-time which is represented through the cosmological line interval or what is called a metric, this is essentially allows us to calculate distances and intervals between evens that is the crux of the whole matter and today we learned how to interpret it.

We saw that you can the space can be of 3 different kinds, it could be flat or the cosmological space the universe at we are leaving could be spatially flat or it could be space of constant positive curvature or of constant negative curvature, we then took a closer look at we mean by a space of constant positive curvature it is a 3 dimensional sphere.

And finally I explained to you how to calculate the redshift of light from a distance source, how that is related to the scale factor and we saw that the wavelength essentially scales with the expansions of the universe which gives rise to the observed redshift of distant sources, let me end today's lecture here.