Astrophysics & Cosmology
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Lecture – 25
The Expanding Universe

Welcome. In the last lecture, we learned about Hubble's law of expansion.

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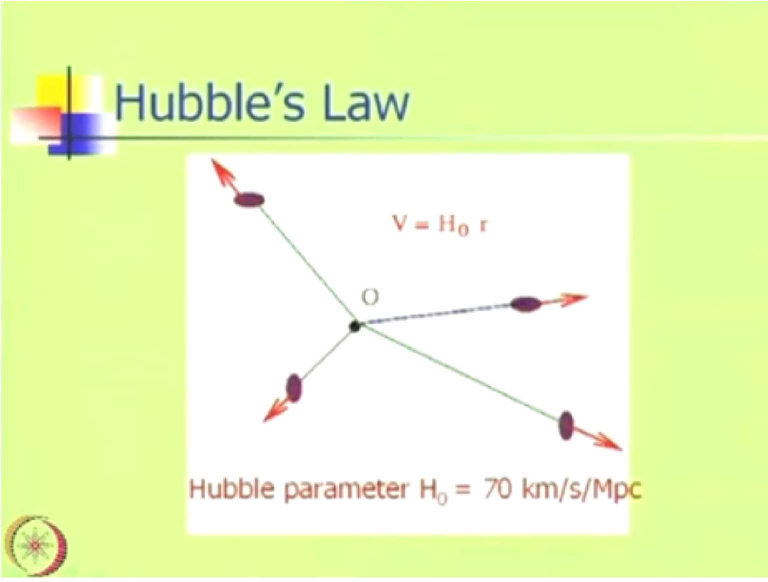

Hubble's law, which tells us that the galaxies are all moving away from us with the speed that is proportional to the distance, this is what Hubble observe. This is an observe fact, which was first noticed by Hubble. That the galaxies are moving away from us with the speed that is proportional to the distance and the constant of proportionality H0 is called the Hubble parameters.

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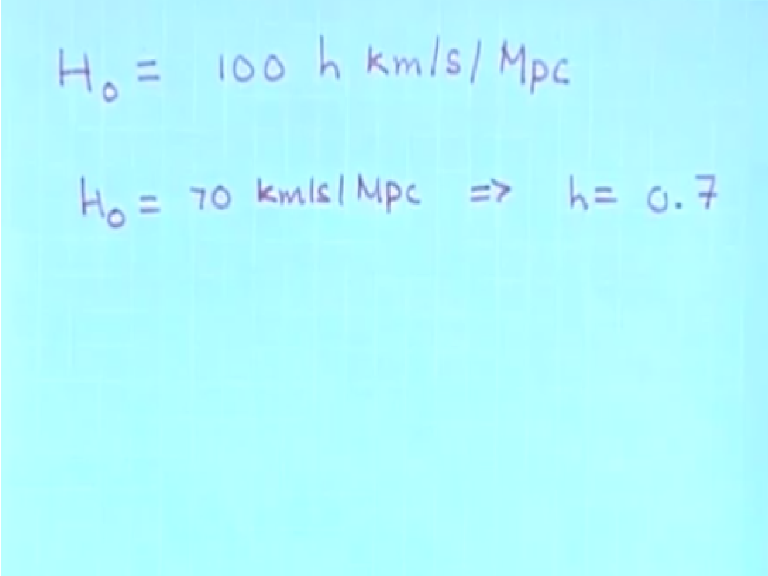

And it is useful and customary to parameterised the Hubble parameter H0 as 100 H km/s/Mpc. That is how it is usually parameterised. So current observations seem to indicate that the Hubble parameter has a value 70, which you would represent by saying that the parameter H, so H0=70. You would say that H=0.7. So this small h is a dimension less number that parameterises the Hubble parameter.

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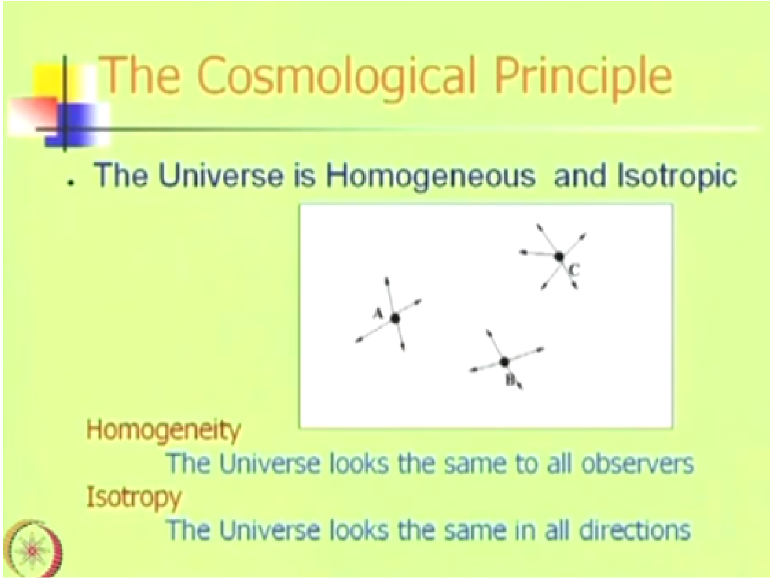

In the last lecture, we also learned about the cosmological principle that the universe is homogeneous and isotropic. There is no preferred position, there is the preferred direction in the universe.

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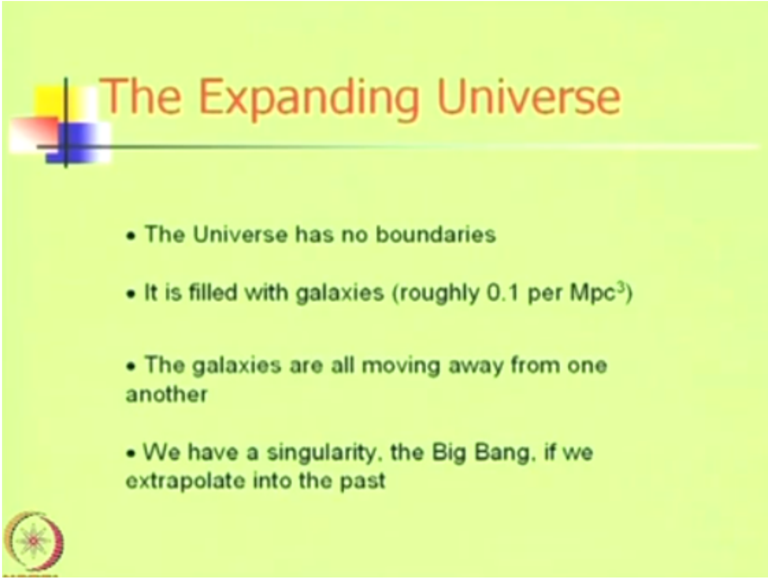

And then I told you that if you combine these 2 things. You are led to a model, which has no boundaries. So the model is in the simplest situation the universe is infinite. The universe can have no boundaries, so the simplest model is that the universe is infinite. And it is filled with galaxies.

Because the universe around us is also filled with galaxies and since the universe is homogeneous the entire universe is filled with galaxies and the galaxies are all moving away from one another. This was the basic picture. So if you combine these 2 things, the observational fact that the galaxies are all moving away from us. And the theoretical input that we expect the universe to be homogeneous and isotropic, we are led to a model where the universe has no boundaries.

It is filled with galaxies, uniformly filled with galaxies and the galaxies are all moving away from one another. Now this is the model of the expanding universe and today we are going to learn a little bit about the nature of this expanding universe. So the first thing that I should mention is that it is convenient to introduce a coordinate system called the Comoving Coordinates.

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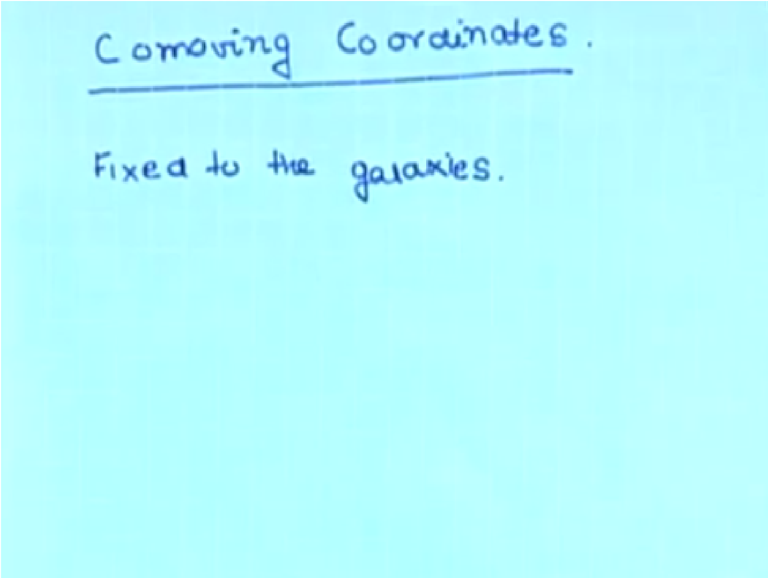

So if you are going to describe the expanding universe, it is convenient to work in a coordinate system called the Comoving Coordinates. So this is a coordinate system that is fixed to the galaxies. The galaxies I have told you are all moving away from one another, so you can think of it like this.

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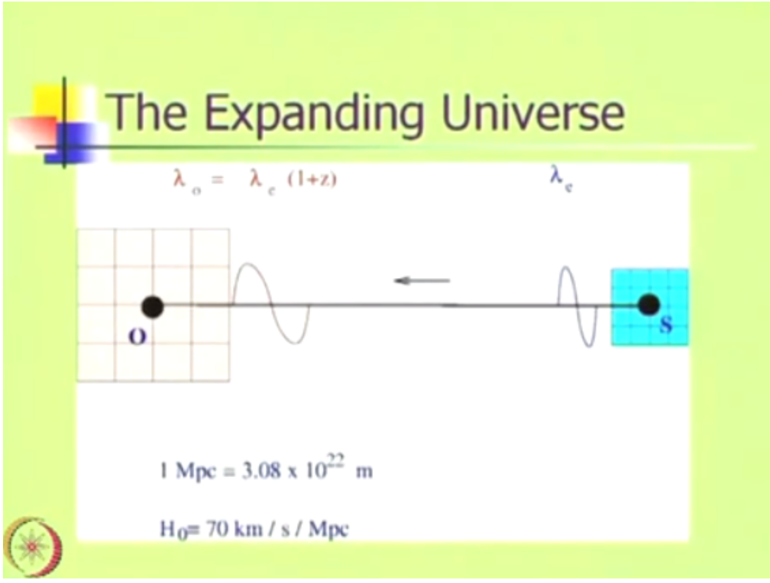

Imagine a galaxy sitting at each point of grid point in this graph paper that fills this part of the universe, it is a part of the universe. And there is a galaxy sitting at every grid point. And at these galaxies are all moving away from one another. So at a later time, the same part of the universe will look like this, a gird spacing has become larger. And the separation between the galaxies has increased. This is the basic idea of the expanding universe.

So the Comoving Coordinate system is a coordinate system that is attached to these galaxies.

So let me draw picture and explain this to you.

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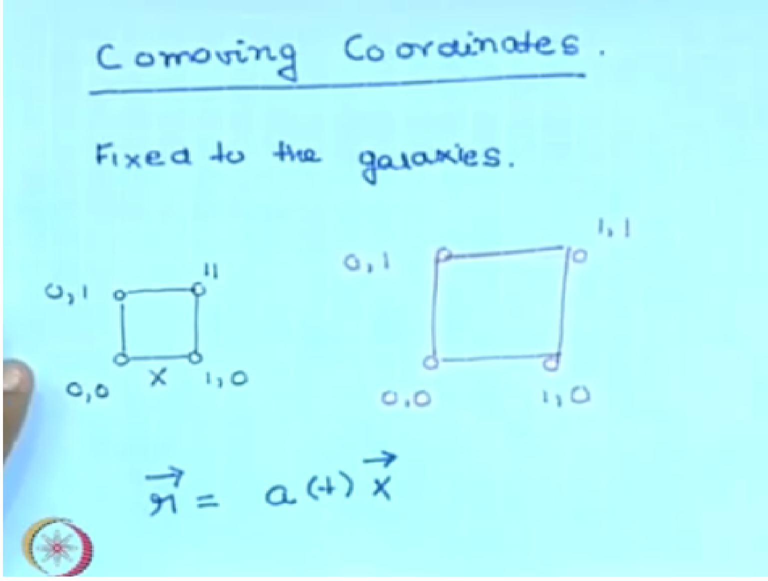

So let me draw a picture, so these are let us say this is a small part of that picture and these are 4 galaxies. And this is my Comoving Coordinate X. I denote my Comoving coordinate as X. And in this Comoving Coordinate system, this has a coordinate 0, 0, this has 1, 0, this is 0, 1 and 1, 1. Now at a later instant of time what will happen, these 4 things are the 4 galaxies over here.

They are more or less uniformly distributed, for convenience I have represented them as being equally distributed at equal intervals. Now just imagine at a later instant of time what will happen is that the separation between these galaxies has increased, so what it looks like is like this. The Comoving Coordinates of these 4 galaxies continued to remain the same. So this still remains 0, 0, this is 1, 0 and 0, 1 and 1, 1.

The Comoving Coordinates of these 4 galaxies continues to remain the same, but you can see that the physical coordinate R, so the physical coordinate R has increased right. The physical coordinate is the actual separation between these galaxies. So We write the physical separation as A of T*X okay. Here X is the, X refers to the Comoving Coordinate system. R is the actual physical coordinate, A is something called the scale factor, which actually tells you that whether that the separation is increasing.

So these are 2 galaxies, their Comoving Coordinate remain same, so this is a still at X=0, 0 X=1, 0. It continues to be that there. The X coordinate of these galaxies remain fixed as the universe expect. What increases with time is the scale factor A of T. Okay and the expansion of the universe the fact that the galaxies are moving apart has gone into this factor function A of T, which is the scale okay. So let me write these things here, so we have a 3 things.

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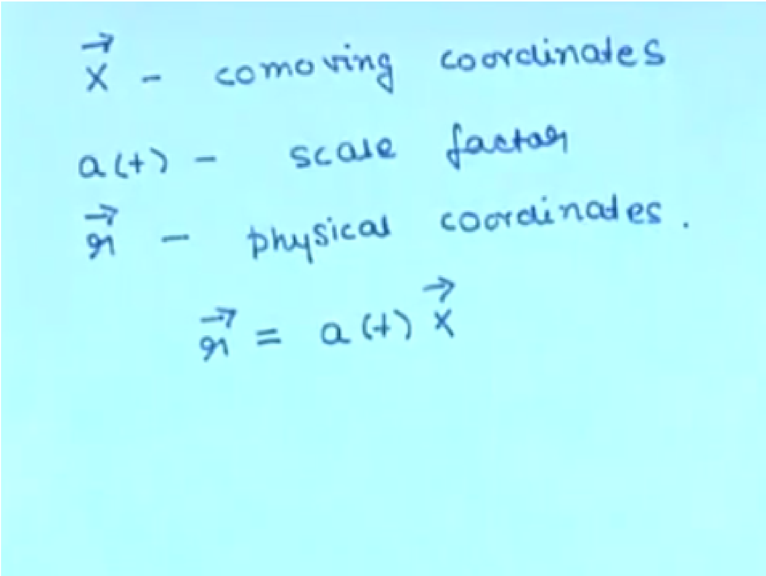

We have the Comoving Coordinates X. We have the scale factor A of T. And we have the physical coordinate R. Physical coordinator or distance. And the relation again let me as write it here is R=A D*X. So the galaxies in this coordinate system sit at fixed values of X. They are uniformly distributed in X and as the universe expands this A of T is keeps on increasing essentially okay. So this is the way you, this is a convenient coordinate system to describe the expanding universe.

(Sir cannot the universe all like this Comoving Coordinates, the scale factor is changing and that coordinate is not changing that means they are following a linear path). Well no, who says they are following a linear path. (In this we have a picture). The question is that X is fixed and A is changing and so it follows a linear path that is not so, a could be some arbitrary function of time.

We do not know what time it is right, what the function of time is. The basic thing is this is you have a universe, which is homogeneous and isotropic. Right that is what we believe and you want to also have the universe follow, you have also incorporate Hubble's law into this picture. So the way to do that is to assume that the galaxies are all moving apart from one another okay.

So if since the universe is homogenous, the galaxies are uniformly distributed okay. So this is a picture, this picture, in this picture just imagine, this is just a picture okay do not take it literally, it is a picture, so it is a kind of cartoon, so in this picture there are galaxies uniformly distributed. And this fills the entire space okay. It is isotropic because the galaxies are all moving away from each other without reference to any special direction okay.

So this is a model which is consistent with Hubble's public law and also consistent with homogeneity and isotropy. Where the entire universe is filled with galaxies like this and they all moving away from each other that is the picture okay. So now you can have a coordinate system that is fixed to the galaxies. And the fact that the intergalactic separation is increasing is there in this function A of T okay.

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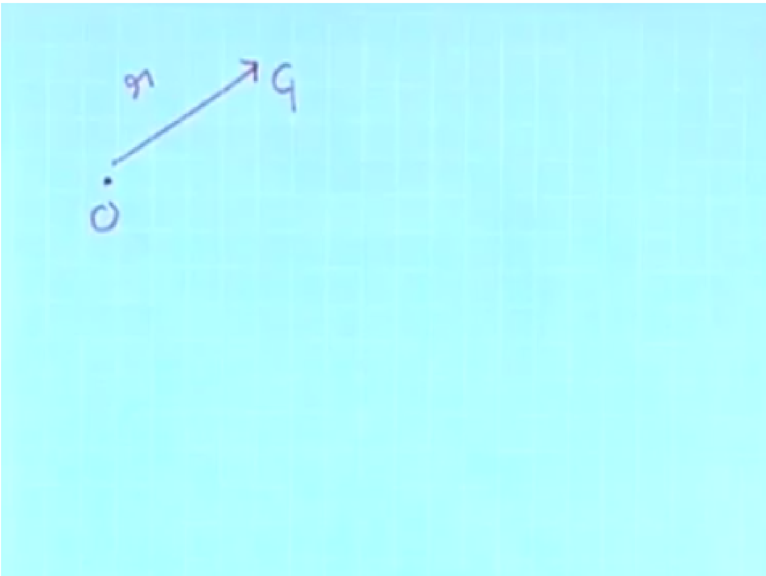

Now let us ask the question, so now we could ask a question that there is an observer sitting over here and this is a galaxy G at a distance R physical distance R a corresponding Comoving Coordinate X. What is the speed with which this observer will see this galaxy moving. To determine that, we have to just differentiate R. Now as the universe expands, the galaxies sit at fixed X coordinate, there Comoving Coordinate is fixed.

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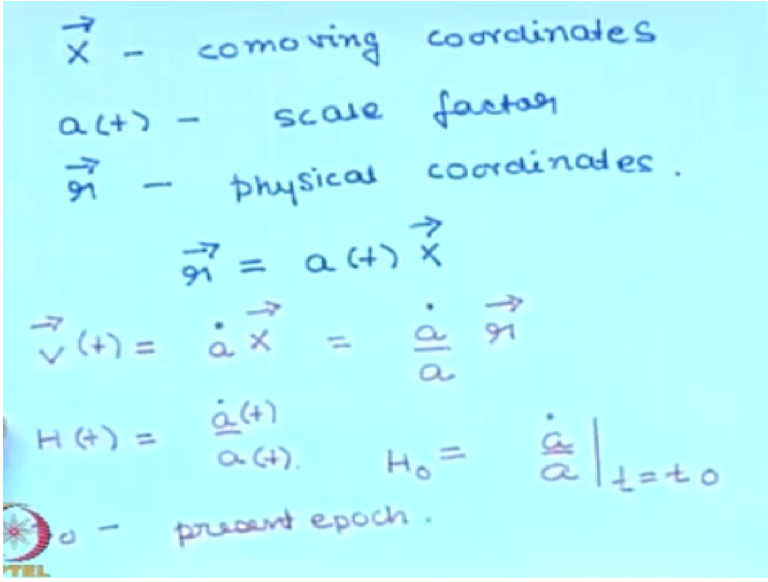

So the velocity of the galaxy is going to be A dot where A dot refers to the time derivative DADT. A dot*X, the X the Comoving Coordinate does not change with time. And this can be written as A dot/A*R right. So you see this is the Hubble's law and A is a function of time, so you can identify the Hubble parameter from this. So the Hubble parameter H which actually now here the function of time.

So you can evaluate it at any time instant, it is related to A dot, it is A dot/A both of these are functions of time okay. So in this picture, in the Comoving if you go over to the Comoving coordinate system, the entire expansion has gone into the scale factor and the Hubble Parameter is basically with the rate of change of the scale factor divided by the scale factor itself okay.

And the present value of the Hubble parameter H0 T=To A dot/A evaluated at T=T0. T0 refers to the present epoch. The value of T at present okay. So H0 is the present value of the Hubble parameter, it is the rate of the change of the scale factor with divided by A at the present epoch okay. So let me again recapitulate what we have done we have.

So I told you in the last class that we are led to a model where the entire universe is filled with galaxies and they are all moving away from one another. So now we go over to coordinate system, which is also moving expanding with the galaxy, which is fixed to the galaxies. And the fact that the separation between the galaxies is increasing is gone into this function called the scale factor.

Now we write the Hubble. We look at the velocity in terms of this model, so the Comoving coordinate of each galaxy does not change as the universe expands as the galaxies move away from one another, all that changes is the scale factor, so we are led to a relation where the velocity of the galaxy is proportional to the distance into some function of time, this function of time is A dot/A.

And comparing this with the Hubble law, we see that we can identify this with observe fact see by Hubble, we can identify this with the Hubble parameter and the present value of the Hubble parameter is the value of the derivative at the present epoch okay. So all that we have done till now is that we have just changed the coordinate system nothing else okay.

We have gone over to a coordinate system, which is fixed to the galaxies, the galaxies are homogeneously distributed, so I can imagine them to be equally spaced along different direction in the universe, so I have a coordinate system attached to that and they are all moving away so the scale factor is increasing with time. And we saw them that we write the Hubble parameter is A dot/A.

Let us now determine what this A of T is. What is the functional form of this A of T. We still do not know what that is. So far this, let us take a very simple model, the simplest model that one could adopt is that we have free expansion. There are no forces okay, so that is the simplest model, which we shall take up now.

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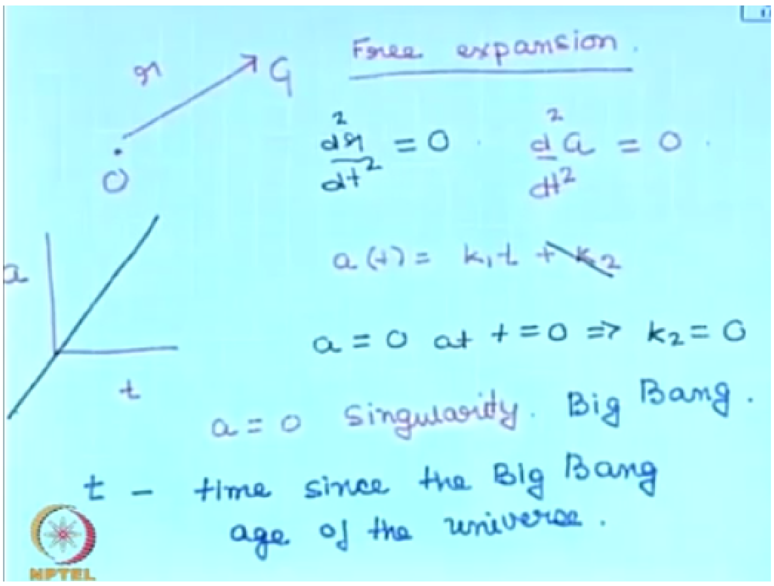

So this is the model of free expansion. So in this model there are no forces acting on this galaxy. It is moving freely okay the whole universe is expanding like a set of free particles. All the galaxies are moving away from us and their motion is just like free particles moving away from us okay. We have ignored all any effect influence of enforce. So let us write down the equation of the motion for this galaxy. I am the observer sitting here, let us write down the equation of motion for this galaxy.

The equation of motion we all know is D square R BT square=0, in the absence of any force. And the physical coordinate as been written in terms of the Comoving Coordinate and as the galaxy moves away from us the Comoving coordinate remains fixed because the coordinate system moves with the galaxy, so all that changes is the scale factor. So this essentially tells us that the second derivative of the scale factor is 0.

So we can straightaway write down the solution, the solution is A of T=K1T+K2. These K1 and K2 are constants of which you obtain constants of this. They are basically determined by the boundary conditions. They are constants of motion okay. Now this K2 is just an additive constant. You see the behaviour of A of T, let us plot A of T. So this is A, this is T. And the solution is a straight line.

That is free motion particle moves with a fixed velocity, which is basically what we have done. So it is a straight line and I am free to place the K2 decides where the straight line will cut the X access. So it decides where the scale factor because 0. And it is most convenient to choose K2 so that the scale factor becomes 0 at T=0, so A=0, we choose the scale factor, so that A=0 at T=0, which implies that K2 is 0.

So this thing is not there okay. So that the time T=0. The scale factor collapses is becomes 0. Now you see what is the significance of the scale factor becoming 0. The scale factor becoming 0 means that the separation between different galaxies. Whatever be the Comoving coordinate, they all are at 0 separation. The entire universe is filled with galaxies and different galaxies have different coordinate, Comoving coordinate so not only I have galaxy here with Comoving coordinate 2, 0.

And one beyond etc but when the scale factor becomes 0, the distance between all of these galaxies is 0 and the entire infinite universe and the separation between any 2 galaxies in the infinite universe becomes 0, this is a singularity. Right so this is singularity, so this is a singularity and the singularity was interestingly given a name Big Bang by Fred Hoyle who do not believe in this evolving universe model, he you wanted to make fun of it, but somehow the name stuck, so this is also called the Big Bang.

It was given by a person who did who did, who did not wanted to basically make fun of this theory but anyway the name stuck, so the singularity is also called the Big Bang. So is essentially a singularity so we do not know what happens over there, the density becomes infinite. And we can assume for the time being that the universe started at the beginning of the universe that is the time (()) (21:28) starts from there. So T is essentially, the time T is essentially the time that elapsed since The Big Bang.

That is the time that elapsed since the Big Bang, since the epoch when A became 0. This is also called the age of the universe, this is the cosmic age, cosmic time or the age of the universe. If you believe that the universe started at the Big Bang and the singularity then this is the age of the universe okay T, so the time T, here it is called the cosmic time. So T is also the cosmic time and it is the age of the universe okay.

Now in this model, so we have worked out this model, we have looked out everything in this model, this is a simple model where the universe is expanding freely for this model let us see what the Hubble parameter is, what is the Hubble parameter at any instant of time, so how does one calculate the Hubble parameter to calculate the Hubble parameter what one has to do is, one has to calculate A dot/A. So let us calculate A dot/A remember A is constant K1*T. So we want to calculate A dot/A.

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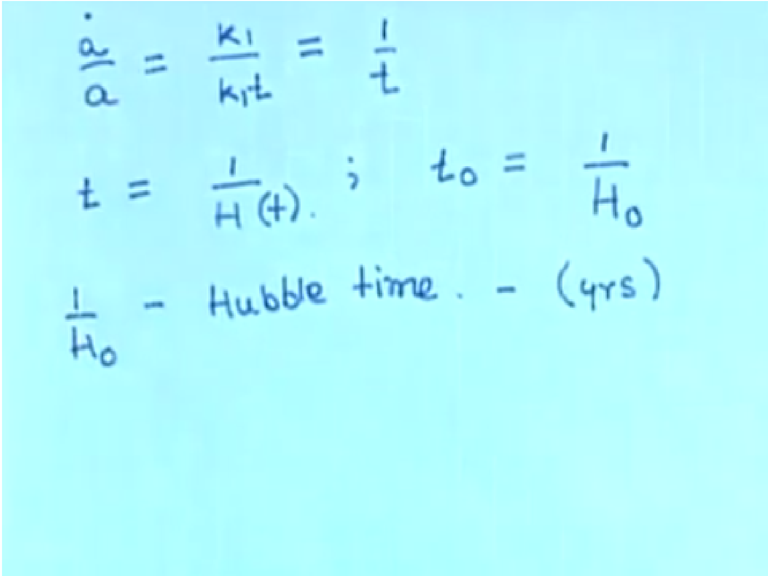

So A dot/A in this freely expanding model is K1/K1T. So we see that this is 1/time. This is the Hubble parameter at any epoch. So if you can measure the Hubble, you see that at any epoch the age of the universe is inverse of the Hubble parameter okay. And the present age so if you can determine the present the value of the Hubble parameter, you can determine the present age of the universe.

This gives us the age of the universe, 1/the Hubble parameter gives us the age of the universe. In this simple model where the universe is expanding freely okay. And just tell you that this makes sense let us look at the Hubble parameter. The Hubble parameter can be parameterised as 100 h km/sec/Mpc. What is the dimension of H0.

So in the numerator, you see we have length by time divided by length again. So the length cancels out, so the Hubble parameter has dimension inverse of time right. The Hubble parameter has dimension inverse of time. And this is called the Hubble time. So this time 1/H0 is referred to as the Hubble time.

And I leave it to you as an assignment to work out what this is in years, so the assignment is determine the Hubble time assuming that H0 100 h km/sec/Mpc determine this in years, so determine this in years. How many what is the Hubble time, what is the age of the universe if it were expanding free. But we know that the universe is not expanding freely.

So this gives you just an estimate an order of magnitude estimate of the H, it does not give you a real estimate of what you expect the age of the universe to be okay. As we shall see shortly this the assumption that the universe is freely expanding is not a correct, it is not correct so we have to also take into account forces acting on the galaxy okay, but it does give us some idea of the nature of the expansion and the Big Bang and the Hubble time scale okay.

So the Hubble time scale gives us an order of magnitude estimate of the age of the universe. Let us now incorporate the effect of gravity. And strictly speaking one should be using the general theory of relativity, but in this course we shall not use the general theory relativity we shall use simple Newtonian physics. And let we tell you that the results that we will get will be correct. So the results match okay.

So there is no difference in the results, if you were to use the general theory of relativity. With this (()) (26:51) let me get down to calculating the dynamics of the expanding universe. So we are going to basically study now the dynamics of the expanding universe. So let us right down the same thing again, so we have an observer over here.

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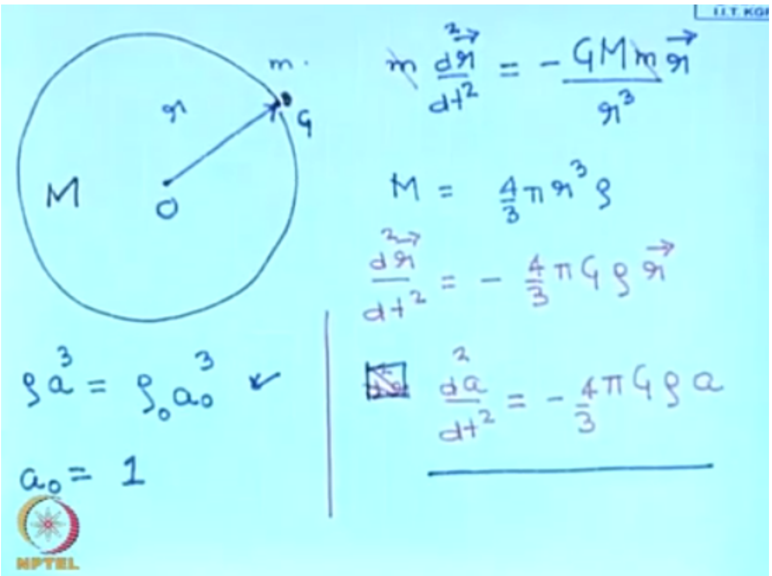

This is the observer. And there is a galaxy here G. And we would like to write down the equation of motion for this galaxy. Let us say that the galaxy has mass m. Now since the universe is homogenous and isotropic, the mass distribution, the density of the universe is everywhere the same. So you can think of it as a spherical mass distribution around the observer because it is isotropic.

So the observer around this observer, the mass has a spherically symmetric distribution. And it is well known that if you want to calculate the force on a mass, which is inside a spherically symmetric mass distribution then only the mass inside sphere will contribute to the force. The mass outside will not contribute okay. So we can write down the equation of motion of this galaxy, it is very simple, the only force here is gravity and at these large scales the only force that acts is gravity only force which is known is gravity.

The universe we assume is charge neutral on large scales so there are no net charges, so it is neutral on the average. So we can then write down the equation of motion for this galaxy, what you have to do is, we have to just consider the force due to the mass inside a sphere of radius R. So this is my galaxy and the force on this galaxy is due to the mass inside a sphere of radius R.

And so the equation of motion is quite straightforward m D square R DT square is equal to so I can write it as a vector if you wish or you could write it as a scalar, it does not matter because it the forces are all radial, the acceleration 2 is radial, this is =-G into the mass within this sphere of radius R into the mass of the galaxy divided by R square or I can write it as divided by R cube*R.

So this is my equation of motion for the galaxy. In the previous example of free expansion, we had ignored the effect of gravity so that gravity here is going to slow down the expansion of the universe that is the main thing okay. Now we can write this in terms of the density of the universe remember that the density is the same throughout because we have assumed that the universe is homogeneous.

So if I write this in terms of the density, we know that the mass, inside a sphere of radius r is 4/3 pi r cube*rho okay. So we have these 2 equations and then if we substitute this here, this r cube cancels out from the denominator and the other point to note is that the mass of the galaxy also does not figure anyway this is the equivalent's principle.

The gravitational acceleration is independent of the mass of the galaxy that cancels out from the inertial mass and the gravitational mass are the same. So this cancels out and it does not depend on the mass of this particular galaxy. So we have then the equation, further what we could do, is we could write in terms of the Comoving Coordinate system and the scale factor okay. We will do that in step, so let us write down this equation first the question, then becomes D square R, DT square=- 4/3 pi G rho*R.

So that is the equation and that we have. And this equation can further simplified and we can write, this is for a particular galaxy, now we could write it in terms of the scale factor. So let us replace the physical coordinate in terms of the scale factor into the Comoving Coordinate. The Comoving Coordinate of the galaxy is fixed, it does not change so I can take it outside the derivative and you see that it cancels from both the left and the right hand side.

So we are left to an equation for just the scale factor and the question is D square A, DT square=- 4/3 pi G rho*A. So this is the equation that governs the evolution of the scale factor. So the point here is once you solve this equation and get the scale factor, you know how the entire universe is expanding. So by just studying the equation of motion of a single galaxy, you can work out how the entire universe is expanding because the scale factor is the thing that has the encodes information about the expansion of the universe.

Once I know how (()) (33:39) I can use this to determine the motion of any galaxy. Different galaxies correspond to difficult Comoving Coordinates okay. The other point to note here is that the density of the universe does not vary from place to place. But it does vary with time. Why does it vary with time. Basically, the mass is conserved. So as the universe expands you can see as the universe expands, the mass inside any volume, physical volume has to be fixed. So the length, the volume of the universe goes up as a cube.

But the mass has to be constant, mass is density into length cube. So we can see that the density into the scale factor cube has to be a constant right. As the universe expands, the volume increases as A cube, the mass is conserved, mass within any volume, any region is conserved, mass is neither produced not destroyed. So in addition to this we also have rho A cube is constant.

As the universe expands, the density goes down as 1/A cube. And we can write this as the present, so this refers to the present. A subscript 0 always refers to the present. So this is the present density into the present value of the scale factor cube. And we will use a Comoving Coordinate system, so that it matches with the physical coordinates at present. So the value of the scale factor at present is assumed to be 1. That is how we shall work. So we shall assume that the present value of the scale factor is 1 okay.

So now we have to simultaneously solve these 2 equations, one equation being the equation over here. And the equation over here. So we have to simultaneously solve these 2 questions. So what we could do is you see there are 2 functions of time in this equation, one function of time being the scale factor, other is the density. So what we could do is we could just eliminate density from this whole thing by replacing density by the present value of the density divided by A cube okay.

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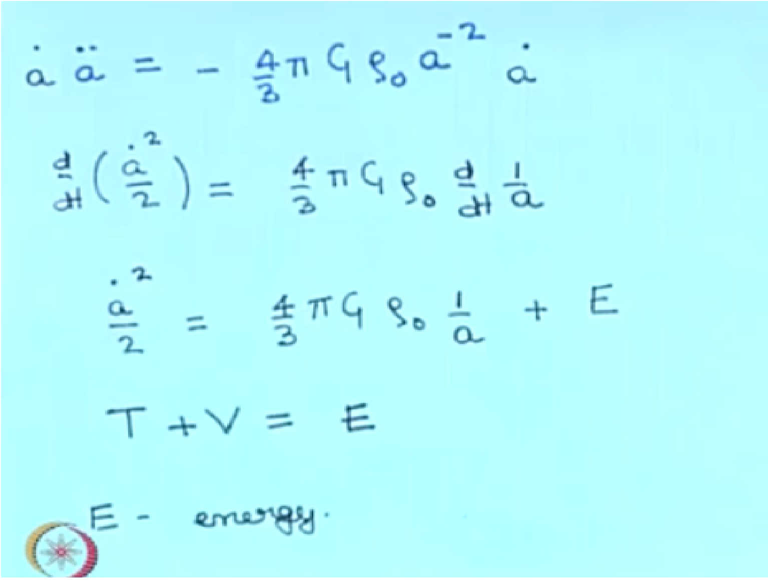

So let us do that so what this equation then becomes is A double dot=-4/3 pi G and we are replacing the density with rho 0/A cube. A 0 is 1. So this will be replaced by rho 0 divided by A cube. So I will have a 1/A square here okay. So what I get here is rho 0 A -2. And the way to solve this equation is as follows, what you do is you multiply the left hand side and the right hand side of this equation with A dot.

So this you can see is a time derivative D/DT of A dot square/2. If I differentiate A dot square, I will have 2 terms one of these terms will be A double dot, the other term will be A dot. So I would get A double dot*A dot. And there will be 2 such terms so this 2 will be cancelled out okay. And so this same equation can be written like this.

Now if you look at the right hand side, A to the power -2*A dot this can be written as 4/3 pi G rho 0*D/DT of 1/A. Because if I differentiate 1/A I will get A dot*-A to the power -2 okay. So this same equation can be written like this and now I can integrate both the left hand side and the right hand side of this integration. And in doing this in integration, I will then have to introduce if I do the integral I have to introduce a constant of integration.

So the solution to this equation is A dot square/2=4/3 pi G rho 0*1/A+ a constant of integration E. This is the solution to this equation where E is a constant of integration that we introduce when we integrate this okay. So we worked out the solution, the first integral of the equation that governs the expansion of the universe.

And you have to do one more integral in order to solve this. Now let us just spend a little time and look at this equation okay. This equation if you notice is essentially the left hand side the term on the left hand side of this equation is essentially the kinetic energy of the galaxy. Consider a galaxy at unit Comoving Coordinate. Let us consider a galaxy at the Comoving coordinate x=1.

Then its velocity will be A dot and its kinetic energy, let us say its mass is 1 so its velocity is A dot its kinetic energy is half A dot square. So if I have a galaxy at unit Comoving distance and unit mass then its velocity its kinetic energy is half A dot square. Similarly, let us look at the potential energy of that same galaxy so the potential energy of a galaxy here is going to be –GM/R.

And for a galaxy at unit Comoving Coordinate, the mass inside is 4/3 pi into the volume into the density. The density inside is rho which can be written as, so the density inside, so the mass inside here is pi 4/3. So let us just look at the mass inside this, the mass inside this is here 4/3 pi R cube rho. R cube for a unit Comoving Coordinate is basically A cube*rho. A cube*rho can be written as rho 0 okay.

So the mass inside this sphere is 4/3 pi A cube rho 0, basically 4/3 pi rho 0. And the potential energy is the mass divided by the distance – mass times the distance, so this is the potential energy okay. So this is essentially the kinetic energy T+the potential energy. This equation essentially same as this okay. And this E is a constant okay. So this E can be thought of has been the energy of the galaxy, which we are discussing, which is conserved in the motion.

Here it appears as a constant of integration when I do the first integral okay. Now the question is how to determine the value of this E from observations. I have a universe, I am living in a universe. And how will I determine to integrate this equation once more and determine the solution for the scale factor, I have to determine the value of E. The whole thing determines depends on the value of E.

How to determine the value of this E okay. To determine the value so let us discuss that. So to determine the value of E let me write down the same equation in a slightly different way let us put back the value of the density over here. So rho 0 is rho*AQ. If I put back the value of the density over here, this becomes rho*A cube/A so rho*A square.

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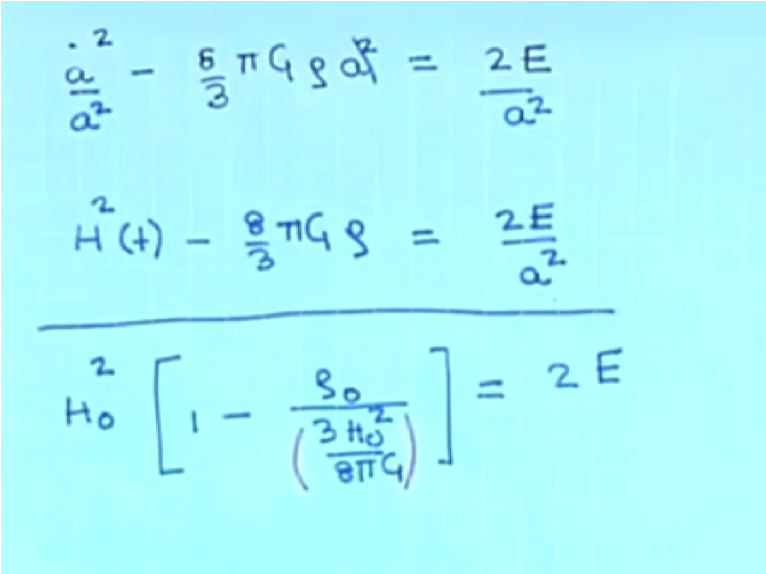

So if I put back the value of the density over here, this equation then reads A dot square -8/3 pi G rho A square=2E. So I have multiplied that equation by 2 and I have replaced rho 0 as rho*A cube. Now let me divide this equation by A square throughout. So if I divide this equation by A square throughout what I get is 1/A square here, this A square is gone and I have A square here okay.

So this is the equation that we have. Now look at this equation carefully. This equation essentially reads that the value of the Hubble parameter at any instant of time. Because A dot/A is the Hubble parameter, this is the square of the Hubble parameter= -8/3 pi G rho is equal to this constant 2E/A square okay. This is the general equation. If you apply this equation at present, so we want to determine the value of E from present day observations.

To do that what do you do then you apply this equation at present T=T0, at T=T0 this becomes H0 square so at present what we get is H0 square and I can take H0 square common over here on the left hand side then I have 1 – rho 0 divided by 3 H0 square by 8 pi G=2 E.

So if I apply this equation to the present epoch, present epoch means T=T0 then this becomes the present value of the Hubble parameter, this becomes the present density and this becomes A 0 which is 1, which we have chosen to be 1. So we have this equation.

Now the point to note here is that in this equation we have this quantity over here in the ratio of these 2 quantities which has to be compared to 1 okay. So the first thing that you can see here is that this combination 3 H0 square/8 pi G is something that is of dimension of density okay and this is called the critical density, so the present value of the critical density.

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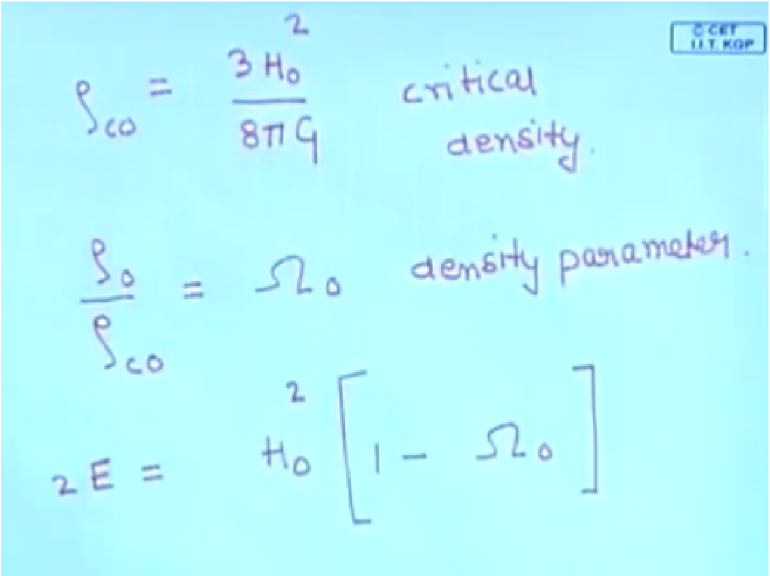

So this is rho c0. This is called a critical density. So another homework determine the value of the critical density right, you know the value of H0 and the others are all known, so determine the value of the critical density of the universe okay the critical density is a very crucial density scale in the universe. So the ratio of the, if you look at this expression, it is H0 square*1 - the ration of the actual density now rho 0, that is the actual density of the universe now. To the critical density now.

This is what determines the constant E okay, constant of the integration E. So 2E and this ratio is given a name it is very important in cosmology. There is a special name for this, so this is the ratio that occurs over here. It is denoted by omega 0. It is called the density parameter. And the equation, the constant E is now can be written in the following way the constant E okay.

So let me again remind you want these things that we have discussed are, we see that the

Hubble parameter defines a density scale in the universe. So the present value of the Hubble parameter defines the present value of the critical density, which is rho c0 and the critical density is 3 times the Hubble parameter square/8 pi G.

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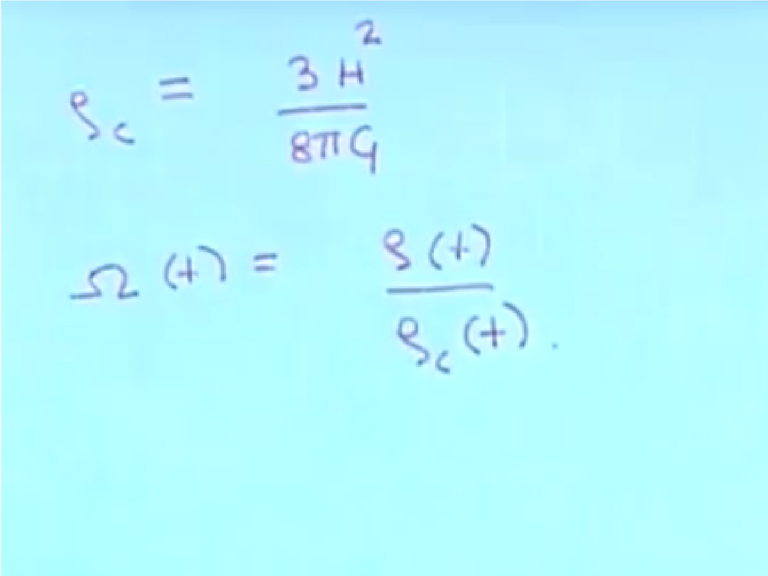

So this is at present at any instant of time you can write down the critical density rho c, as 3 H square/8 pi G. This is true at any time instant not only at the present, but at any instant of time okay. So the Hubble parameter define the density scale and the ratio of the actual density of the universe to the critical density is called the density parameter of the universe omega and its present value.

So this omega can also be defined at any instant of time. So mega at any instant of time=rho, these are all actually functions of time possibly function, because H also is the function of time. So the critical density is the function of time and density parameter also is a function of time. We are interested here in the present day values, so the present day value of the density parameter omega 0 is the value of the actual density of the universe to the critical density of the universe okay.

Now what is the specialty of this ratio density parameter the value of the density parameter decides the value of the constant of integration E.

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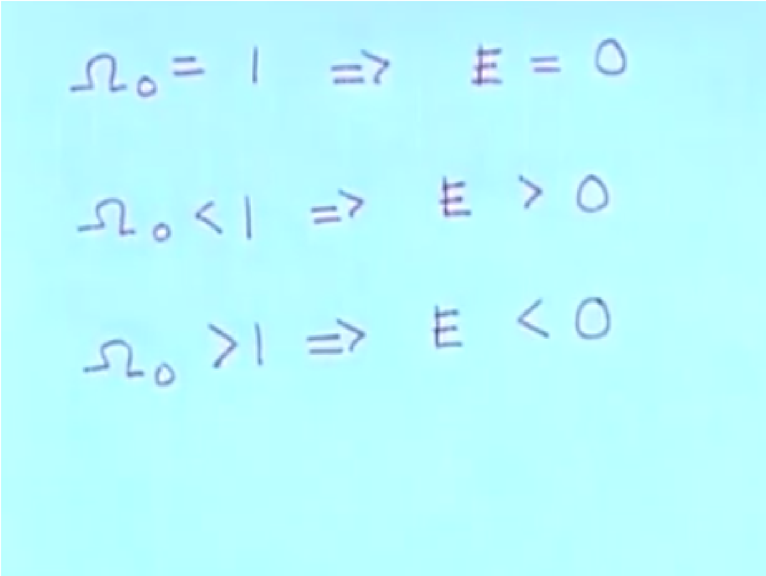

If the density parameter has value 1. This implies that the constant of integration is 0 okay. This is the critical value. It divides between 2 different kinds of cosmological models okay If omega 0 is < 1, it implies that the constant of integration is > 0, if omega 0 > 1 it implies that the constant of integration E<0 and this constant of integration E, I told you can be thought as being the energy total energy of this galaxy.

Let me end today's lecture over here. In tomorrow's lecture, we shall interpret what the significance of these and then look at some of the solutions. Yeah, it could be right, it is a function of time why not. That is the present value of the critical density H0 square, but H is the function of time, so the critical density is also function of time okay.