Astrophysics & Cosmology
Prof. Somnath Bharadwaj
Department of Physics and Meteorology
Indian Institute of Technology - Kharagpur

Lecture - 26
Dynamics of the Expanding Universe

Welcome, let me remind you that in the last class we had started discussing the expanding universe and I told you that it is convenient to go into a co-ordinate system which is expanding with the galaxies which is fixed to the galaxies and this is called the comoving co-ordinate system.

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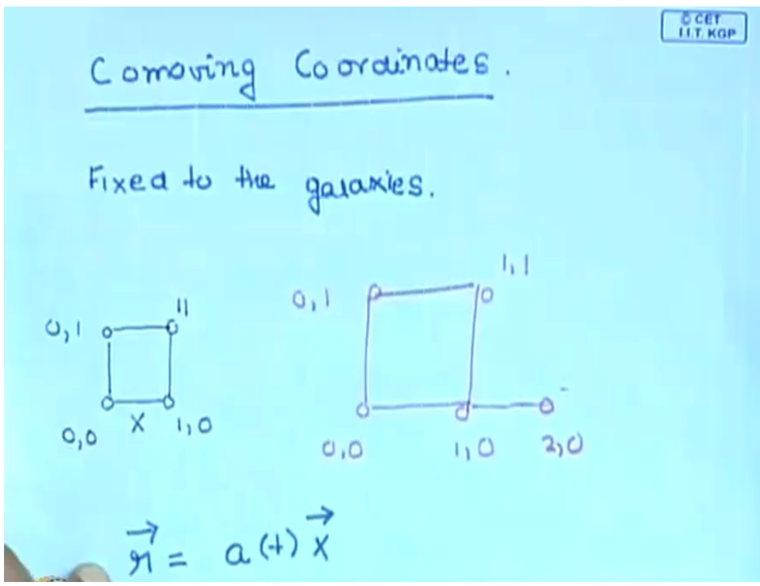

So this co-ordinate system X is fixed to the galaxies, the intergalactic the galaxies are all moving away from one another, so that is there in the scale factor a(t) which is an increasing function of time and the physical coordinate r between 2 galaxies then which keeps on then keeps on increasing as a function of time.

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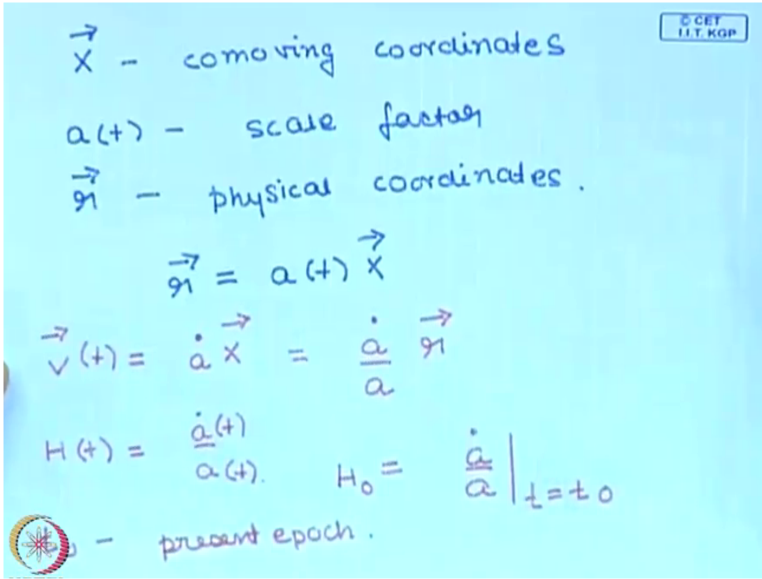

So with these definitions the Hubble parameter was the time derivative of the scale factor divided by the scale factor itself and the present value of the Hubble parameter is then a dot/a at t=t0, t0 refers to the present time.

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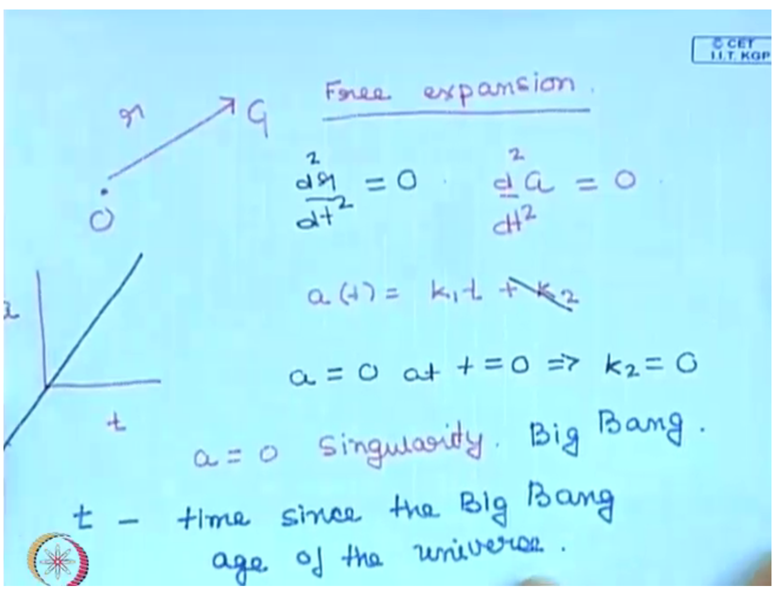

Given this, so given the scale factor and the comoving co-ordinate system we first considered a very simple models where the universe is freely expanding, there are no forces acting on between the galaxies and in this model we walked out the behaviour of the scale factor as a function of time, so we found that the scale factor increases linearly with time and at a certain time instant the scale factor reaches 0.

And we chose the constants there were different constants of integration that come about when you integrate when you write down a(t) and we chose the constant of integration suitably so that the time origin for time is at the instant when the scale factor becomes 0, these scale factor becoming 0 is a singularity which is referred to as the Big Bang, the entire universe collapses to a point.

So the time is measured from that event and the age of universe then at the cosmic time is given by t.

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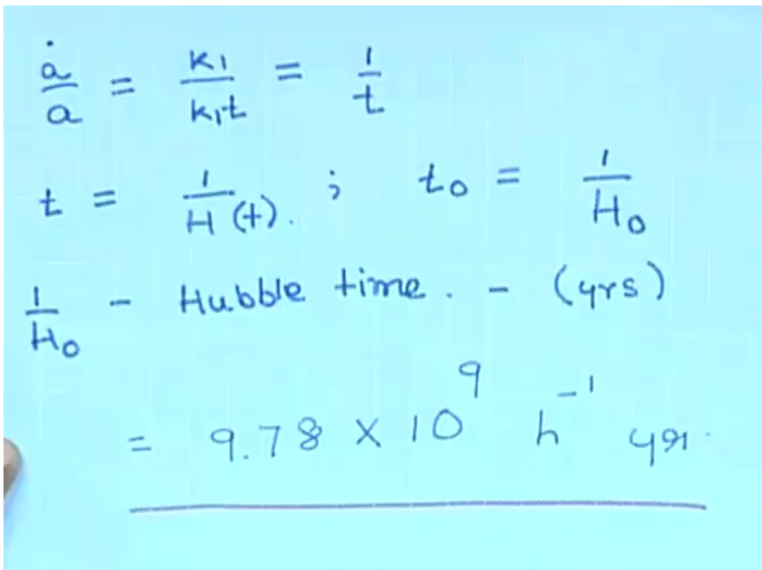

And in this model we saw that the age of the universe at any instant of time is related to the inverse of the Hubble parameter at the same instant, so the present age is 1/H0, 1/H0 is referred to as the Hubble time and in the last class I had asked you to estimate this in years, so I hope we have done this I have given the value over here.

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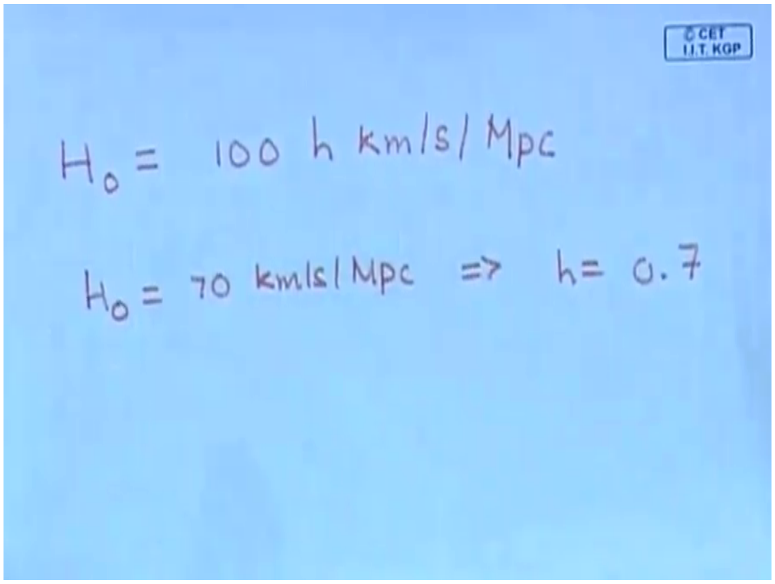

So if you assume that H0 is hundred h kilometers per second megapersec, the Hubble time comes out to be 9.78*the 10 power 9 h inverse years, so the order of magnitude of the age of the universe, whatever cosmological model you use is of the order of 10s of billions of here billions right this is 10 and this is approximately 10 billion years so it is of that order, now I also told you that this only gives you an order of magnitude estimate.

Because here we have ignored the influence of gravity which will slow down the expansion of the universe and the relation between the Hubble parameter and the age will be modified.

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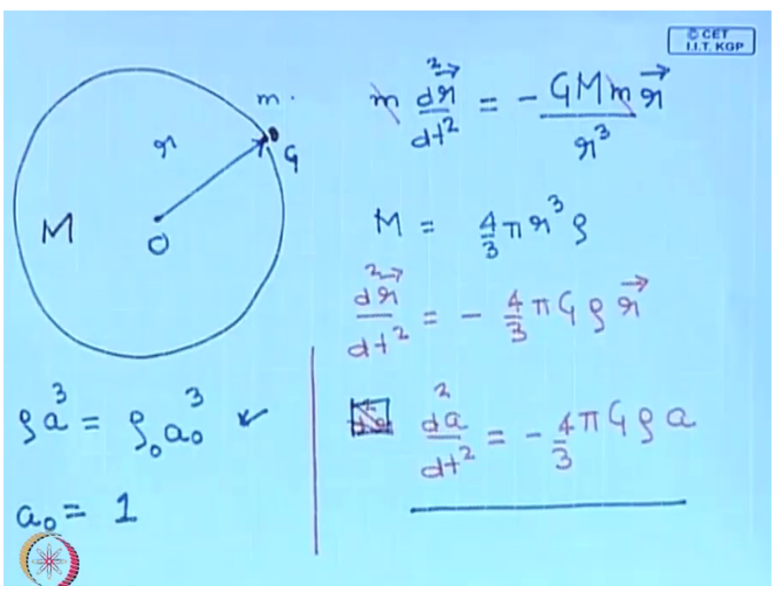

We next put in the effect of gravity and if you include the effect of gravity, you then have 2 equations which are written over here 2 equations for the scale factor, the first equation tells you the second derivative of the scale factor is –4/3 pi G rho a the physics of this is essentially that the motion of the galaxy over here.

This is the observer who is observing this particular galaxy the motion of this galaxy, the forces acting on this galaxy are due to the mass enclosed within a sphere encircling the galaxy that is all, so starting from this you can arrive at this relation for the scale factor in addition to this you also have the fact that rho*a cube is the constant, this is the conservation of mass.

As the universe expands the length scale increases proportional to a, the total mass in a volume has to in some region has to be conserved, so that volume the volume of that region increases as a cube so the density has to fall has 1/a cube because the mass has to be conserved, further so this is also=rho 0 the present density into the scale factor at present cube, further we assumed that the scale factor at present has a value 1.

We are free to choose whatever value we wish for a scale factor at present, so we have chosen this to be 1, so at present the comoving coordinates system and the physical co-ordinate system exactly same, in the past the comoving coordinates would be different okay from the physical coordinates or in the future also.

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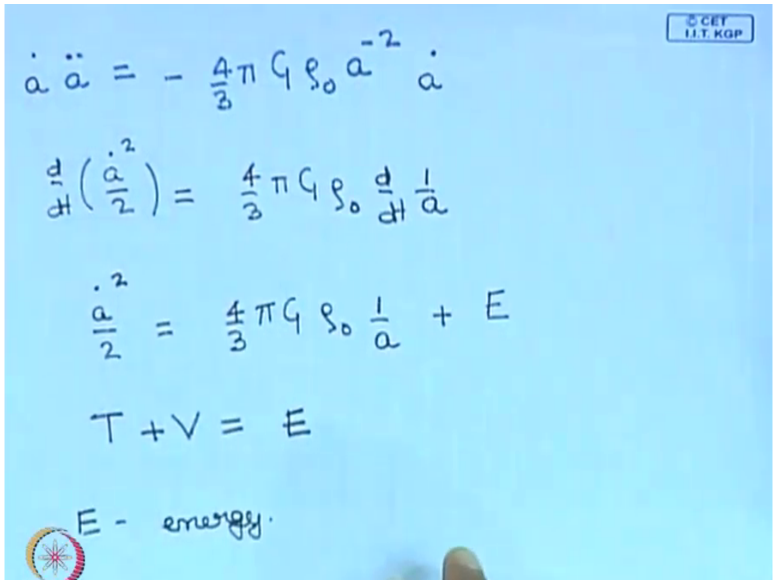

So these are the 2 equations that we had to solve and we preceded by integrating by multiplying the equation with a dot time derivative of the scale factor on both sides and then we obtain this equation half a dot square+4/3/G rho*1/a+ = E+E okay this is equal to this +E, E is a constant of integration that comes about when you integrate this equation 1, so this equation is the first integral of this equation.

And we interpreted this equation as follows this term could be identified with the kinetic energy of this galaxy, this term could be identified with the potential energy of this galaxy gravitational potential energy due to all the mass inside –GM/r essentially is this term and then we know that a Newtonian mechanics if I have a system like this then the total energy of this particle is going to be conserved okay.

So this is the total energy which is a constant of integration a conserved quantity for this motion, so this equation could be written as T+V the kinetic energy + the potential energy is equal to the energy of that galaxy, remember this is a purely Newtonian interpretation okay, now before so this is the equation that we had and before proceeding further let me point out one fact.

The fact is that the motion of this galaxy is exactly the same as the motion of a particle thrown from the, throne radially outwards from the surface of the earth, because a particle thrown out radially outward from the surface of the earth will be governed, the motion of that particle will be governed by exactly the same equation, now for a particle which is ejected or projectile ejected radially outward from the surface of the earth.

The mass that is inside the earth remains fixed the mass that contributes to the gravitational acceleration. Similarly, for this galaxy the mass that contributes to the gravitational acceleration remains fixed because the matter inside this is not never going to take this galaxy according to Hubble's law the matter inside this sphere is moving slower than the galaxy because it is at a smaller distance.

So this mass is, the mass inside the sphere will never overtake this galaxy and go out, so this M is basically a constant independent of time, so this equation is essentially the equation of a projectile which is being projected outwards and from the surface of the earth and we very well know what the solutions are okay, the solutions are determined by the value of the total energy of the particle.

If you have a total energy which is negative then if the particle has an energy which are the particle which is being ejected out has which is the projectile has a negative energy, then what happens is that the particle will go up to a certain distance and then where the kinetic energy becomes 0 and then it fall back again.

If the total energy is exactly equal to 0 that is the situation where this particle has the escape velocity and we just imparted the escape velocity to this particle right, so it will reach infinity true but when it reaches infinity it will be at rest it will come to rest at infinity okay and the third possibility is that the particle has positive energy in such a situation the particle will reach infinity and have a non 0 velocity.

So these are exactly the same situation the same things will happen here also depending on the value of E, so it is quite clear that the evolution of the scale factor is crucially dependent on the value of E okay, then I address the question how can we determine the value of E at least in principle.

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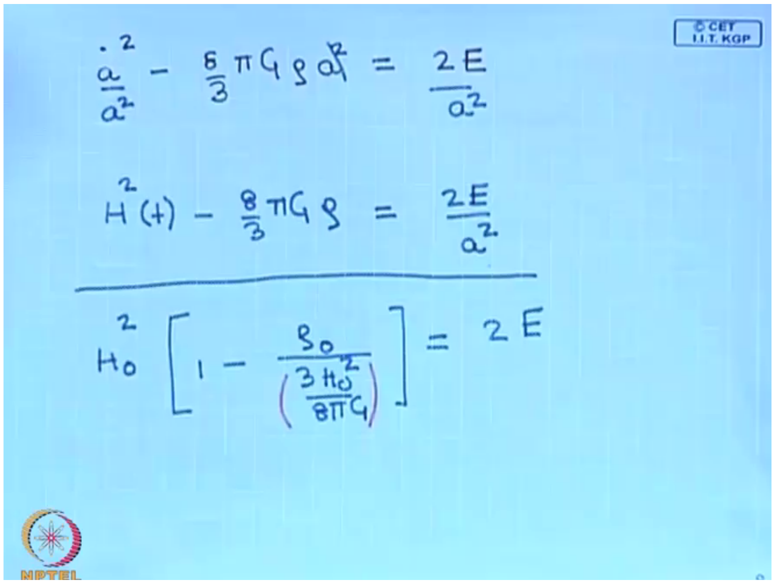

So to address that question what we did was we wrote the same equation in a slightly different form, we replace rho 0 here with the density into a cube and we then obtain this equation which is essentially this, and if you take this equation and write it at present then you have this relation between the constant E the energy E and H0 square and this ratio over here the dimensionless ratio over here.

So we see that in this dimensionless ratio we have the present density of the universe in the numerator, in the denominator we have a density something of the dimension of density which is determined by the value of the Hubble parameter this density is called critical density.

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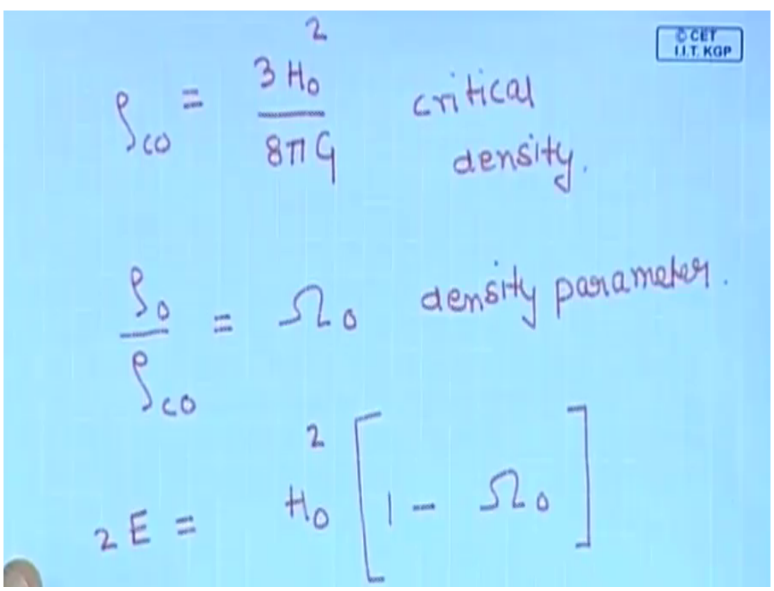

And the present value of the critical density is 3 H0 square/8 pi G and the ratio of the actual density to the critical density of the universe we called the density parameter, it is this ratio which determines the value of E

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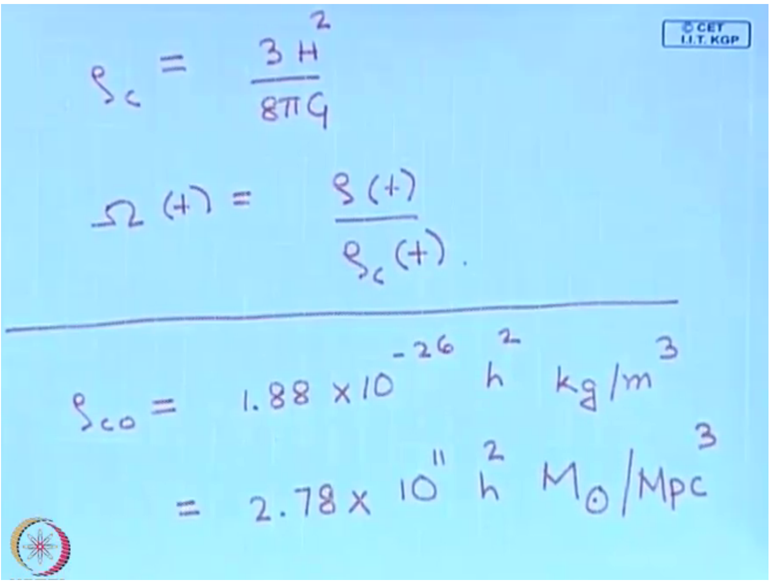

Further I also told you that this ratio that the critical density is a very general definition it is not only that you can define it at present you can define this at any epoch, so it is 3 H square/8 pi G at the epoch in question and the omega density parameter also can be defined at any epoch it is a ratio of the 2 densities at that epoch, I had also asked you to estimate the value of the critical density.

So I hope you have done this if you put in the value of the Hubble parameter and the other constants it comes out to be so the value of the critical density comes out to be 1.88*10 to the power-26 h square the h square is there because we have parameterized Hubble parameter in terms of this h kg per meter cube, so what does this tell us, this tells us or okay.

Before that you can also write this in units which are sometimes more convenient in astrophysics you can write it in terms of solar masses per megapersec cube and it then turns out to be 2.78 10 to the power 11 solar masses h square solar masses per megapersec cube, what is the significance of this density?

And the significance of this density is that if the actual density of the universe at present is more than this value then the density parameter is more than 1 and the value of the energy is negative okay, if the actual density is just equal to this critical density then the omega density parameter has value 1 and the value of this energy is 0, this corresponds to the particle that is being ejected having just the escape velocity.

If the density of the actual density of the universe is less than the critical density then the density parameter is less than 1 and E turns out to be the energy turns out to be positive, so all of this is summarized and it is summarized over here.

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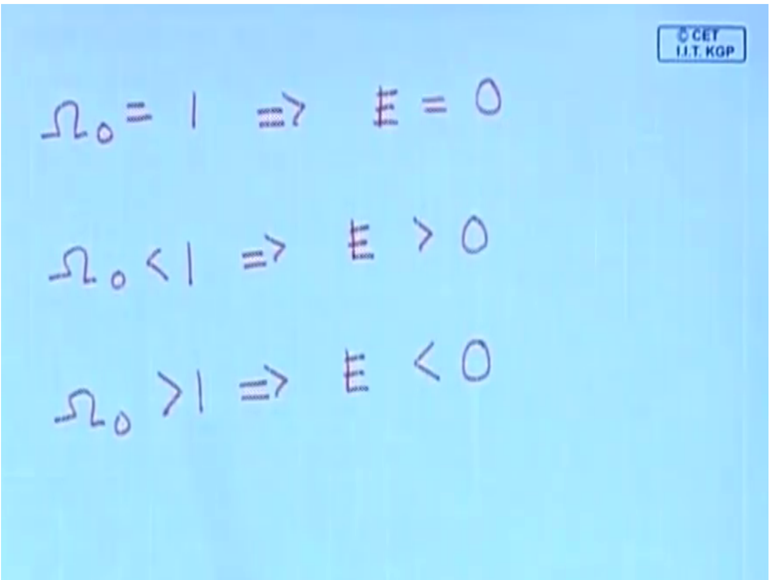

Then I will recover the free expansion model which essentially corresponds to omega 0=0 okay, so that is the case where the constant is H0 square/2 free expansion which we have already worked out, so the main thing then is that if you can determine the actual density the Hubble parameter I have told you is quite well determined, we shall discuss later how it is determined if you cannot this determine the actual density of the universe.

You can then determine this constant of integration E and you can determine then you can work out the solution to this equation and you know the complete evolution of the universe okay, let us first before we discuss this any further let us first work out the solution to this equation what does it look like, so let us first take up one situation which is the simplest where the value of this energy is 0 which is the critical model okay.

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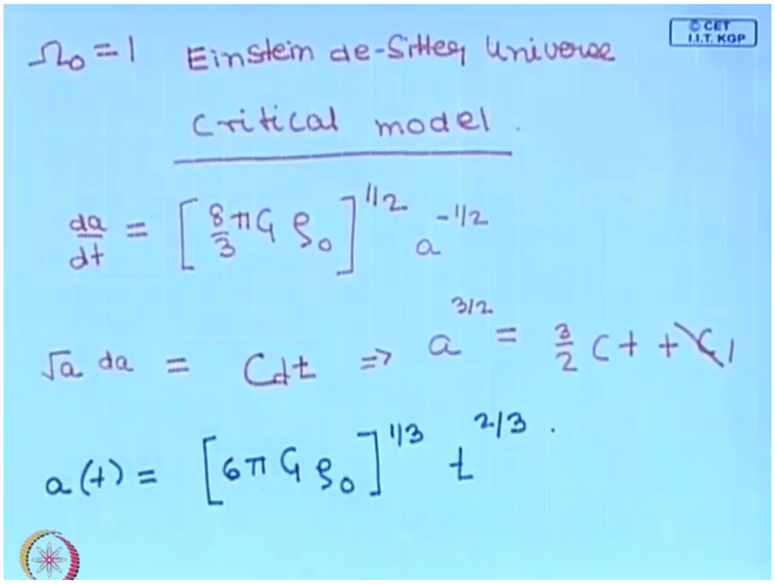

So if the density parameter is 1 the energy is 0, so the density parameter being 1 essentially corresponds to a situation where the kinetic energy of this particle exactly balances the potential energy of the matter due to the mass inside that is the density parameter 1, whereas if the density parameter is more than 1, so the density is more so the potential energy is more than the kinetic energy.

Whereas if the density parameter is less than 1, the potential the density is less so the potential energy is less than the kinetic energy, the situation that we have already considered which is free expansion essentially corresponds to putting the density parameter to 0, because in that model there is no gravitational force and if you have no mass, no density in the universe, no matter in the universe so the galaxies are just test particles they do not exert any gravitational force.

Let us just work out the value of the scale factor what does it look like, so the scale factor corresponding to omega 0=1, this is the critical model this a critical universe it is also called the Einstein de-Sitter universe, Einstein de-Sitter model or universe model okay or just the de-Sitter universe also the critical cosmological model.

It is critical because it is just the dividing, it is the dividing line between the 2 models one with the positive value of E and other with negative value of E, for which the behaviour are quite different completely different, so for this particular model let us work out what the solution is and here the equation is rather simple, so we can start off with this equation rather it is more convenient to work with this, so let us start off with this equation put E=0.

So we want to integrate this equation with E=0, so the equation so what you do is you just take the square root of the right hand side multiplied by 2 and take square root of the right hand side, so what do you have is da/dt=8/3 pi G rho 0 to the power half and you have a to the power-half over here, so this is a very simple equation to solve, what all that you have to do is you have to take this on to the right hand side and integrate this.

So what we have is that da*square root of a=some constant let us give it a name I will just call it C*t I have just put this over here in this constant and there can be a constant of integration here, but we would like the scale factor to be 0, into dt sorry that is the thing that we have.

So let us integrate this equation if I integrate this equation what we will get is a to the power 3/2=so if I have a to the power 3/2 here I have 2/3 so I can take it on to the right hand side and what I will get is 3/2 into this constant into t + another constant of integration which I will set=0 because I would like the scale factor to vanish when t=0 not at some other instant of time okay.

So we can then write down the scale factor, the scale factor a as a function of time=how much is it=so we have 3/2 over here and if I take 3/2 inside the square root sign then I will have 3 will cancel out and I will have 3 left out because I take square of this, so I will have 9 3 on top and if I take this inside I will have a factor of 4.

So I will have 4 6 sorry 6 pi G rho 0 to the power of so here I have to the power 3/2 here I have to the power half so I will have to the power 1/3 t to the power 2/3 so that is my solution, in the situation where omega 0=1 the density parameter is 1, the crucial point here is that the scale factor is proportional to t to the power 2/3, the constant is some number which comes which is there okay.

That basically determines the normalization of a we would like a to be 1 at present okay, so let us for this model now determine the value of the Hubble parameters, so the scale factor is something that is not physically measurable here the quantity that is measurable is the Hubble parameters, so let us determine the value of the Hubble parameter in this model right, so how does the Hubble parameter evolve with time for this particular model.

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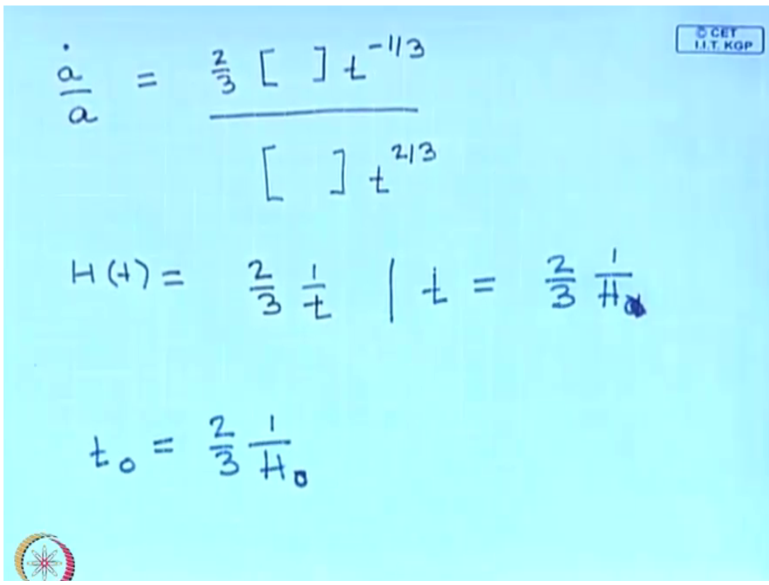

So in this model if you calculate the Hubble parameter a dot/a, so if I differentiate this I will have the constant 2/3 t to the power –1/3, so let me write it here 2/3 the constant t to the power –1/3 that is the a dot divided by the same constant t to the power 2/3, so now you see that the Hubble parameter H(t) in this model comes out to be 2/3*1/time or at any instant of time the age of the universe so let us say the present age=2/3 of the Hubble time.

So the presence present age of universe sorry at that instant not at that instant of time and the present age of universe is 2/3 the present value of the Hubble parameter, so what do we see, we see that the as far as the Hubble constant is concerned the main consequences that the gravitational slowing down due to gravity will essentially reduce the age of the universe, so if I had no gravitational slowing down then the age of the universe is exactly the Hubble parameter.

Whereas if I am living in a cosmological model where the density is exactly balancing the critical is exactly=the critical density omega 0 is 1, then the age of the universe is 2/3 of this value for the same value of the Hubble parameter, so the age is smaller so we have worked out 2 cosmological models already, we have worked out the solutions for 2 cosmological models let me draw these solutions schematically.

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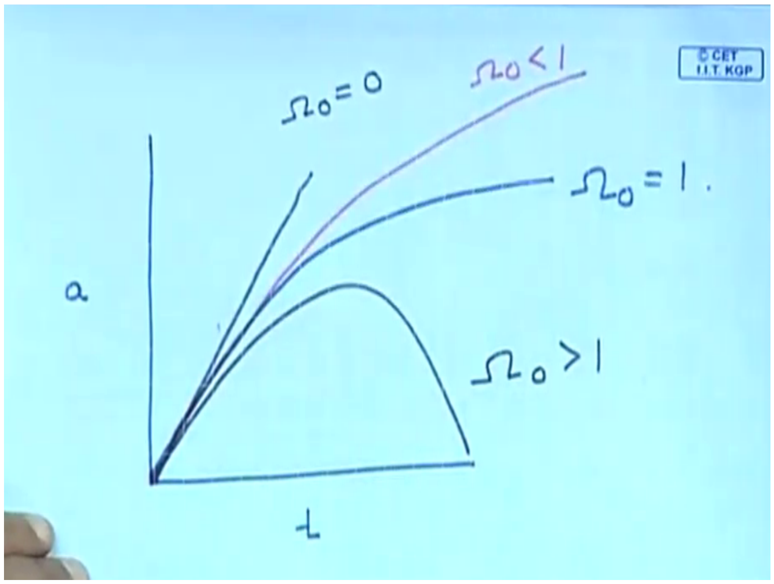

The first solution that we worked out a as a function of time, the first solution that we have worked out we have linear expansion of the universe, so the universe expanse like this that is omega 0=0 and then we worked out another model where omega 0=1 and we saw that it is proportional to the scale factor grows as t to the power 2/3 and we can draw the nature of the curve, so let us draw the nature of the curve, the nature of the curve will look like this.

This is omega 0=1 and it will become flat as you approach infinity because the slope of the curve a dot is 1/t to the power 1/3 and as t goes to infinity that goes to 0, so the slope will approach 0 as t goes to infinity, so that is the omega 0=1 model, now the question is one has to now work out the other cosmological models corresponding to different values of E, but we know that the other values of E.

We know the problem and the solving this equation is rather straight forward for different values of E it is not a very difficult thing it will may involve some amount of more lengthier calculation that is all, but it is not a very difficult conceptual it is not a difficult calculation, it is essentially the same problem as that of a projectile thrown from the surface of the earth, it is exactly the same problem projectile which is already familiar to all of us hopefully.

So we know that if E is negative or if omega 0>1 the particle will return back after some distance, so and the model for will look like this so for omega 0>1 the universe which is expanding will not continue to expand forever reach a maximum value of the scale factor and then it will collapse back you will have what is often referred to as the big crunch, so you have the Big Bang at the start and you will the universe will end with what is called a big crunch okay.

The third possibility which the other possibilities omega 0<1 for those cases the slope will reach a finite value we know that the particle has a finite velocity when it reaches the infinity, so the model will look something like this it will have a finite slope at infinity okay and here this is the extreme situation omega 0 is 0 which is from the start right from the start it is a free particle.

All of these models will approach a free particle at infinity this will be a free particle at rest, this will be free particle with finite a velocity, this is a free particle to start with I will not work out the mathematical forms of these you can look it up or work it out yourself, but this is the broad nature of the behaviour of the scale factor.

Now let us so you see the crucial point here is that the behaviour of the universe of the expansion of the universe depends crucially on the value of the density parameter, so it is essentially it is very essential to determine which cosmological model do we live in, which of these is our universe that is the crucial question, we would like to know what is the evolution, what is the fate of our universe, what is the past evolution of our universe.

After all cosmology we would like to study the whole universe determine its past evolution, its future etc. okay, so this is determined by this density parameter and density parameter I have told you is the ratio of the actual density to the critical density, so critical density is known the main problem lies in determining the actual density of the universe, how to determine the actual density so the difficulty lies over there.

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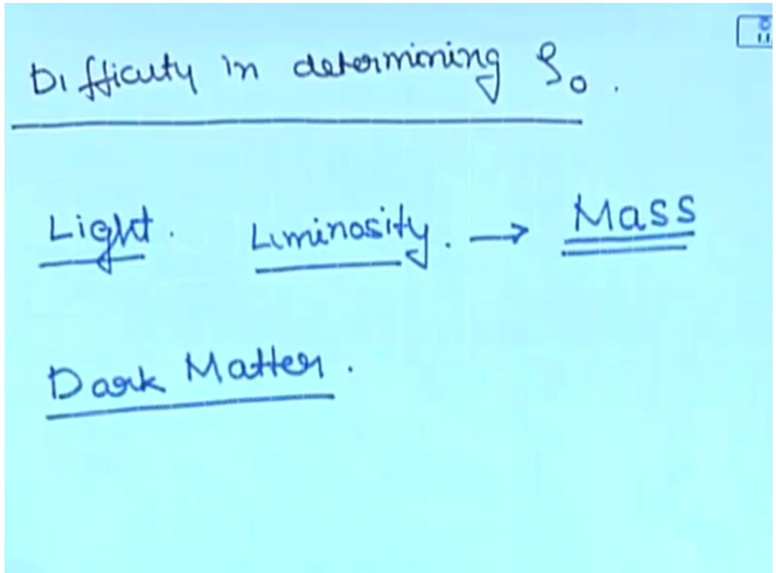

So the difficulty lies in determining rho 0 because if you know the density you can straight away determine what the value of the density parameter is, and the main difficulty lies over here in determining the density, why what is the difficulty let me briefly just try to give you an idea of what a difficulty is, what you can measure here and directly from earth for example is the light coming from different astronomical objects.

So you can measure what you can measure here is the luminosity, you can measure light electromagnetic radiation, so you can measure the amount of light coming from different galaxies for example and you know the number of galaxies number density of galaxies, so if you know the luminosity so what you know is the luminosity of objects this is what you can measure, you did not know what the mass how to convert this into the mass.

And you can determine how many objects there are but question is what is the relation between the luminosity and the mass how much does it corresponds to, now we have seen for example for stars the relation between luminosity and mass is not linear it is complicated relation right, so this is the one of the big uncertainties that you have, how to convert luminosity into mass, this is if you believe that all that you see is just a luminous matter.

But the problem is much more severe than this because we have already seen that there are components of the matter which are referred to as a dark matter, what is dark matter? It is matter which emits no light which we do not see, we know it is there because it exerts a gravitational attraction, if it exerts the gravitational attraction it will contribute to this equation.

It will contribute to the mass density inside but we do not see it, it is dark it does not emit radiation directly, for example we discuss the rotation curves of galaxies and I told you that there is evidence that you require dark matter to explain the flat rotation curve, but and that is not the only situation where you need dark matter, so a large component of the density of the universe is believed to be dark and we do not know what this that matter is.

So this is another problem and that we have okay, so we do not know, there is no way of directly measuring the density okay, so this is something which cannot be directly measure the density of the universe, so if you think that you could actually measure the density and determine the value of the density parameter that is not feasible, so one has to adopt indirect means for determining the value of the density parameter and that is the first thing that we have to realize.

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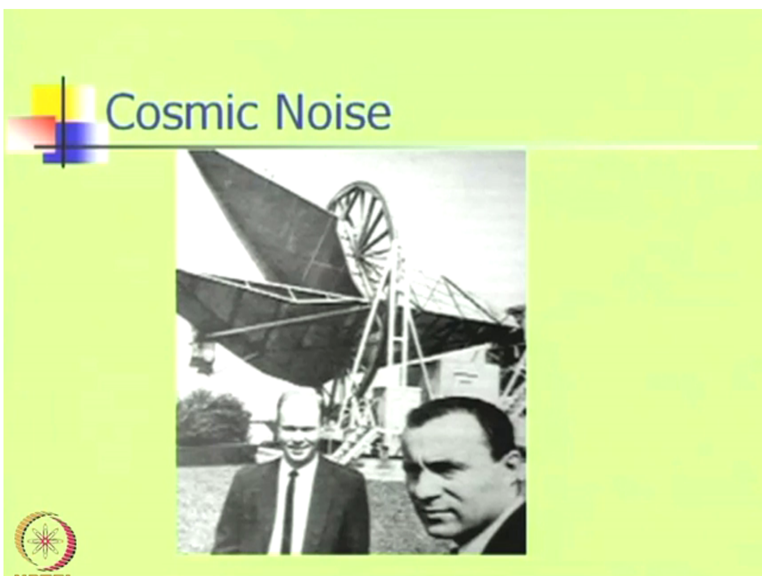

Discussion till now has entirely been based on the observations of galaxies and the idea that the entire universe is filled with galaxies and these galaxies are moving apart from one another, let us now jump to a very remarkable discovery that was made in 1964 observational discovery in the year 1964.

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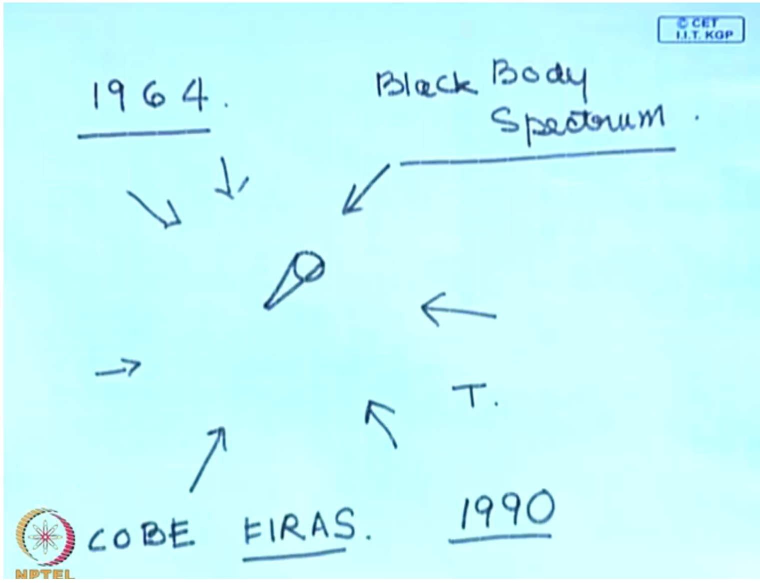

And there were quite before that there were speculations that one expects to discover such a thing we shall learn about these later in this course okay, let me just jump straight away to the discovery, so at that time 1964 1960s satellite communication, communication through electromagnetic waves was being developed and the there were 2 radio astronomers Arno Penzias and Robert Wilson who were working at the AT & T bell labs, bell laboratories in the USA.

And they were using this radiometer, so this is a radiometer to study the noise for satellite communication through electromagnetic waves and they were working at around centimeter wavelengths somewhere over there and so what they were doing what they were using this radiometer to characterize the noise from different directions in the sky and you expect for example the sun to produce some electromagnetic radiation at those frequencies at those wavelengths.

The bulk of the radiation we have seen comes at in the visible but it will there will be some part of the spectrum which I have already told you in those wavelengths in addition there will be some radiation coming from our galaxy, but both of these things we know they rotate in sky, the sky is rotate, the earth is rotating, so their position in the with respect to the say the vertically upward direction will keep on changing with the time.

And any such radiation you expect the contribution to vary with time, but Penzias and Wilson found that in addition to all of this there was some component some radiation which was coming quite uniformly from all directions in the sky, so this was a radiation which was coming from all directions, so if you have radiometer and point it they found that there is a component of radiation which is more or less the same from all directions.

The radiation was so they could not identify this radiation with any known astronomical or terrestrial source and Penzias and Wilson were worried about this radiation they were very sure that there radiation was not generated inside radiometer it was from outside and they could not identify any source for this.

At the same time there was a group of theoreticians and observers observationalist that Princeton University who are trying to detect such a radiation which was coming from all directions which had roughly the same intensity as what as the radiation that was discovered by Penzias and Wilson.

So when they heard on Penzias and Wilson discovery they immediately realized that the radiation that Penzias and Wilson had discovered is actually cosmological in nature its cosmological, so that is why it is referred to here as cosmic noise, so the radiation that Penzias and Wilson discovered is not only is the radiation isotropic around us it is so the radiation comes the amount of radiation coming is the same from all directions not only that.

But we believe that the radiation is also homogeneous it fills the entire universe, so this radiation fills the entire universe and this cosmological radiation was expected to have so it is expected to have a Black body spectrum, Planckian spectrum so Black body Planckian Spectrum okay, so remember that this black body Planckian spectrum arises when radiation is in thermal equilibrium with matter.

And they expected this radiation to have a Planckian spectrum it if it is cosmological in origin a black body spectrum and its temperature a black body radiation is completely characterized by at temperature was expected was around a few Kelvin, now obviously they Penzias and Wilson made the observation only at a particular wavelength or frequency and they could not determine the spectrum, the fact that it was a black body spectrum would not be determined by them.

So you required the observation that other wavelengths to establish the fact that this is black body spectrum it is now quite well established and well accepted that the radiation that was originally discovered by Penzias and Wilson is indeed a black body spectrum.

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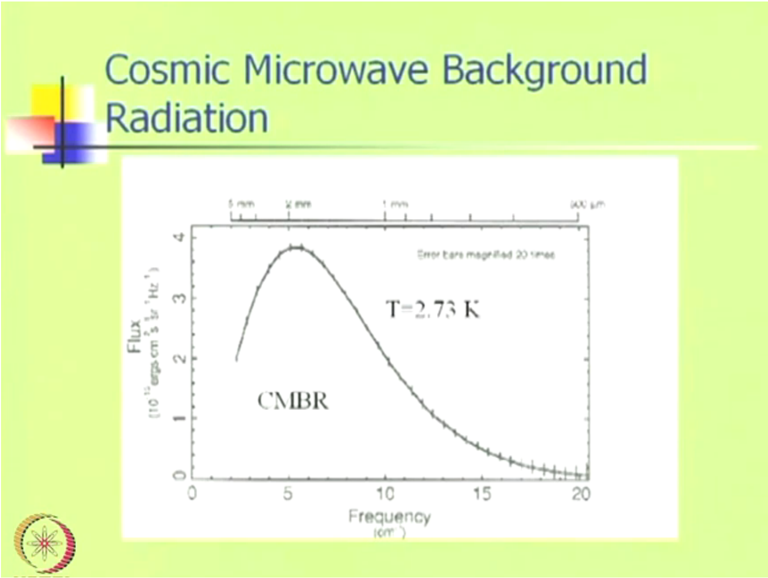

There was a satellite the COBE the NASA Satellite called COBE Cosmic Background Explorer which had an instrument called FIRAS or infrared spectrometer which measured the spectrum of this background radiation, so this background radiation is called the cosmic because it is cosmological it fills the entire universe, Microwave because this is in the microwave part of the spectrum it peaks in the microwave part of spectrum background radiation CMBR.

So this is a radiation that fills the entire Universe it has a temperature, the temperature is measured by COBE FIRAS is 2.73 kelvin and the fit to a black body spectrum is so good and the measurements was so precise that there error bars you can see some errors bars here, these error bars have been so the data points are shown over here the data points.

And the smooth curve is a fit so is actually a black body spectrum not a fit it is a black body the best fitting black body spectrum which corresponds to a Planckian at 2.73 kelvin, the errors on this data point have been magnified the one sigma errors have been magnified hundred times so that you can see them, they are actually hundred times smaller they have been magnified hundred times so that you can make them out in the graph.

The measurements was so precise and this is the most accurate black body spectrum that has ever been measured even on earth okay most precise measurement of a black body spectrum, so and both so these are very for path finding works in cosmology, so both Penzias and Wilson they received the Nobel Prize for this discovery.

And the John Mather who was the leader of the team that measured the spectrum also received the Nobel Prize quite recently in 2006 if I am correct okay, this satellite this COBE satellite was launched in 1990, the measurement was made in 1990, so this black body so one so before that there were points there were discrete points which had been measured but they were still scope for deviations of a black body spectrum.

It is now well established that over this entire region these spectrum is blackbody, it is a Planckian spectrum okay and noticed that the peak occurs at around 2 millimeters okay, that is where the peaks of this spectrum occurs around 2 millimeters and here you have the frequency in units of centimeter inverse okay, so it is 5 okay, so this is the cosmic microwave background radiation.

So for the time being we know at least we know that there is one more component in addition to the galaxy and that component is a black body radiation, so we have to also take into account the black body radiation in our discussion, we shall study we shall come back to the cosmic microwave background radiation in more detail in later lectures okay, for the time being let us just study its effect one aspect of this radiation

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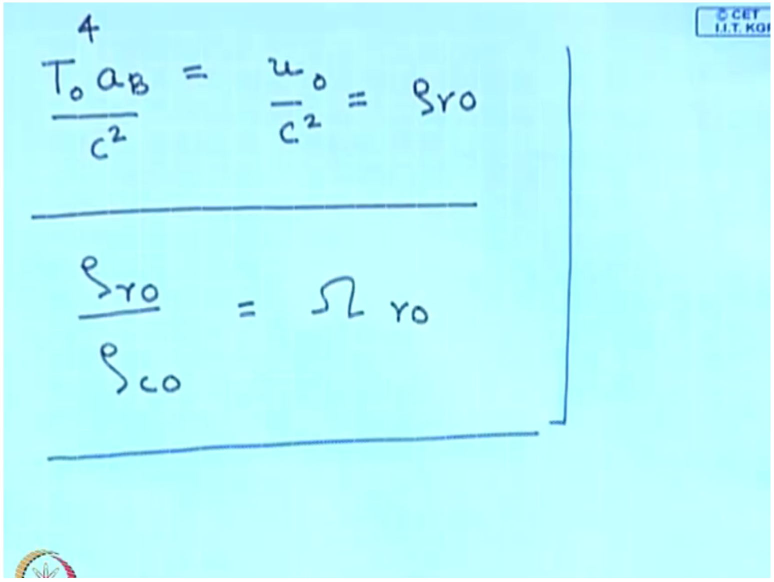

So the cosmic microwave background radiation has a temperature T, so it is a black body radiation with temperature T and you know that if there is a black body radiation with temperature T, then it has an energy density which is T into the Stefan Boltzmann constant aB to the power 4 this is the energy density epsilon or u let us call it u corresponding to this black body radiation.

And I will request you to calculate the energy density corresponding to this okay calculate the energy density corresponding to this 2.73 kelvin not only can you have an energy density you can also think of a mass density equivalent mass density if I divide this by c square rho okay and so there will be a component of so let us call this rho gamma component in the CMBR and this will give us the value of the rho gamma at present.

If I take the present temperature of the CMBR T0 u0 okay, so not only should you calculate the energy density but you should also calculate the ratio of the density in the CMBR to the present value of the critical density, so this is the density parameter omega gamma 0 or omega CMBR if you wish, so this is the contribution from the cosmic microwave background radiation to the density of the universe okay.

Now the question that we are interested in is what happens to the density of this component of the universe as the universe expands, for galaxies and other such matter we have seen that the density into a cube is constant, so as the universe expands the density falls as a cube, let us see what happens to this okay.

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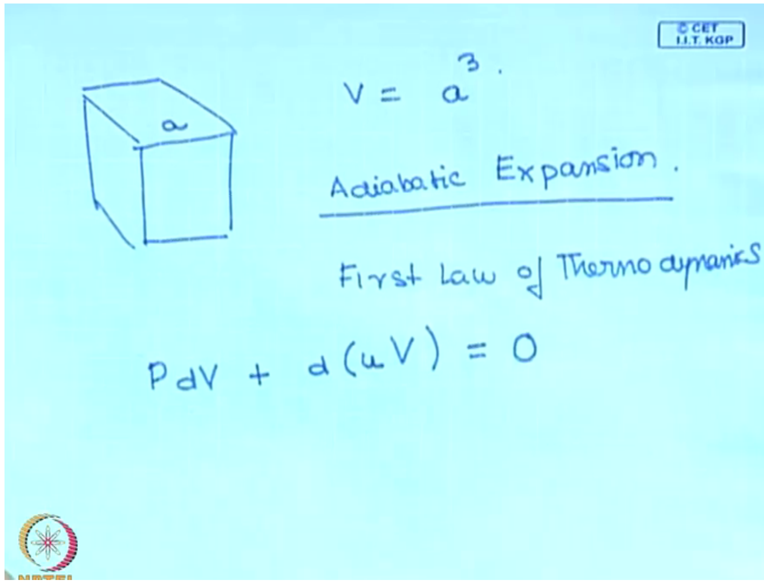

So the expansion of the universe let us look at a finite volume of the universe, let us say it is a unit comoving volume, so the volume is scale factor a basically that is the volume a cube, so the volume of this is a cube its unit comoving distance, volume of unit comoving distance okay, now we are assuming that the universe is undergoing adiabatic expansion, so this is like a volume which is undergoing adiabatic expansion.

So this is an assumption which is quite well justified this so a volume some any process maybe assume to be adiabatic if it does not exchange heat with the surrounding right and it is quite a reasonable assumption that in the expansion of the universe one part of the universe does not exchange heat with any other part of the universe okay.

So we are going to assume that the expansion is adiabatic, so if I take this volume and consider adiabatic expansion then we know that it has to satisfy the first law of thermodynamics which is that the pressure into P dV+ the change in the internal energy the internal energy we know is the internal energy per unit volume into the volume this should be=0.

So we are going to use this to determine how the density of the cosmic microwave background, the energy density of the cosmic microwave background radiation changes as the universe expands, let me bring today's lecture to a close over here and take up this for discussion in the next lecture.