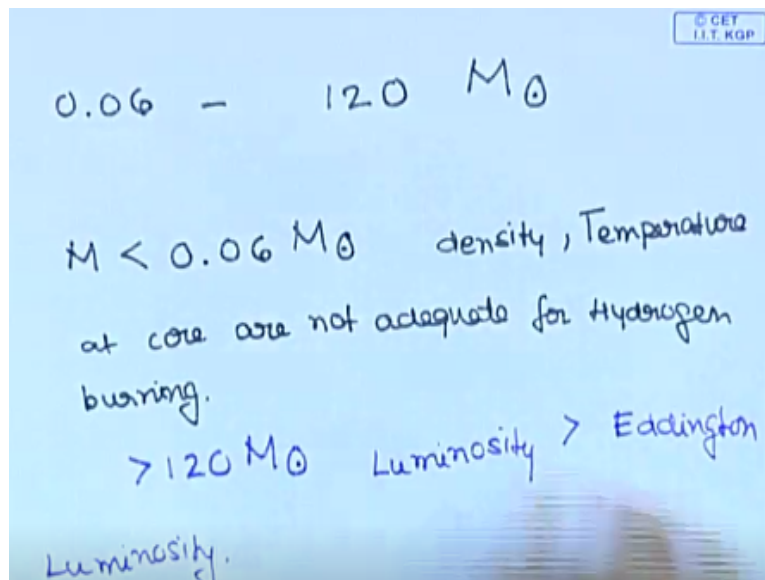


Lecture - 22
White Dwarfs and Neutron Stars

And particularly the H.R. diagram we have already discussed the equations that govern a star which is burning hydrogen and these equations yield solutions.

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For the mass range 0.06 to 120 solar mass so what happens for masses which are smaller or larger than this. Suppose I take a star whose mass is less than 0.06 solar masses and apply the equations of hydrostatic equilibrium. The mass distribution the luminosity distribution etc. and solve them it turns out that the density and temperature at the core are not adequate for hydrogen burning.

So if you have a mass smaller than this the nuclear fusion process does not start up that is the basic thing that you do not have a star. If you have a mass which is > 120 , the solar mass then what happens is that the luminosity exceeds the Eddington luminosity and let me remind you we have discussed the Eddington in a problem earlier and if the luminosity exceeds the Eddington luminosity.

The radiation pressure adequate to blow out the outer layer of the star the gravitational attraction is not sufficient to hold in balance the radiation pressure. So, if you solve those equations for a star of mass $>$ this the luminosity is so high that the outer layer of the star the star gets blown apart due to the radiation pressure and if you do not have a stable solution okay, so you find solutions in this mass range now corresponding to this mass range.

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$$0.1 - 150 R_{\odot}$$

x 150

Luminosity - sensitive to mass

$$0.011 - 8 \times 10^5 L_{\odot}$$

$$x 10^8$$

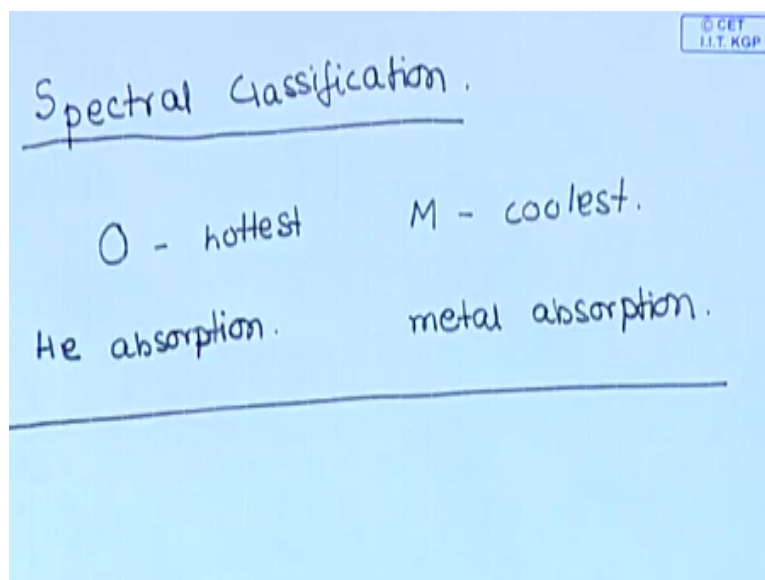
The radius of the star varies between 0.1 to 150 sorry 15 times the solar radius, okay so the point of note here is that the mass range the ratio of the largest mass for which you can have a hydrogen burning star to the smallest mass is a factor of 2000 whereas for this mass range the radius does not change its not very sensitive to the mass the radius changes only by a factor of 150, okay does not increase proportionately it is a very somewhat vague dependence.

But if you look at the luminosity so you saw the equation and determine the luminosity of the star solve the set of equations and for the questions which we had discussed and look at the luminosity of the star the luminosity range between 0.011 to 8×10^5 solar luminosity so the luminosity is extremely sensitive to the mass and you change the mass by a small amount the luminosity goes up enormously.

This is a factor of 10^8 variation in the luminosity. So, the brightest star is 10^8 times brighter than the faintest star which you can have provided the stars are burning hydrogen because these are all what we have been discussing till now are all solutions of the 4 equations which we had written down and they cover this mass range you have solutions for the mass range which I told you for this mass range.

And the corresponding range of radii is this range in the luminosity is given over here. Okay now the temperatures okay so that temperature look at these stars can be classified let us now have a look at the classification of this of the stars, so for the different masses, you get stars of different temperature surface temperature.

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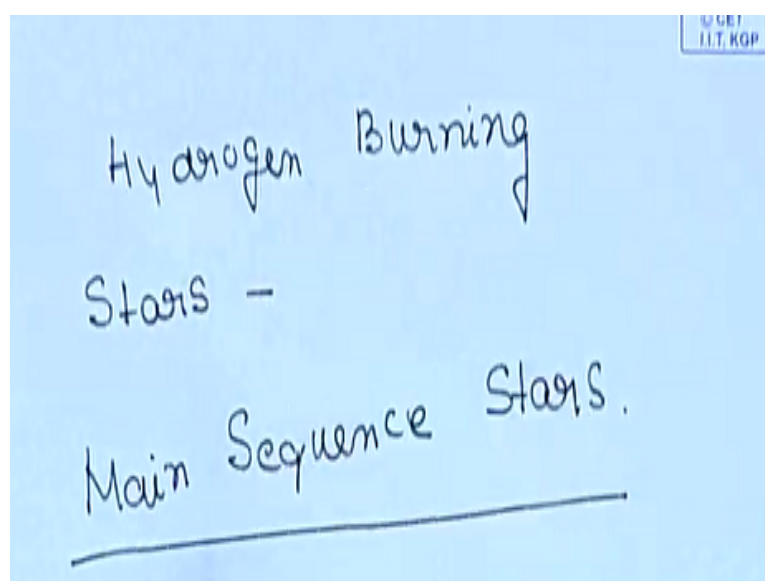


And the stars can be classified based on the temperature which is also indicated by the spectrum, so this is called Spectrum spectral classification, so the spectrum is a direct indication of the temperature and the hottest star is classified as O, the O stars are the hottest and the M stars are the coolest and observationally this manifests itself in the spectrum of the stars so in the O stars, you get helium absorption spectral.

You get helium absorption lines to observe to excite helium you have to produce ultraviolet radiation. So, these stars O stars produce copious amounts of ultraviolet radiation which is adequate to ionize helium, so you can observe helium absorption lines whereas in the M stars you get metal absorption lines, and these are the 2 extremities the whole spectral classification is as follows.

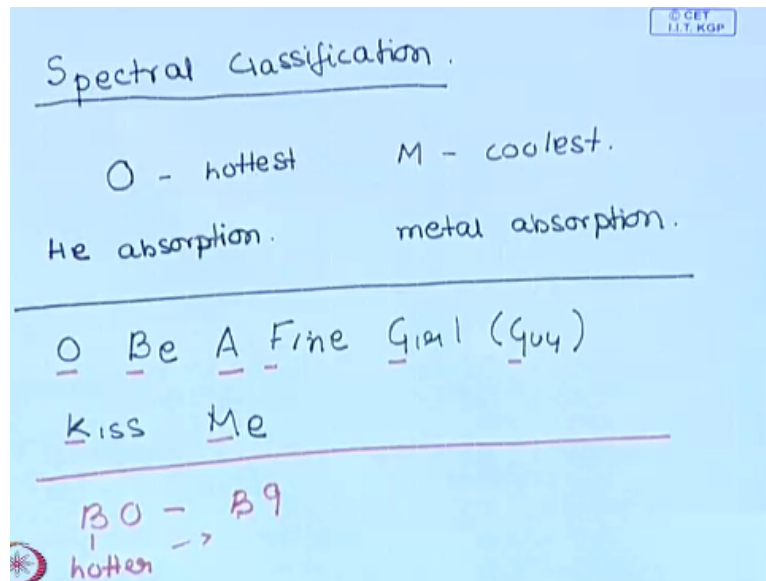
So the hydrogen burning stars okay, before I proceed any further a point which I should have mentioned earlier.

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Stars which burn hydrogen stars these are referred to as main sequence stars and why it is called the main sequence I shall explain to you shortly okay, so we shall interchangeably we are terming them as hydrogen burning stars our main sequence stars and the equations which we have developed are for these classes of this class of stars.

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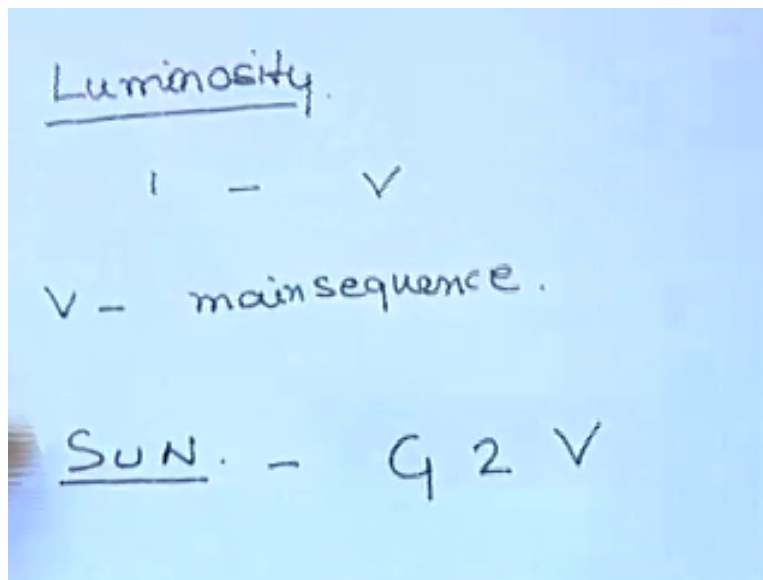


Now in general and stars can be classified into the this based on their spectrum and the spectral classification scheme is as follows O Stars are the brightest hottest stars M stars are the coolest stars and you have different spectral classes in between so let me write down the entire space spectral classification scheme it is as follows O and there is a pneumonic. So let me tell you the pneumonic also O Be focus only on the capital letters O Be A fine is a fine girl or guy whichever you prefer Kiss Me.

So this is the pneumonic by which the astronomers remember the spectral classification scheme and the different spectral classification is O B A F G or it could be this K M the O stars are the hottest and the M stars are the coolest. Okay and this is a pneumonic by which you can remember the spectral classification scheme there is a further subdivision inside this so the B stars for example, so you have you can have a classification a sub classification.

So the subclass say is B0 to B9, okay so this holds for all of them. So, there are 2 9 is a subclass for each spectral type again you have a subclass which is labeled by the number 0 to 9 where this is hottest hotter than this okay, so the temperature is essentially inferred from the spectrum straight away okay now given the same temperature it turns out that you can have classes of different luminosities also so to indicate this there is another index which indicates the luminosity.

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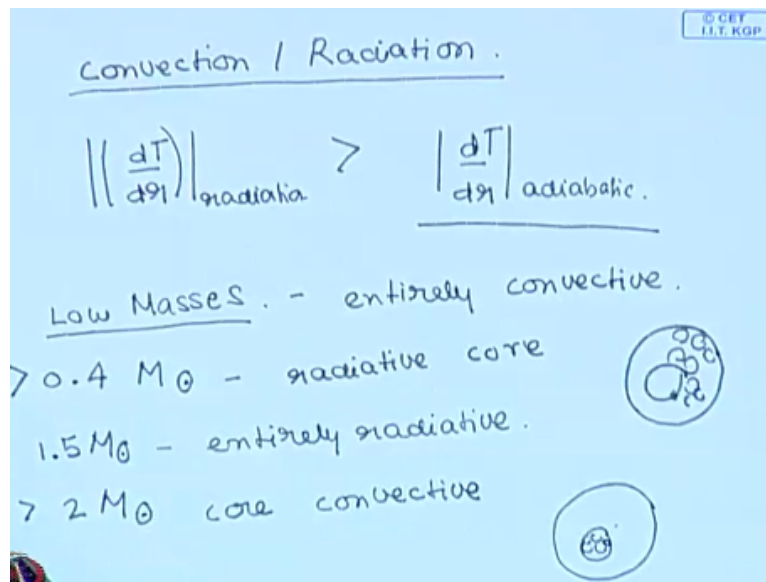


And this runs from 1 to 5 and 51 are the giants so the 1 are the most luminous stars 5 are the lesser luminous star so 5 refers to the main sequence stars 1 of us 2 giants which we shall come to okay, super giant. So this is a luminosity classification in addition to the spectral classification you also have a luminosity classification which tells you the luminosity so the sun.

For example, the sun is classified as G 2 5 because that is the classification of the sun just have a look where does the sun lie. The sun is a typical star very ordinary kind of star and it is not a very hot star neither is it a very cold star it lies somewhere in the middle of the spectral classification scheme it is G star there is a sub classification too which is somewhat towards the hotter side amongst the G stars and it is a main sequence star burning hydrogen which is indicated by 5.

So you could have a G star which is not on the main sequence also, okay so the G 2 is an indication of the temperature. This is an indication of the luminosity this is how stars are classified observationally and this is an outcome also of the equations so the equations also if you take a mass and solve the equations you will get it from the surface and properties you will get the spectral classification class and you will get luminosity also okay another outcome.

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A very interesting outcome is the convection whether the energy transport is convective or radiative on the diverted 80 again we have discussed this and I have told you that the criteria for deciding which of these will occur is that if the temperature gradient so if we have radiation transport and if that if the radiative temperature gradient if this exceeds the idiomatic transport temperature gradient then we have convection and convection is a very efficient means of transporting energy.

So, the temperature gradient then does not exceed this value, it remains at this value. So with a lower temperature gradient, you are able to transport more energy, okay. Now, so the question is when you solve these equations, the 40 questions with the extra conditions that we have discussed. It turns out that for a lot mass low masses the energy transport is entirely convective and then as you keep on increasing the mass once you cross 0.4 times the solar mass.

So once you cross this you develop a radiative core and you convection region gets smaller and smaller that is outside so the envelope is convective so the core is radiation radiative then and is transported radiatively in the core and the envelope it looks like this so you have radiative transport here and then you have convection in the outer parts just drawn a little part of it then as you keep on increasing the mass.

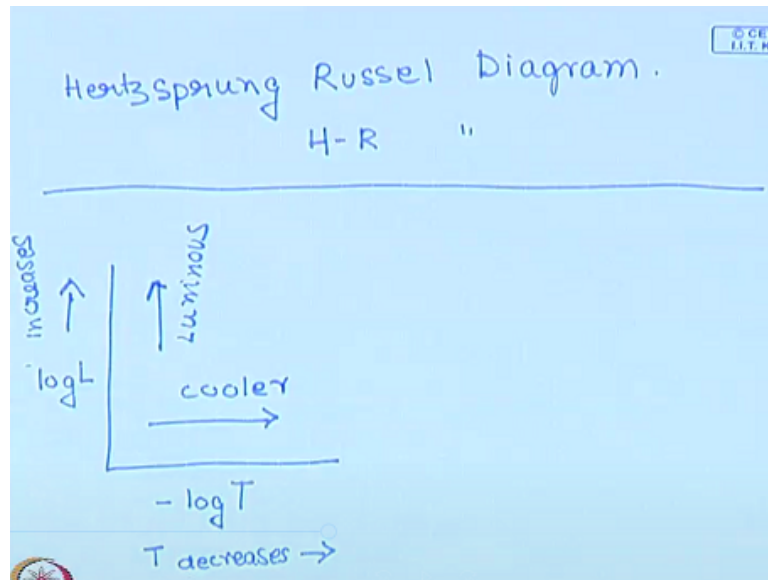
If you look at stars of more and more mass the radiative core keeps on increasing. The sun we have seen that 70% from the inside the energies transported radiatively and 30% out of 30% energy transport is conducted okay when you reach 1.5 solar masses the energy transport is entirely radiated so here the energy transport is entirely radiative.

And then when you cross 2 solar masses the core becomes convective the core becomes convective once you cross 2 solar masses the core becomes convective and for more masses

more than this the core becomes convective. So here what happens is the core becomes convective here, the convection is transported radiatively outside for masses more than this so for masses more than this the region where the nuclear fusion is going on.

You also have convection taking place in the same place where the energy is generated okay, so this gives you a picture of the solution and whether energy is transported convectively or radiatively. now let me introduce the HR diagram this is called the Herzsprung-Russell diagram.

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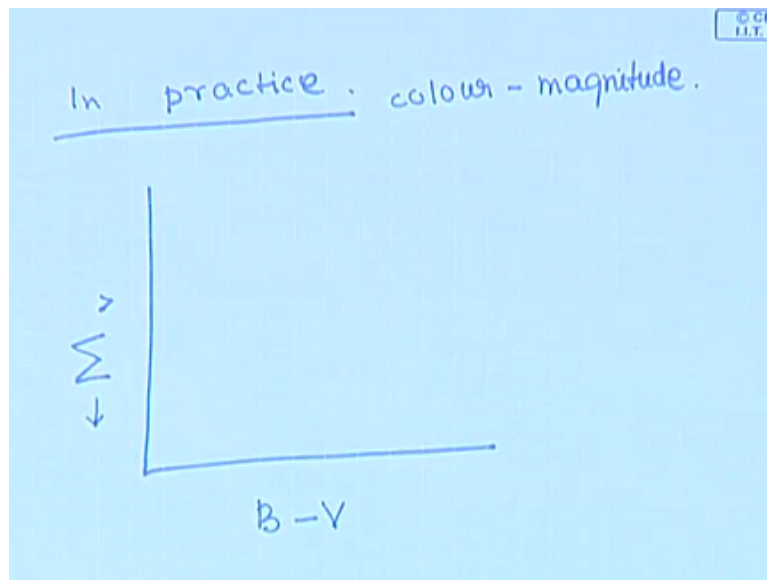


Russel diagram referred to as the H.R diagram the HR diagram is one of the most useful methods for looking at stars for understanding stars and looking at stars this in this HR diagram what is plotted in principle is as follows so in principle what is plotted is as follows and you plot a log of the luminosity the luminosity increasing this way versus the log – log temperature so the temperature decreases in this direction it increases this way.

So, the luminosity let me just put it here it increases this way. The L. L. Increases in this direction T. Decreases in this direction because it is – log T so a more the more luminous an object the upper or higher up in this graph in this diagram it will appear and the cooler an object is the more to the right it will shift okay so let me again make this point clear.

And this side is cooler, and this side is more luminous that is how to interpret the study in practice you cannot observationally determine these 2 things. In practice from the observational point of view what is plotted. Let me show that here so from the observational point of view let me put it in a different diagram.

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So, from in practice what you plot is you plot the absolute magnitude in some band let us say in the V band and it is decreasing in this direction, so it is increasing this way right more the more the luminosity the more negative the absolute magnitude will become which is $-2.5 \log$ fluxes okay, so it is decreasing in this direction the magnitude is decreasing or the luminosity is increasing so this these the sense over here remains intact.

Okay so the luminosity is observationally quantified to the absolute magnitude of the logarithm of the luminosity and the temperature as I have told you is observationally quantified to the color, so you could use the B-V color for example okay could use U-B color and these are the 2 things that are plotted along the 2 different axis so let us say B-V so this is a color magnitude diagram it is a magnitude color like the magnitude diagram so bear in mind the color.

Let us look at the color, if the larger the value of the color, it means, if this value is larger, it means that the blue is fainter than the visual band visual band is at a lower frequency higher larger wave length, so a larger wave length having larger radiation tells us that it is cooler so the more I go in this direction the larger the value of the color the cooler the stars okay the more I go in this direction the less the more luminous the stars.

Now a point which I should make that early on when this diagram this diagram was introduced in the beginning of the in the end of the 19th century or beginning of the 20th century when this diagram was introduced this people had no method of determining distances to stars except for parallax even now there are no diet very few direct methods but the parallaxes were measured only for very nearby stars.

So there was no way you could determine the absolute magnitude what you had to do was. So you do not know what this number is for a star you do not know the luminosity but what you

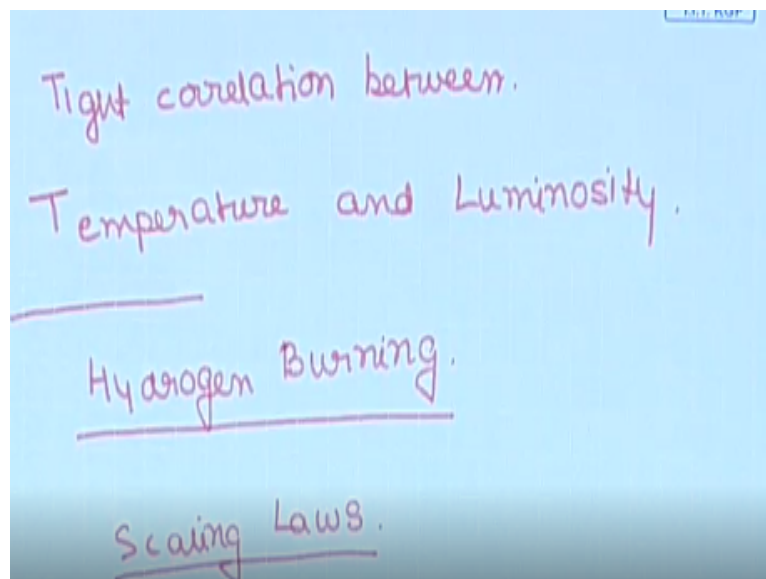
could do was you could look at stars which appear in the cluster so it could be a globular cluster or it could be an open cluster a cluster but a cluster and so that the distance to the cluster is approximately the same from us.

So if you look at a collection of stars to which the distance is approximately the same then the distance introduces a factor which is the same for all the stars. Okay so up to a multiplicative constant we know the luminosity the multiplicative constant distance affects all the stars equally or if you look think of it in terms of the absolute magnitude what you could measure was the apparent magnitude.

But the apparent magnitude differs from the absolute magnitude/5 log the distance right and the distance was the same for all the stars we are looking at them in a cluster or in or in some kind of a cluster in some kind of a group which is physically associated with one another. Okay so this is what people looked at and what people found was that the stars the bulk of the stars lie along a band which runs diagonally across this diagram.

So the bulk of the stars it was found lie in a band which runs diagonally roughly like this across the diagram okay and if you had a different distance all that would do is it would shift this and diating there because the distance would shift the sun. So, the curve would go up and down for different clusters different groups of stars. But it is found that for all of them the bulk of the stars lie on band like this and this is called the main sequence okay. Now, later so this explain so what it tells us.

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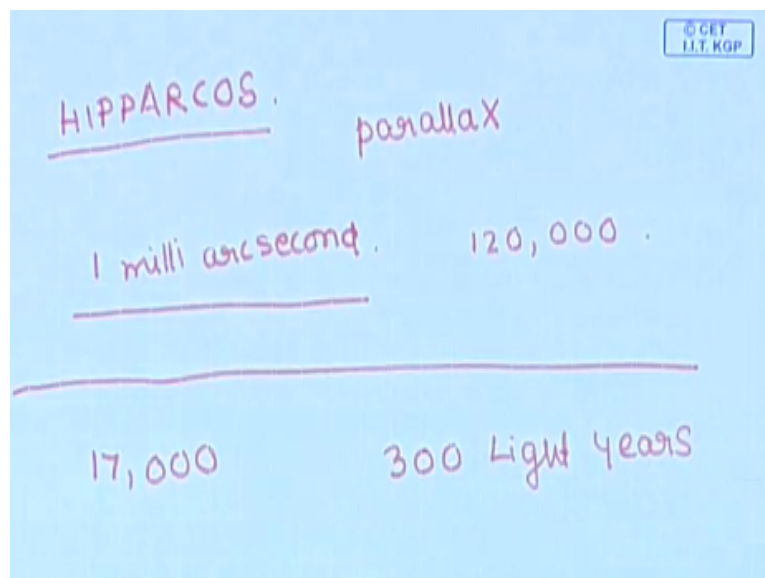
It tells us that there is a tight correlation it tells us that there is a tight correlation between the temperature and luminosity of a star and understanding this was a very big issue what gives rise to this tight correlation between the temperature and luminosity of a star and bulk of the

stars turned out line of band like this. There are stars elsewhere also okay, but this was a very dominant feature.

And this was called the main sequence it was later realized that these are the stars which are burning hydrogen. These are the hydrogen burning stars which lie on the main sequence all the hydrogen burning stars in the HR diagram they lie on a band like this called the main sequence okay and it was later realized also that this there are we have discussed the equations there are fundamentally questions governing this that has been discussed.

But it is not clear why they should lie on a straight line like this we shall see later that there are simple scaling laws which give us some idea why the hydrogen burning stars should lie on a region like that okay this is something that we shall discuss later.

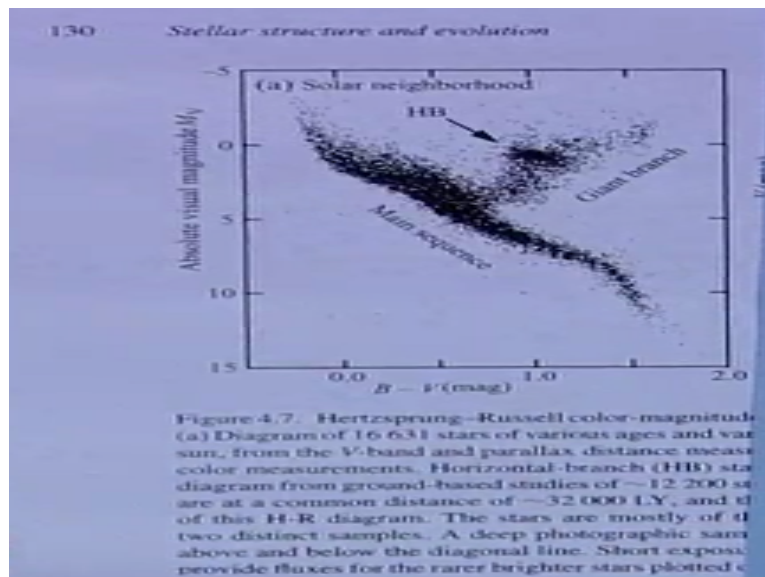
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But things have changed since then and there was a satellite called HIPPARCOS this satellite was launched with the explicit aim of measuring parallaxes okay to nearby stars' parallax distances to nearby stars and it can this satellite could has measured parallaxes to an accuracy of one million arc second okay and it has measured legs distances to around one hundred thousand stars in the solar neighborhood.

So let me show you a HR diagram made with the HIPPARCOS data it has around 17000 within 300 light years.

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And focus on this diagram if we just draw your attention to this diagram this HR diagram so this is the HR diagram which has been measured by the heat of the stars, so this shows you all the stars that have been whose paralyze distances have been measured using Hipparcos it has got 17,000 stars that cannot all the star 17,000 stars within 300 light years the luminosities are what have been measured.

So, this is the absolute magnitude of what has been measured using the Hipparcos data the HIPPARCOS satellite measured the distances okay which goes into determining the absolute magnitude apparent this is the difference of the apparent magnitudes in the B band and the V band and this has been measured from ground based observations okay Hipparcos those basically give the distances.

So notice that the bulk of the stars lie on a band that runs from left to right across the HR diagram and this is the main sequence this so all these are all stars which burn hydrogen and these are the low temperature cool stars relatively cool stars as you move to the left you go to hotter and hotter stars bluer that you go blue and you basically as you move left the color gets less so you are going towards bluer stars which are hotter.

And you find that the hotter stars are also more luminous and the hottest stars that you have the most in this. These are all the stars in the solar neighborhood within 300 hundred is that all stars within the solar neighborhood within 300 light years there shows you the stars on the main sequence okay now and these are not the only stars that you have there other stars also, okay Before we move on, let me show you another HR diagram.

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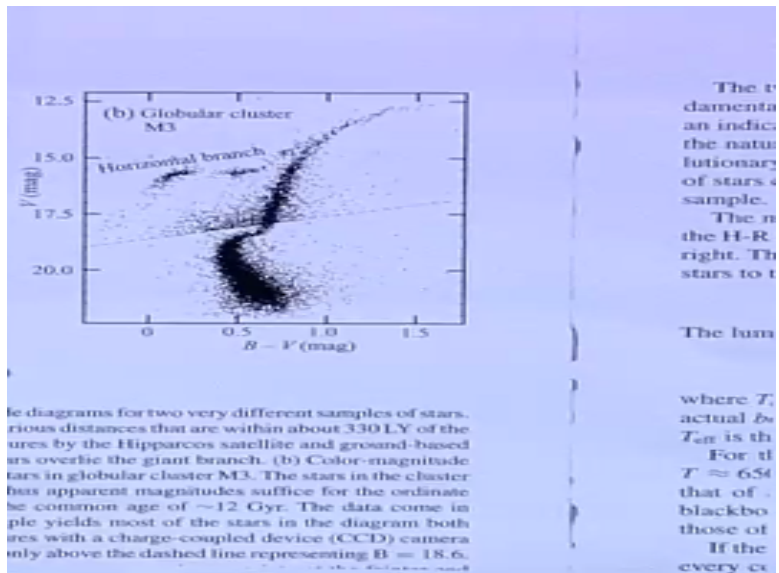


Figure 4.7. Hertzprung-Russell color-magnitude diagrams for two very different samples of stars. (a) Diagram of 16 631 stars of various ages and various distances that are within about 330 LY of the sun, from the V-band and parallax distance measures by the Hipparcos satellite and ground-based color measurements. Horizontal-branch (HB) stars overlie the giant branch. (b) Color-magnitude diagram from ground-based studies of ~12 200 stars in globular cluster M3. The stars in the cluster are at a common distance of ~32 000 LY, and thus apparent magnitudes suffice for the ordinate of this H-R diagram. The stars are mostly of the common age of ~12 Gyr. The data come in two distinct samples. A deep photographic sample yields most of the stars in the diagram both above and below the diagonal line. Short exposures with a charge-coupled device (CCD) camera yields the rarer brighter stars plotted only above the dashed line representing $B = 18.6$.

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This shows you another HR diagram here the HR diagram is off a globular cluster a globular cluster is a collection of stars is a distinct collection of stars and now here the distances to the individual stars are not known but it does not matter because they are all of those roughly the same distance. So I can interpret the apparent magnitude itself as an indicator of the luminosity the distance has the same effect on all these stars.

Now here again notice that you have the main sequence over here this is the main sequence. But it extends to a much smaller distance along the HR diagram and notice that there are deviations.

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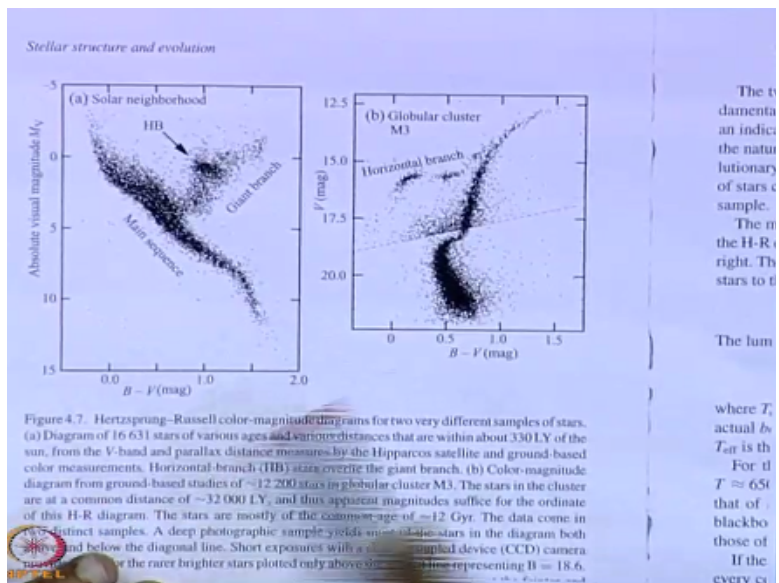


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So, by as compared these 2, so right over here the main sequence kind of stops and you find stars which are off the main sequence to this turning off from the main sequence is a very important is a very important effect and we shall discuss this when we did discuss the determination of this is age cosmological ages the age of the universe this is something that is very important for that okay we shall come back to this point later.

When we discussed the evolution of stars okay now let us go back to our discussion of stars so I have told you how stars can be classified using represented and classify using the HR diagram okay.

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L R

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4$$

SUN 5800 K. - $T_{\text{eff}} < T$.

T 6500 K. $\tau = 1$.

T_{eff} - other stars also.

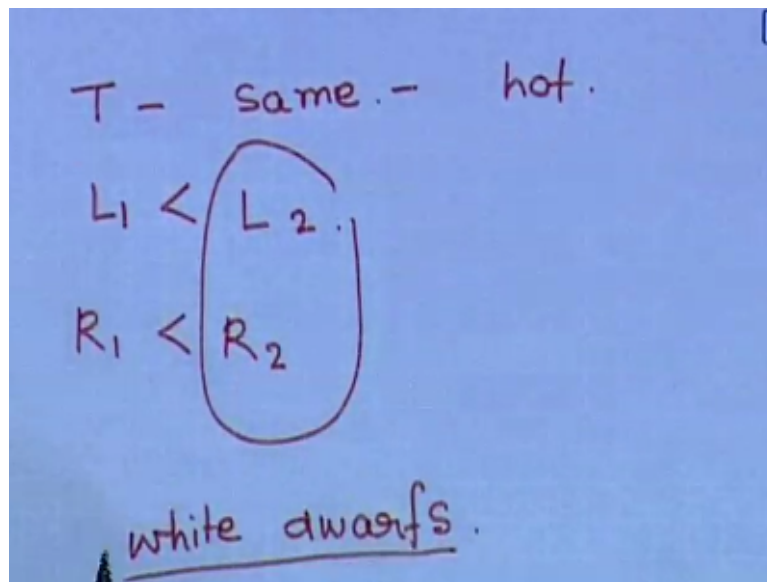
Now given for any star given the luminosity and the radius you can define the effective temperature and for given a star if you know its luminosity and if you know its radius you can define what is called the effective temperature and we have already discussed this for the sun for the sun the effective temperature. comes out to be 5,800 Kelvin okay now if you compare this with the actual temperature of the sun at the photosphere.

So for the sun, this is the effective temperature. We have done this exercise now for the sun if you solve the model the solar the Steller model which we have discussed and if you determine the actual temperature at the photosphere discovers out to be 6500 kelvin photospheres remember is the region where the optical depth becomes one that is where the radiation originates from that is the part of the sun that you actually see the temperature.

There is higher than the effective temperature and this happens because some of the radiation it passes through cooler material it gets absorbed in the process typically the effective temperature is lower than the temperature at the surface that you are looking at for stars okay, but it gives you some idea of the temperature of the surface so one could define this effective temperature for other stars also.

And this is a very useful thing suppose I find a star, let me discuss in a different sheet, suppose I find a star.

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I find 2 stars at the same temperature but 2 stars at the same temperature one of them L1 has a luminosity which is much less than the luminosity of the second star then you would conclude that the radius of the second star is much smaller than the radius of the first sorry the radius of the first star is much smaller than the radius of the second star.

Okay so if I have stars and I plot them on the main sequence the main sequence along the X Axis I have the temperature along the Y Axis I have the luminosity so I just tried to visualize so this is the star on the main sequence let us see and I have another star which has the same temperature as the main sequence. Let us say a very hot star it is quite very hot, a very hot star in which part of the diagram would it appear it would appear to the left.

So hot star would appear somewhere over here to the left now there is one star which lies on the main sequence so there is a main sequence star over here and there is another star which is much fainter so what would you conclude you would conclude that the second star which is much fainter has a much smaller radius okay. So it does turn out that you do get some stars like this over here in this part of the HR diagram of the main sequence.

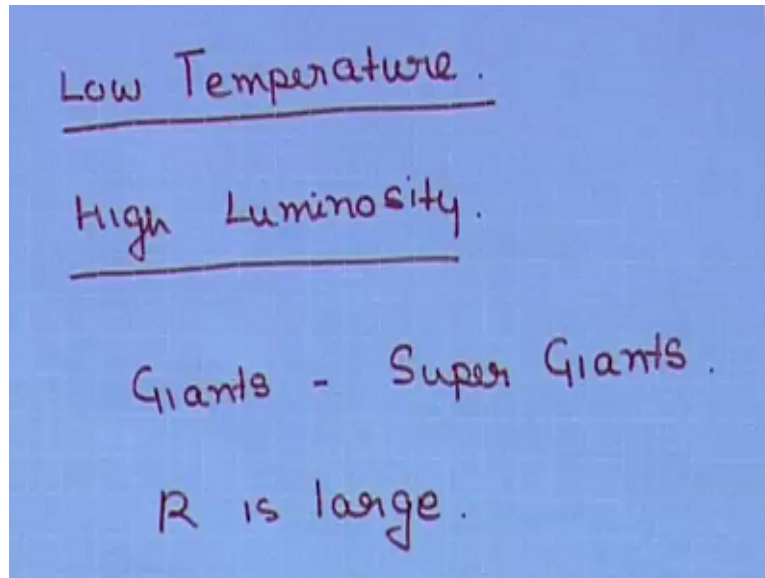
So these stars are called White Dwarfs. So now, you start to appreciate the utility of this H-R diagram you take any star and put it on the H-R diagram if you know it is a distance or if you know it is in a group then you compare it with the other stars you do it just by placing it on the H.R diagram okay and if it lies somewhere and much below the main sequence but it is hot then it is white dwarf.

It is an extremely hot star but of extremely compact size such stars are not hydrogen burning stars they are what are called white dogs. There are different kinds of stars not like the ones that we have been discussing they do not burn hydrogen at the center to start right all the stars

that burn hydrogen are over here similarly let us look at the stars over here. These stars are the hot or they are cold they are quite cold.

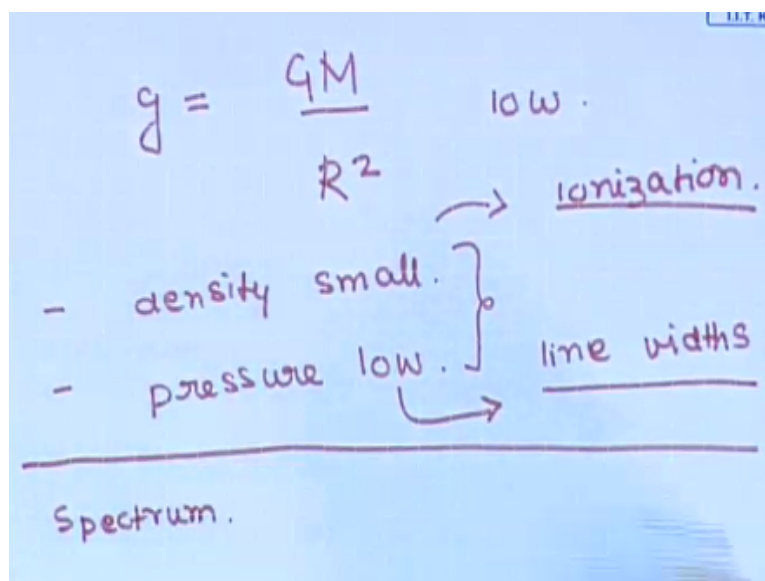
But, they have extremely high luminosities these stars so you have a kind of star which is cold.

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So, these are cool, so you have stars which are at relatively low temperature but high luminosity these stars are what are called giants and super giants okay giants and super giants, so these are stars which are extremely not very hot stars, but they are extremely luminous, so the radius of the temperature is low, but the radius is large basically are is large these again are not main sequence stars okay these giant stars they have a large radius.

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And because of that their surface acceleration surface acceleration or surface gravity which is $G \cdot \text{mass of the star} / R^2$ this is relatively low, so these are not extremely massive stars they are stars of optical mass which have to be going like the sun or maybe somewhere

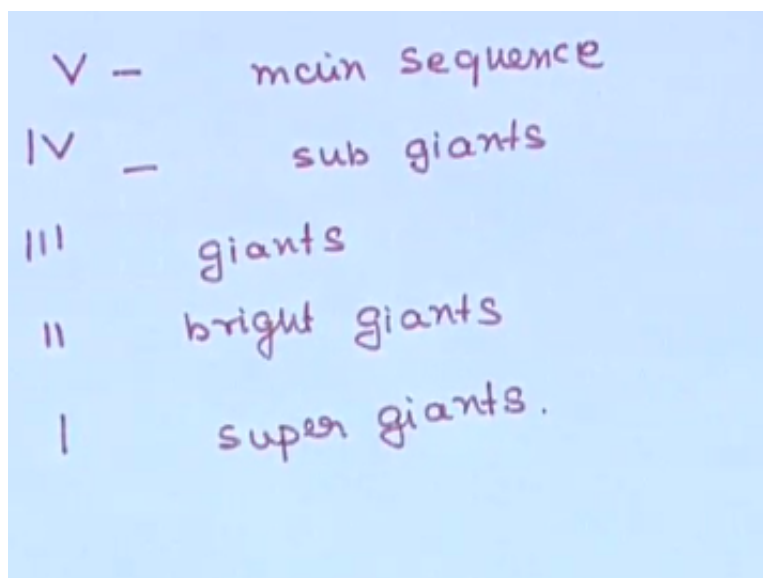
around there which have the radius have increased so the surface acceleration has fallen and as a consequence of that what happens is that the density of the acceleration is low.

And as a consequence of that the density 2 things happen the density is small at the surface and this is obvious because the R has gone up the mass is fix and also the surface pressure is low so at the surface the density and pressure are both small and this manifest itself in who is in the spectrum of the star, so the first manifestation of the density is that the density remember is there Saha ionic equation of any species and, so it affects the ionization of the species.

That is one effect and so the density comes into the Saha ionization equation and it affects the ionization state, so this affects the ionization state that if you go back to your Saha ionization equation you will see that it depends the ionization state depends on the density and the change in the density affects the ionization states the pressure difference the smaller pressure manifests itself in the line width of the lines so if you look at the spectral lines.

From the surface the line width reflects the pressure and the more the pressure you have the broader lines due to pressure broadening so for these giants you will have narrower lines okay so the line width get narrower because pressure is low and from the sky looking at the spectrum it is possible to classify a star as a giant etc., okay. So there is a classification scheme which I had already mentioned let me tell you that.

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So classes of the class 5 are the main sequence class 4 these are the sub giants class 3 these are the giants class 2 these are the bright giants and one these are the super giants. So the main sequence are the ones which burn hydrogen. So let us go back to our HR diagram and take a look. So these are the stars that burn hydrogen here if the local neighborhood. You also find the giant branch over here.

There is another branch over here called the horizontal branch which is kind of overlapping with a giant branch here okay which is more clearly visible over here. So if you look at the globular cluster this is a particular globular cluster M3 okay so the globular cluster is called M Messier on the Messier catalog this is the main sequence here you have the giant branching these are all giant stars off the main sequence.

And here you have these stars which lie on what is called the horizontal branch they are the horizontal branch. Now if you take a look at this you will see that there is a very interesting thing that the brighter part of the main sequence is missing the brighter part of the main sequence is missing. Whereas you have this very extended giants okay this indicates that there is some relation seems to indicate that there is some relation between these.

And if you look at different globular clusters you will then see that the turn off occurs at different places okay so the main sequence is where these stars first lead a large part of their life when they are formed when stars are first form they are formed in the main sequence they lead a large part of their life burning hydrogen once they finish burning their hydrogen. Then they depart from the main sequence.

And you then they become these other kinds of stars the giant super giants they go into the horizontal branch some of them become wide over here etc. and that is a very interesting topic the life story of evolution of stars and that is something that we shall take up in the coming class in today's lecture what we learned is that you have solutions for stable solutions and of the equations that govern a star for only a finite mass range below that mass the stars do not the pressure the core.

The temperature and density at the core are not adequate for fusion above the mass range the Eddington luminosity is exceeded the luminosity exceeds the Eddington luminosity and the star blows up away parts of the stars the luminosity is so high so there is a finite mass range over the mass range the range of radius is not very large but the luminosities vary enormously. So if I go from here to here, the luminosities have changed by 10 to the power 8 or something like that okay.

The radius has not changed very much. The temperature also has not changed by an enormous amount the colors do not change all that very much okay they do change the temperature does change and may be of order or somewhere of that order but not like the luminosity the luminosity is the thing that really increases drastically and that is quite clear it should be so.

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L R

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4$$

SUN 5800 K. - $T_{\text{eff}} < T.$

T 6500 K. $\tau = 1.$

T_{eff} - other stars also.

Because the luminosity scales T to the power 4 temperature increase and by sum amount will be a much more increase in the luminosity so an combined increase in the radius and temperature will give rise to a large increase in luminosity okay and then I discussed how from the observational point of view how the stars are classified you have a spectral classification and you can get real understanding of the stars.

If you look trace them on the HR diagram. So let me stop here we shall resume on this tomorrow in the next class.