Astrophysics & Cosmology
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Lecture - 19
Stellar Physics - IV

The structure of a star and I have told you that there are 4 equations which need to be considered.

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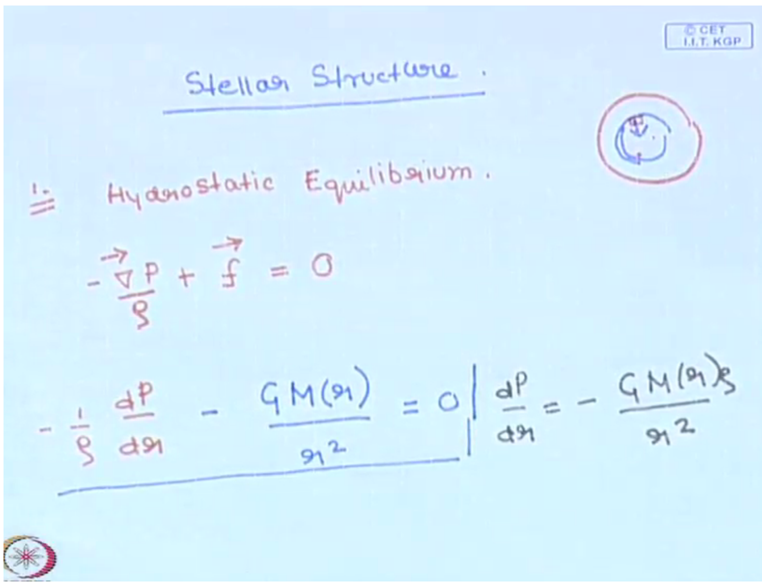

And let me remind you that the first equation is the equation of hydrostatic equilibrium which we discussed in the last class.

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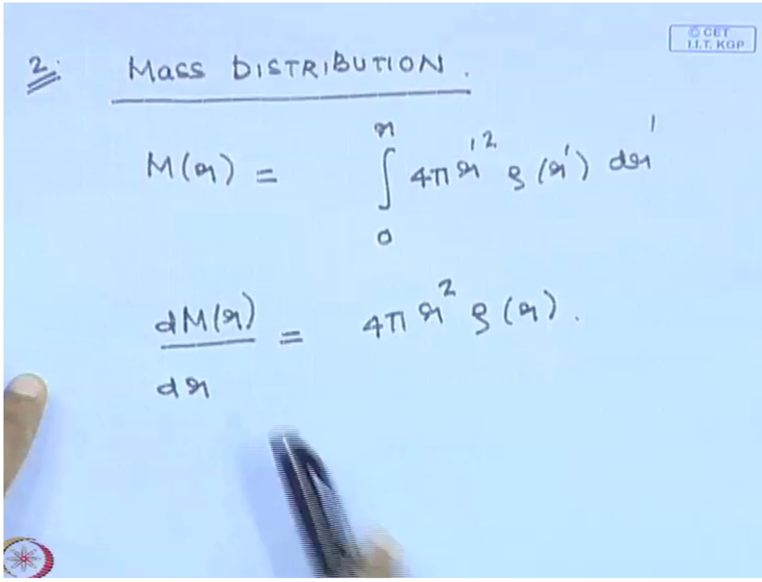

The second equation tell us how the luminosity. so the first equation tells us how the pressure changes as we go along the radius of the star, the second equation tells us how the luminosity of the radiation changes as we go along the radius of the star.

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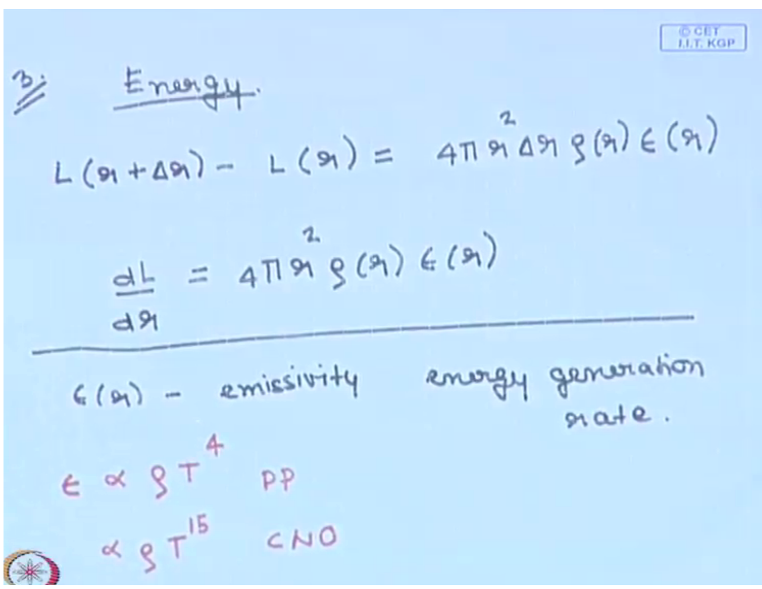

The third equation sorry this was the third equation that we had considered, the second equation tells us how the mass changes as we go along the radius of the sun.

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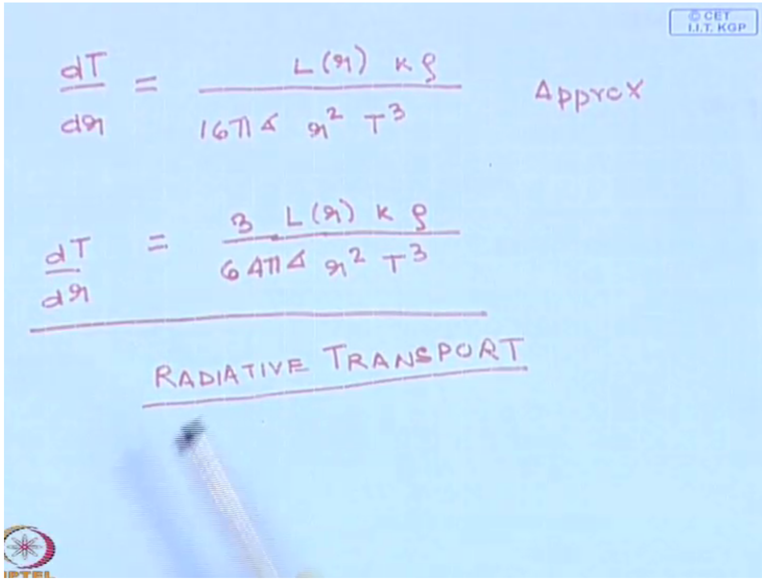

The third equation tells us how the luminosity changes as we go along the radius of the sun and the 4th equation which we were discussing is tells us how temperature changes along the radius of the sun and this we had discussed is determined by the energy transport mechanism and in the last class we were considering a situation where the energy is transported through radiation.

And in such a situation the temperature gradient at any point is determined by a combination of these factors it includes a luminosity at that distance from the center and in the opacity at that position, the density and the temperature involves all of these factors, now this is only one of the energy possible energy transport mechanism this is where the energy is transported by means through the radiation which is propagating outwards.

There is another possible energy transport mechanism which is convection, so let me now take up for discussion convective energy transport.

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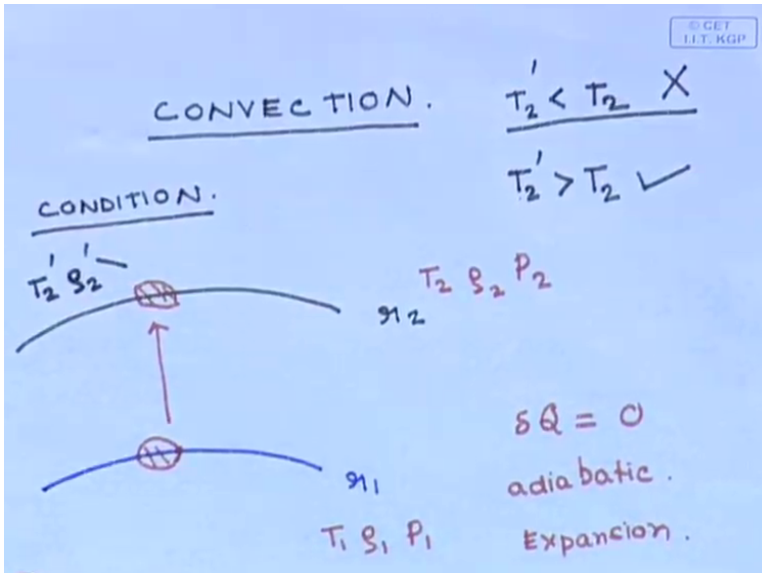

So the, and in this energy transport mechanism what happens is that bubbles of hot gas, so gas is heated in the interior of this Sun or the any star so bubbles of the hot gas rise up carrying the energy and when the bubbles reach the surface they quickly radiate away the energy and they again come down, the cool they radiator away their energy they cool and they come down again and this is the process of convection.

You see, it is quite common when you boil water for example you have convection taking place, so when you have convection going on you have structures that look like this

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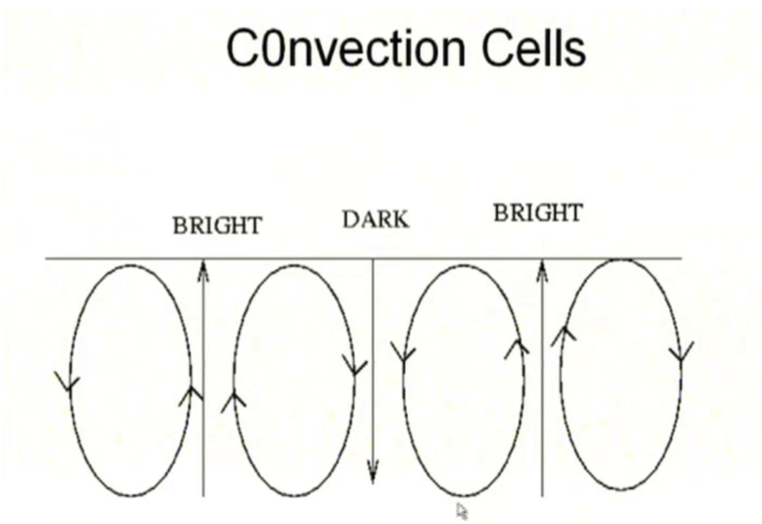

So here hot a hot bubble of gas so it is heated over here a hot bubble of the rises up carrying the energy and when it reaches the surface it quickly radiates the energy and then it cools in that process and it again comes down gets heated again and then rises up again, so you have this kind of cyclic motion for these bubbles and you have structures you expect to have success that look like this.

So these are convection cells this is what happens when you have the convection taking place, now when you have such convection cells just look at this region over here the in this region there is fluid coming up hot fluid coming up and then it radiates the energy over here and it gets cooled and then here in this region and you have cool fluid going down. Now the amount of radiation that comes out and depends on the temperature that the Stefan Boltzmann law T to the power 4.

So you will see a bright region here, a dark region here and bright region here, so you will see this kind of structure of bright and dark regions if you have convection you expect to see such dark and bright regions on the surface okay.

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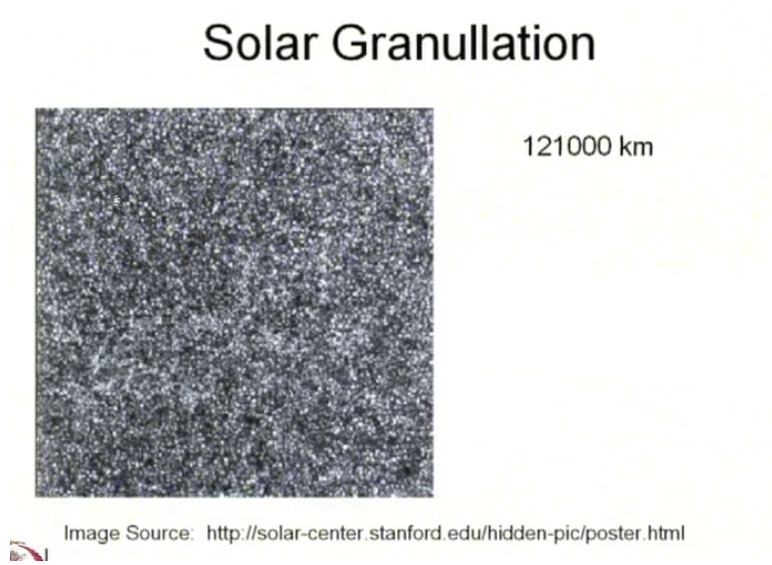

And this is a picture of the surface of the sun of the Solar surface and you see that there are this granules, this granular structure that you see on the surface this picture is an extremely large region if you think of it in absolute terms, it is 1,21,000 kilometers across so this side is 1,21,000 kilometers, this side is 1,21,000 km okay, so this is itself 20 times the earth's radius is 6,000 km, so this is roughly 20 times larger than the earth radius of the earth okay.

So you have these granules this bright spots and this dark spots, this granular structure on the sun is produced by convection that takes place at near the surface of the sun okay and this is another energy transport mechanism and today we are going to this energy transport mechanism that is convection okay, so let us first address the question what is the condition for convection? When will radiation be transported through when will energy be transported radiatively.

And when will energy be transported through convection, so what is the condition to appreciate the condition for convection let us consider a star where the temperature profile is set by the radiative transport of energy okay, so let us consider a star where the temperature profile is set by the radiative transport of energy. So energy is being transported outwards through radiation and we have seen that that will produce the temperature profile.

It will produce a temperature gradient and you can once you know the temperature gradient you can calculate the temperature profile, so let us assume that we have a star where the temperature profile is determined by this and let us consider a radius r1 of the star, so a position r1 in the star. And we will focus our attention on a small piece of small volume of gas, so this is the small volume of gas on which we are going to focus our attention.

And let us say that this gas is at a temperature, so this gas is what we shall referred to as our bubble this is the bubble okay we are going to consider this bubble okay and now this is another okay let me first in words describe to you what happens. So there is this bubble over here and consider a situation where it is slightly perturbed, slightly disturbed okay to another position, so let us say that the bubble is disturbed to a slightly different position r2.

So the same bubble now comes here due to some random so there will be random disturbances inside let us say and this bubble is shifted over here. The temperature at this point let us say is T1, the density here is rho 1 and the pressure here is P1 and the bubble to start with this in equilibrium with the rest of them surrounding material so it has the same temperature pressure and density as the surrounding okay.

Now, it gets disturbed to another question which is slightly above, now we know that the temperature falls as you go outwards as the height increases the temperature falls. So the bubble when it moves from here to here moves to a cooler region, so when the bubble has moved from here to here the temperature outside is smaller is lower okay, so let us say that the temperature here the ambient temperature of the surrounding is T2.

It has density rho 2 and pressure P2 that is the temperature of the surrounding and these temperatures are determined by the radiative transport of energy. So this is temperature ambient temperature here this is the ambient temperature here, now the bubble when it moves up we will assume that it is in mechanical equilibrium with the surrounding okay, so when it moves up when it comes here it now experiences the same the pressure P2 it was earlier experiencing the pressure P1.

When the bubble is move slightly up it now experiences a pressure P2 in the surrounding which has a different pressure P2, the temperature also is lower than where it was earlier okay, and we will assume that it moves from here to here sufficiently fast so that you can it does not exchange any heat with its surrounding, so we will assume that it moves from here to here sufficiently fast so that it does not exchange any heat.

So in this process whether bubble moves from here to here the exchange of heat delta Q is 0, so it is an adiabatic process and since the bubbles move from a place where the pressure is higher to where the pleasure is lower that it has moved up, the pressure is lower outside so it has moved to a region of lower pressure the bubble is going to expand so it is going to be adiabatic expansion. So this motion is going to be adiabatic the bubble is going to undergo adiabatic expansion.

And as a consequence of this adiabatic expansion to another pressure P2 its temperature is going to fall it is going to be cooler is going to be cooler than what it was earlier, now the question the main question is it cooler than the surrounding or is it hotter than the surrounding that is the main question okay. The bubble going to go cooled compared to T1 when I move it from here okay and it is going to come to another temperature let us call that temperature T2 prime and it will have a density rho 2 prime where these are these pertain to the bubble.

So the surrounding has the temperature T2, density rho 2. The inside of the bubble it expands adiabatically the outside temperature is determined by the radiation transfer the bubble moves from here to here and expand adiabatically its temperature will be different and its temperature is T2 prime and its density is rho 2 prime okay, now the question is what happens to the bubble when it comes here and what happens essentially depends on whether T2 prime.

So there are 2 possibilities if T2 prime < T2. So if the bubble when the hidden atmosphere is cooler than the surrounding, the density air is going to be more than the surrounding density, if the density of the bubble is more than the surrounding density then the buoyancy force is going to act downwards and the bubble is going to come back to its equilibrium position and there will be no convection, so in this position if this condition is holds then there is no convection.

Whereas if the bubble when it moves from here to here is hotter than the surrounding medium then the density here in the bubble is going to be lover and the bubble is going to experience an upward buoyancy force so if T2 prime >T2 we are going to have its going to experience an upward buoyancy force it is going to move up and it will continue to move up and you will have convection setting in okay, so you will have convection okay.

So let me remind you again the important criteria is that if I move the bubble from here to here, the temperature of the bubble should be higher than the temperature of the ambient medium surrounding when I go from here to here this is the condition for convection in general okay and in this case the temperature different that profile, the temperature here and here are both the surrounding temperature are both determined by the radiative transport of energy okay.

So we have to, so to determine the condition for convection what we have to do is that we have to compare 2 things we have to compare the rate so what are the 2 things that we have to compare, we have to compare the temperature gradient.

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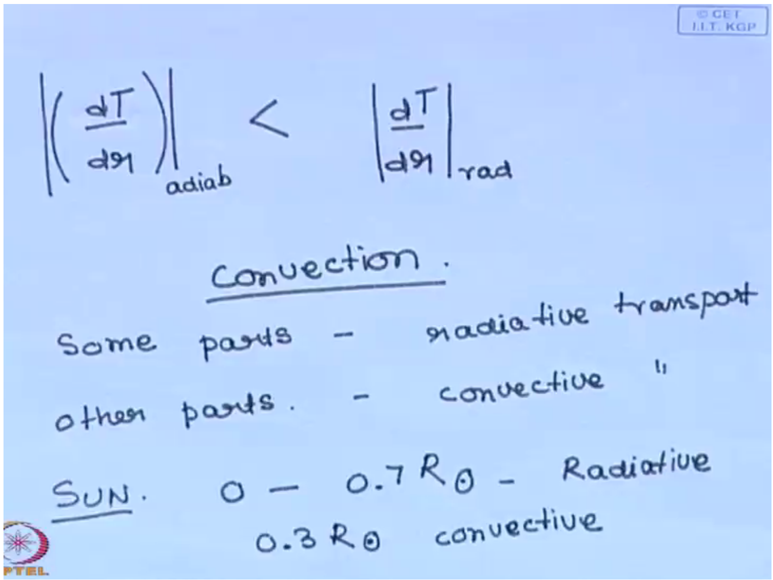

This is the rate at which the temperature changes if I move the bubble adiabatically, so that is the rate at which the temperature changes if we move the bubble adiabatically and we know that this is going to be a negative quantity the rate of change of temperature as it moves outwards is going to be negative, temperature here is going to be less than the temperature here. So let we will look at the mod of this.

And we will compare it to the rate at which the ambient temperature changes for the radiative energy transport right, so this term determines how much the temperature falls when I go from here to here, this term determines how much the temperature of the bubble falls when I go from here to there and we want the surrounding temperature to fall more than the temperature of the bubble the then that fall in the temperature of the bubble okay.

So the condition for convection is that this should be more than this, the temperature the magnitude of the temperature gradient due to the energy transport radiative energy transport should exceed the magnitude of the temperature gradient due to adiabatic expansion okay and if this condition is satisfied then convection will start, then you will have convection okay. Now let me remind you that the radiative the temperature gradient due to energy transport due to through the radiation is dependent on the luminosity at that point okay.

And so it happens so what happens is that if the luminosity at this point becomes so large that the required temperature gradient exceeds the adiabatic value, adiabatic temperature the temperature gradient due to adiabatic expansion, then you have convection setting in okay. Convection is more efficient energy transport mechanism okay and if convection sets in then the it wins, it takes over okay.

So if once convection starts the then its starts becomes it become once this condition is satisfied then convection becomes the most dominant energy transport mechanism okay, now when you solve the structure of a star then you have to solve all of those 3 4 equations which I showed you just now. So if the luminosity somewhere become sufficiently large that in this situation is this condition is satisfied you have the convection okay.

So when you solve the equation of the structures of the, equations for the structure of a star it usually happens that the condition for convection is valid in certain parts of the star only okay, so in some parts of the star you will have radiative transport of energy this condition will not be satisfied. And in other parts of the same star you will have convective transport and you will know this only when you solve the equations which I just outlined showed you in the beginning of today's class.

So for the sun let me just give you the numbers for the sun it turns out that from 0 to 0.7 R sun, so 70% of the solar interior you have radiative transport of energy and the remaining outer 0.30% of the star of the sun you have convective transport of energy okay. Now let us next calculate this so when convection occurs the energy transport is through the convective through the motion of bubbles once this condition is satisfied the energy transport is through the motion of this hot bubbles.

So let us now calculate the temperature gradient that is set up in such a situation, in such situation usually the temperature gradient reaches this value and it reminds there okay. So convection is a very efficient energy transport mechanism, the moment you have convection the temperature gradient does not exceed this value okay, so the bottom line of this discussion is that when the temperature gradient due to radiative transport is l< this value.

You have the temperature gradient given by this, in regions where the radiative temperature gradient exceeds the adiabatic value it reminds at this value. Because convection is a very efficient energy transport mechanism it does not permit typically the energy the temperature gradient exceeds this value okay, so you have this value in most of the region where convective energy transport is occurring energy gets transported at a rate which maintains the adiabatic and temperature gradient.

Though, in the outer parts of the sun you do have regions where the temperature gradient is higher than these and those are the regions where you have super adiabatic temperature gradients okay but by and large and this is the value that you have okay, so let us calculate this value.

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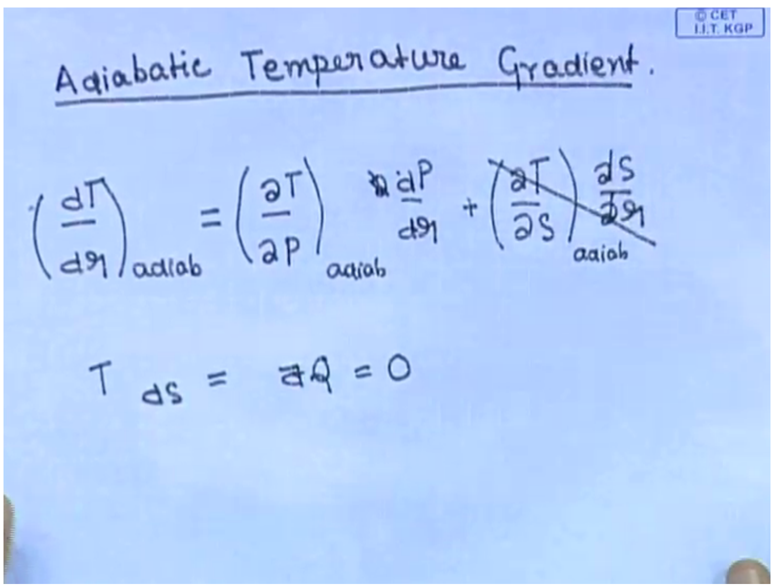

So we would like to calculate the adiabatic temperature gradient okay, so let us again consider this is the bubble that you are dealing with and we would like to calculate what happens to the temperature of this bubble if we move this up some distance delta r, that is the equation that we are looking at so we would like to calculate and that the gas can be described the state of the gas can be described in terms of 2 state variables right that we know in Thermodynamics.

So for this exercise it is convenient to think of 2 state variables as the pressure and the entropy okay, so the temperature of the gas is now a function of the pressure and the entropy, entropy of this bubble of this volume of gas okay, then this derivative can be written as del T del P adiabatic*dP/dr+del T del S adiabatic*dS/dr okay, S and the pressure the entropy and the pressure are the state variables they vary with r.

Now if you have adiabatic expansion of this volume of gas then the heat transport the heat exchange is 0, and we know that the entropy change dS*Td the temperature gives us the heat exchanged and we know that this is 0 in an adiabatic process, so in an adiabatic process this term does not contribute and we have that the temperature gradient is equal to the change the derivative of the temperature with respect to pressure into dP/dr the pressure gradient.

And the pressure gradient we have already have an equation for the pressure gradient, so let me show you that equation that was a hydrostatic equilibrium, so this is the pressure gradient dP/dr is the acceleration into the density, gravitational acceleration of the matter inside into the density, so we can use this to calculate the pressure gradient, let us now calculate the rate of change of temperature with respect to the pressure for an adiabatic transformation.

We have already done this for a gas of photons let me now do it for a perfect gas of atoms okay, so we know that it has an equation of state.

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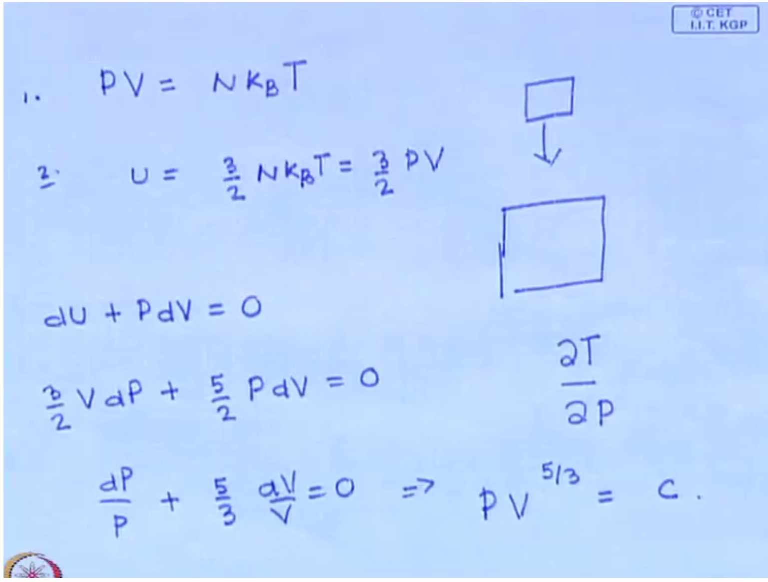

So let us consider a gas and this gas is made to expand adiabatically, we would like to calculate del T/del P for such an adiabatic expansion and the state of the gas we know the gas satisfies an equation of state that PV=N KB T, N is the number of particles, KB is the Boltzmann constant, T is the temperature okay and we also know that the internal energy of the gas U is 3/2 N KB T which we can also write as 3/2*PV okay.

Now for an adiabatic process applying the first law of thermodynamics dU+P dV is 0 which can now be written as 3/2 and here we will have 2 terms one will be P dV when I differentiate this and the other one will be V dP, P dV term can be combined with this so I will get okay let me write down the 2 terms, one time will be V dP and the other term will be+5/2 P dV =0 right. Because this will give us 2 terms when I put this in here.

One of them, I can combine this and I will get a factor of 5/2 okay and here we can divide throughout by P*V and what we get is dP /P+5/3 dV/V=0 and this we know is log P, this is log V, so what this tell us is that PV to the power 5/3 is a constant okay, for in general let me now just write down the general thing.

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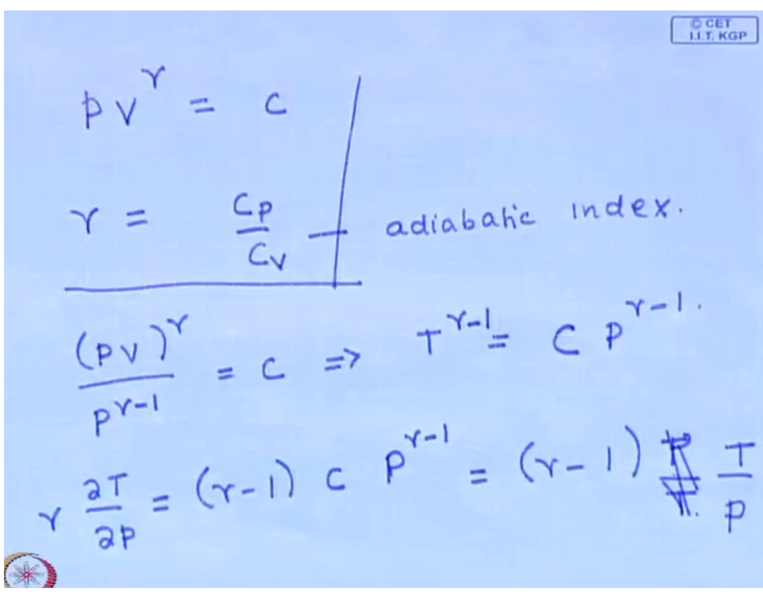

In general PV to the power gamma is a constant where gamma is the ratio of the specific heat at constant pressure to the specific heat at constant volume, and for an ideal gas we have this okay monatomic ideal gas single particles, if this is in general for a general adiabatic transformation and here we can this is called an adiabatic index gamma, now we would like to evaluate del T/del P and so we have to use this so we know that PV=N KB T.

So I can write this as PV to the power gamma/P to the power gamma-1=C which essentially tells us I can replace this by N K T and these factors of N and K are constants so I can observe them inside this constant what it tells us is T to the power gamma I can replace this with T to the power gamma=some constant let us say capital C*P to the power gamma-1 and we can differentiate this now.

So we want to, we can differentiate this del T/del P and if I differentiate it what I will get is that del T/del P*gamma=gamma-1 C P gamma–2, C P gamma-2 we can write, so we can write this as gamma-1*T to the power gamma-1 right this should be T to the power gamma-1 here, so P to the power this whole thing can be written as P/T sorry T/P right, so P to the power gamma -1 we can write as P so we can just eliminate this from here and you will get T/P okay.

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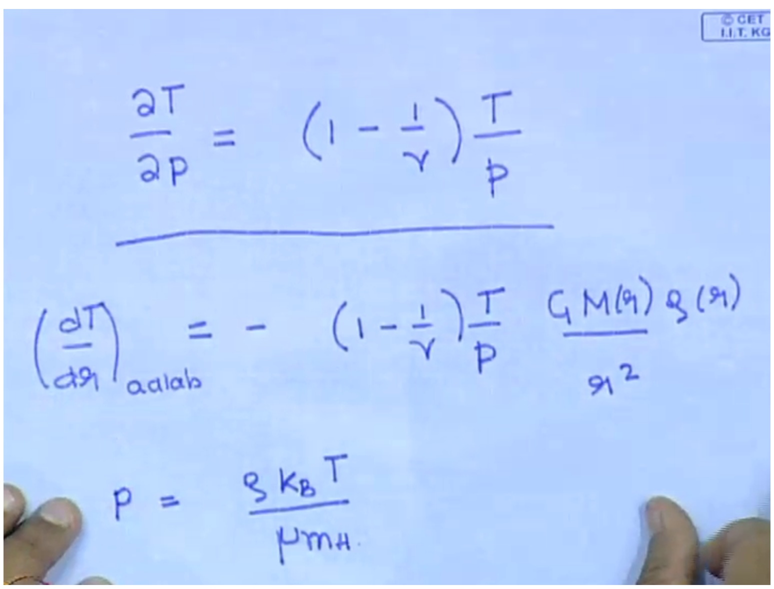

So what you will have at the end of the day is that del T/del P= so we will 1–1/gamma*T/P okay, so that is for an adiabatic process and we use this to calculate the adiabatic temperature gradient dT dr, so this is now equal to so we have – let me put in one by one okay we have 1–1/gamma T/P so this is del T del P and then we have del P del r, del P del r is – GM(r)/r square*rho okay that is the dP dr this step hydrostatic equilibrium okay.

And we also know that P=rho KB T by the average atomic weight, atomic number, atomic weight divided by the mass of one hydrogen atom, this is essentially the same thing here written in a different way that is all okay, so this is the same thing as this PV=N KB T written in a slightly different way it in terms of mass density okay and we can use this to replace P so here we have P rho, so here we also have T rho/P so we have T rho/P.

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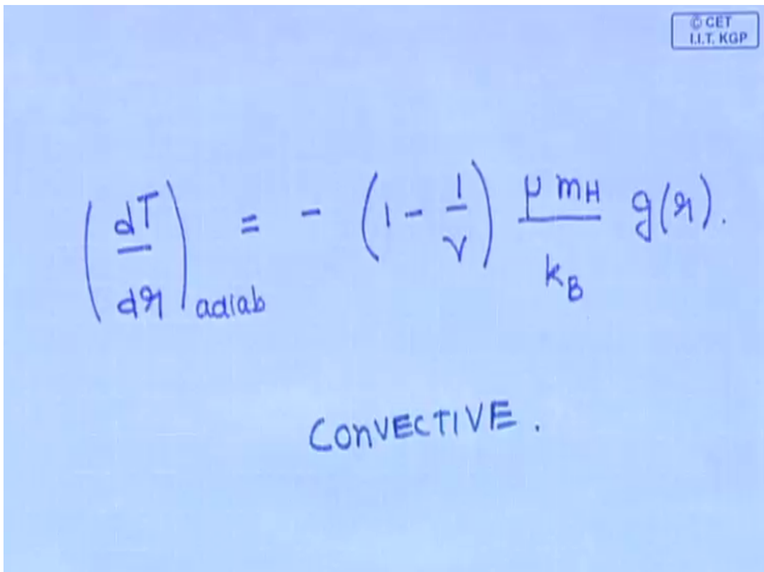

So then substituting those factors in this equation what we get is the final equation dT/dr adiabatic =-(1–1/gamma) and then we have so rho T and P comes here so we have mu MH/KB into the gravitational acceleration at that point at that distance from the center okay, so we have now obtained the 4 equations and that have to be solved to determine the structure of a star and these 4 equations let me show them to you once more, the equations that we have to simultaneously solve.

So the first equation is the equation of hydrostatic equilibrium, this gives the pressure as a function of r, the second is the mass distribution, the third is the way the luminosity changes with distance and the 4th is the temperature gradient which is decided by the energy transport mechanism, if it is radiative transport you have this, if it is convective transport and you have this and which of them you have to use is determined is to be determined by seeing which one is so larger.

If this is smaller than this then you have to use this, the moment this becomes bigger than this you have to use the adiabatic temperature gradient and we know that the convection has set in okay, so you have to simultaneously solve these 4 equations, in addition to these equations there are certain secondary equation which are also required, so let me tell you what this secondary equation for, these 4 equations themselves are not adequate.

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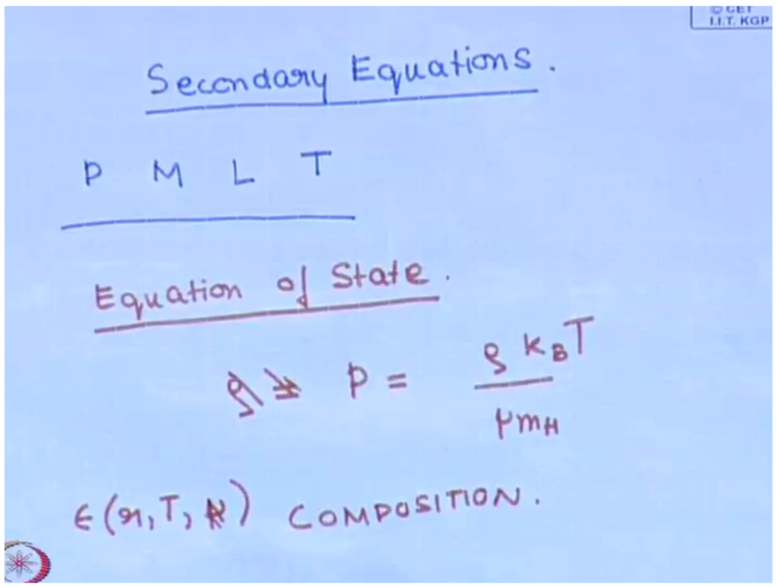

There are certain secondary equations that also have to be solved, it would have to be okay so if you look at this you have 4 unknown quantities, the 4 unknown quantities are the pressure, the mass, all of these are the functions of r, when you have the luminosity and the temperature all of them vary with r, but there are other quantities that appear in the equation, so these quantities for example if you look at the first equation you have the density right.

So you have to for the density one hat to use the equation of state, the equation of state I just wrote it down it is the density equal to, P let me write it differently P=rho/mu MH KB T you have to use this right, so if you use this then the density is no longer an independent quantity it depends on the pressure and the temperature, you also need some other factors so let us look at those, so you need to know okay.

Let us look at the mass profile, the mass profile does not involve anything new it has a density which again you can express in terms of pressure and temperature, but the moment you look at the luminosity, the luminosity has the energy generation rate or the emissivity okay that comes in when you want to calculate the luminosity the rate at which the energy is being generated at different parts so I need to be told the energy generation rate epsilon.

And this has to be calculated as a function you need to know this as a function of r. we know that this is a function and the temperature and also the chemical the composition okay, the composition so there is a composition over here right not only on mu but also on the composition, so that it also depends on the composition and it will change that this is so nuclear fusion and the rate at which energy is being generated will be different if you change the composition.

If you change the fraction of hydrogen we have seen that the energy generation rate changes it goes as a square of the hydrogen fraction we have seen this already okay, so it depends on the composition, so you need to also tell the composition as a function of this position and okay as the position along the star and that will I again change as the star keeps on burning more and more fuel okay.

Let us look at the next equation, the next equation is the temperature gradient, so if you have radiative transport then you also need to know the opacity that is the absorption how much radiation what is the absorption cross section okay or the absorption coefficient per unit volume a unit mass that is the opacity kappa.

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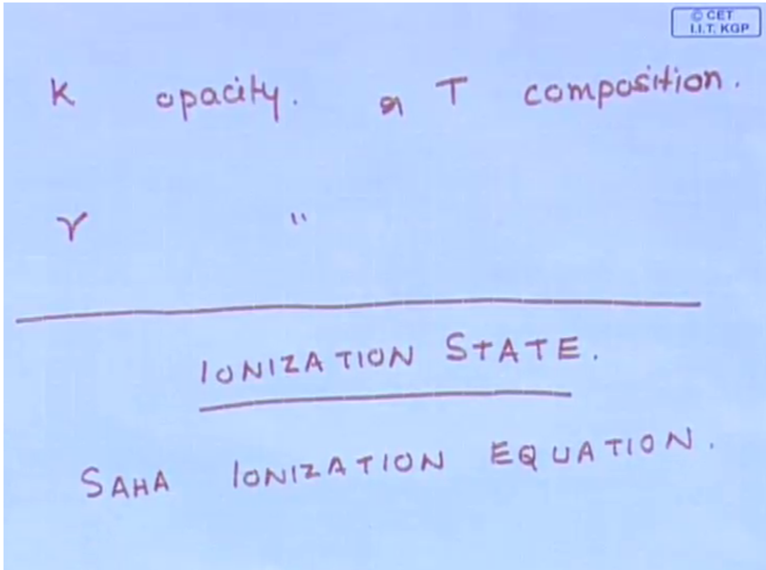

And kappa again so you need to be now told this kappa opacity and this again will be function of r T and the composition, if you have convective transport and to determine which of these 2 is going to be effective you need to also know the gamma the adiabatic index right, so one has to also calculate the gamma, these are all factors that one has to simultaneously solve that you need gamma and it could vary.

Gamma, the value of gamma could in principle vary from position to position, so this again has to be calculated and it will depend on the composition etc. In addition to all of these one also needs to know the ionization state of the gas, one has to solve for this along the star it is not necessary that all the different components of a star will all be equally ionized at a given temperature and pressure right.

So one needs to solve for the ionization state of the material inside a star simultaneously with all of these okay and the ionization state for any species it tells us what is the ratio of a fraction that is ionized to the neutral fraction okay, so you can use this and you have to calculate essentially this and this is given this has to be calculated using the SAHA ionization equation.

So what is the SAHA ionization equation for a gas at a certain temperature and pressure so given at a certain temperature and density you may say it let us calculate what fraction of the gas is going to be ionized and if there are several possible ionization states it tell us the fraction of the gas which is of the atoms which to be in each ionization state okay, so this is something that I am going to take up in the next class the Saha ionization equation.

And let me stop today's class over here, tomorrow we are going to take up the Saha ionization equation and then how to proceed then what are the solutions that one obtains for this stellar structure, today I have told you what the basic equations for determining the stellar structure are, if you want to determine the state of a star you have to simultaneously solve all of these with the appropriate boundary conditions, the boundary conditions are that the pressure should vanish at the surface okay that is the main boundary condition.