Astrophysics & Cosmology
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Lecture – 16
Stellar Physics – I

The structure of a star and I have told you that there are 4 equations which needs to the considered and let me remind you that the first equation is the equation of hydrostatic equilibrium which, we discussed in the last class.

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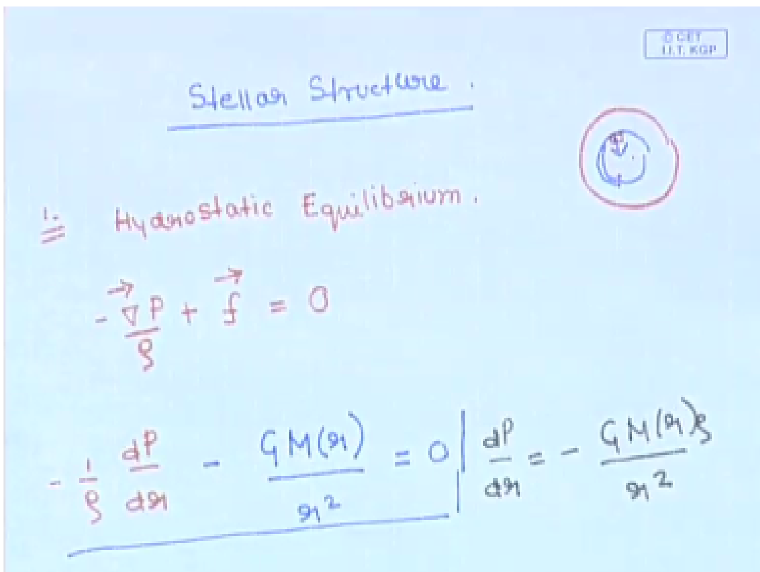

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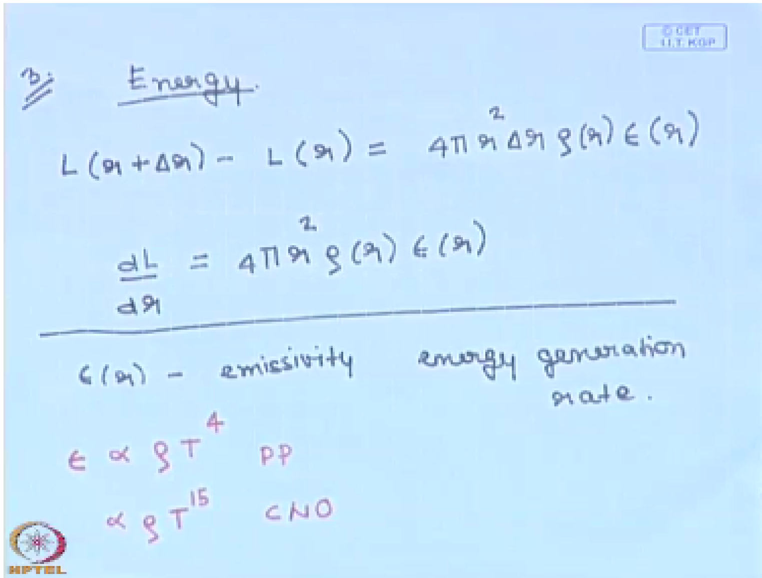

The second equation tells us how the luminosity, so the first equations tells us how the pressure changes as we go along the radius of the star. The second equation tells us how the luminosity of the radiation changes as we go along the radius of the star.

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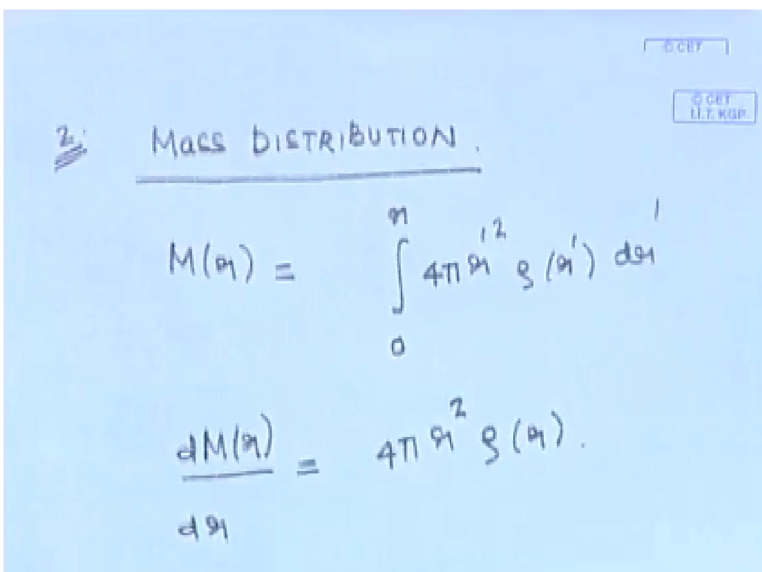

The third equation, sorry this was the third equation that we had considered. The second equation tells us how the mass changes as we go along the radius of the sun.

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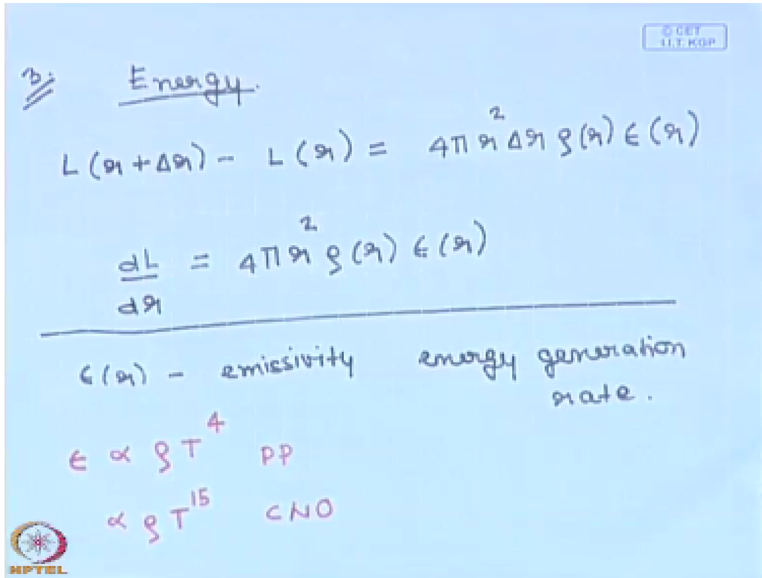

The third equation tells us how the luminosity changes as we go along the radius of the sun.

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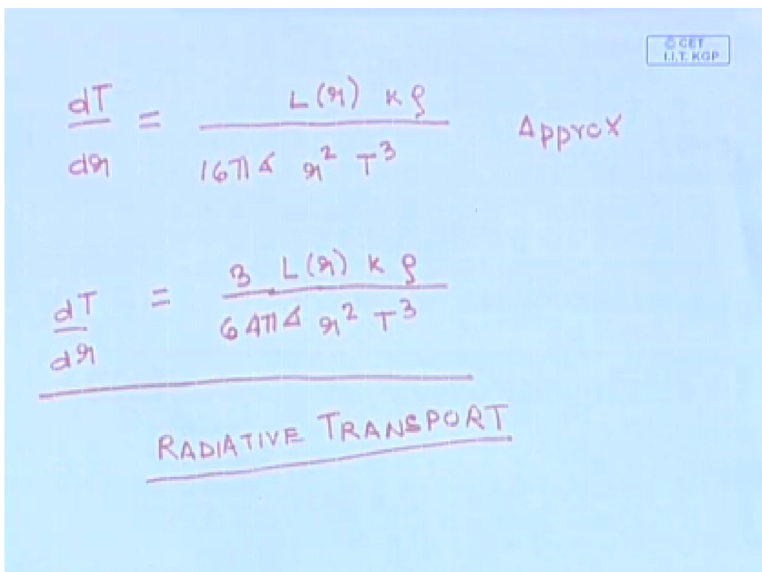

And the 4th equation, which we were discussing is tells us how the temperature changes along the radius of the sun. And this we had discussed is determined by the energy transport mechanism and in the last class we were considering a situation where the energy is transported through radiation. And in such a situation the, the temperature gradient at any point is determined by a combination of these factors it includes a luminosity at that distance from the centre.

And the opacity at that position the density at the temperature it involves all of these factors. Now this is only one of the energy-- possible energy transport mechanism this is where the energy is transported by means through the radiation which is propagating outwards. There is another possible energy transport mechanism which is convection. So let me now, take up for discussion Convective Energy Transport.

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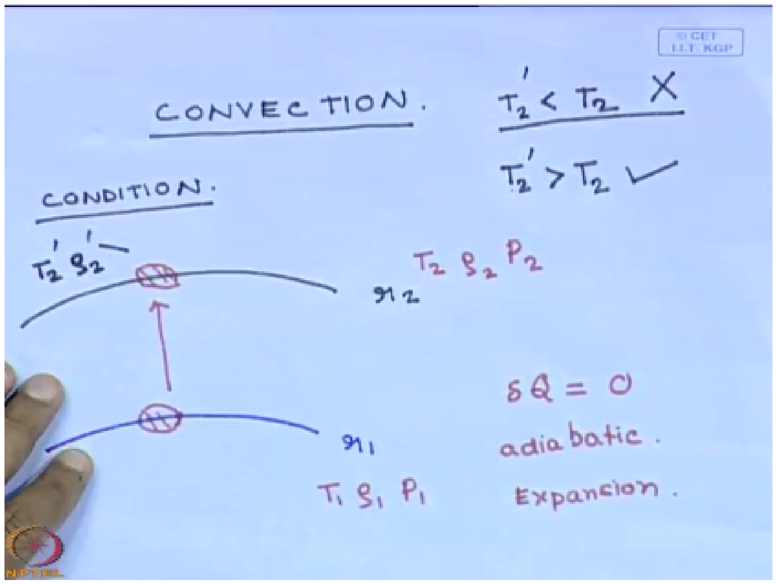

So the, and this energy transport mechanism what happens is that bubbles of hot gas so gas is heated in the interior of this sun or the any stars, so bubbles of hot gas rise up carrying the energy. And when the bubbles reach the surface they quickly radiate away the energy and they again come down. The cool, the energy cool and they come down again. And this is the process of convection you see it is quite common where you boil water. For example, you have convection taking place.

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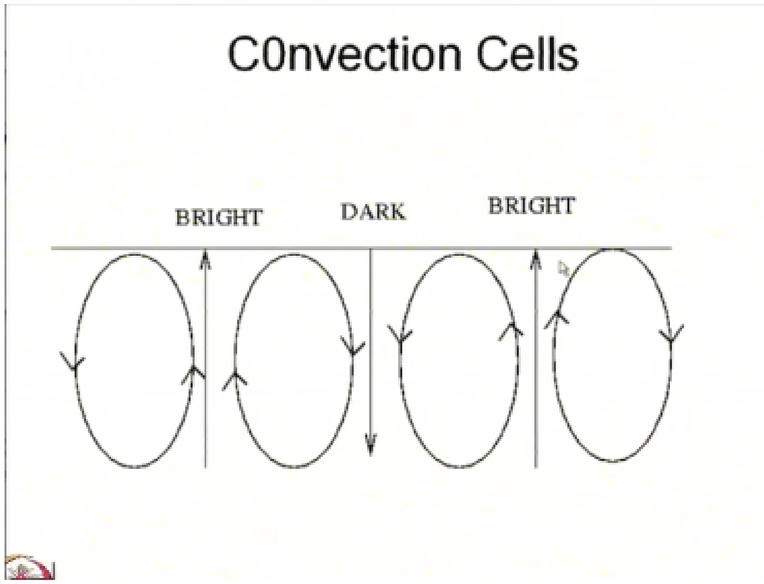

So when you have convection going on you have structures that look like this. So here hot, a hot bubble of gas, so it is heated over here a hot bubble of gas rises up carrying the energy and when it reaches the surface it quickly radiates the energy and then it cools in that process and it again comes down, gets heated again and then rises up again, so you have this kind of cyclic motion for these bubbles and you have structures you expect to have structures that look like this.

So these are convection cells. This is what happens when you have convection taking place. Now when you have such convection cells just look at this region over here the-- in this region there is fluid coming up hot fluid coming up. And then it radiates the energy over here and it gets cooled and then here in this region you have cool fluid going down. Now the amount of radiation that comes out depends on the temperature, that is the Stefan Boltzmann Law T to the power 4.

So you will a bright region here a dark region here and bright region here, so you will see this kind of structure of bright and dark regions if you have convection you expect to see such dark and bright regions on the surface. Okay.

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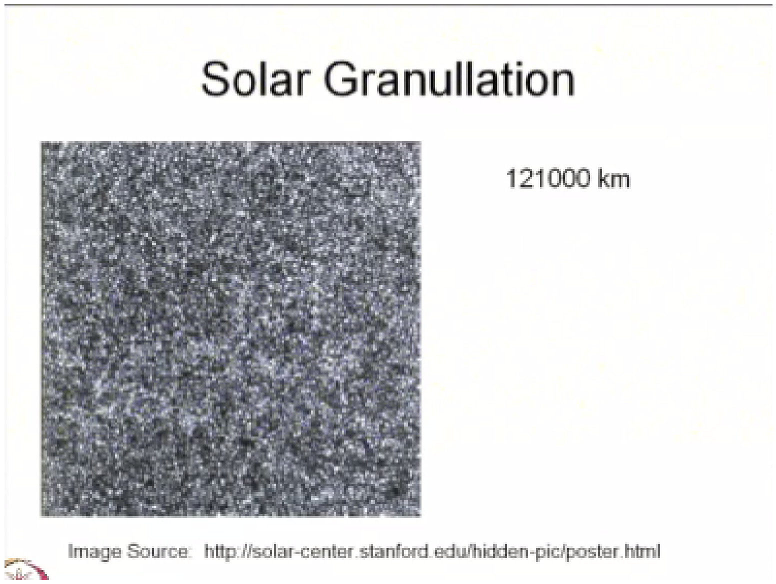

And this is the picture of the surface of the sun of the solar surface, and you see that there are these granules; this granular structure that you see on the surface; this picture is an extremely large region, if you think of it from absolute term; it is 1,21,000 kilometres across, so this side is 1,21,000 kilometres, this side is 1,21,000 kilometres. Okay. So this is itself 20 times the earth radius is 6,000 kilometres, so this is roughly 20 times larger than the earth, radius of the earth. Okay.

So you have these granules this bright spots and the dark spots this granular structure on the sun is produced by convection that takes place at near the surface of the sun. Okay. And this is another energy transport mechanism and today we are going to consider this energy transport mechanism that is convection. Okay. So let us first address the question, what is the condition for convection.

When will radiation be transported conduct through-- when will energy be transported radiatively and when will energy be transported through convection. So what is the condition? To appreciate the condition for convection let us consider a star where the temperature profile is set by the radiative transport of energy, okay. So let us consider a star where the temperature profile is set by the radiative transport of energy.

So energy is being transported outwards through radiation and we have seen that that will produce a temperature profile it will produce a temperature gradient and you can once you know the temperature gradient you can calculate the temperature profile. So let us assume that we have a star where the temperature profile is determined by this and let us consider a radius r1 of the star, so position r1 in the star.

And we will focus our attention on a small piece small volume of gas so this is the small volume of gas on which we are going to focus our attention and let us say this gas is at a temperature, so this gas is what we shall refer to as our bubble, this is the bubble. Okay, we are going to consider this bubble. Okay. And now, this is another okay, let me first in words describe to you what happens.

So there is this bubble over here and consider a situation where it is slightly perturbed, slightly disturbed, okay to another position. So let us say that the bubble is disturbed to a slightly different position r2. Okay, so the same bubble now comes here, due to some random-- so there will be some random disturbance in inside, let us say and this bubble is shifted over here. The temperature at this point, let us say is T1; the density here is rho one and the pressure here is P1.

And the bubble to star which is in equilibrium with the rest of the surrounding material, so it has the same temperature pressure and density as they surrounding. Okay, now it gets disturbed to another position which is slightly above. Now, we know that the temperature falls as you go outwards as the height increases; the temperature falls. So the bubble where it moves from here to here; moves to a cooler region, so when the bubble has move from here to here the temperature outside is smaller is lower okay.

So let us say that the temperature here the ambient temperature of the surrounding is T2 it has density rho 2 and pressure P2, that is the temperature of the surrounding and these temperatures are determined by the radiative transport of energy. So this is the ambient temperature here; this is the ambient temperature here. Now, the bubble when it moves up, we will assume that it is in mechanical equilibrium with the surrounding. Okay.

So when it moves up when it comes here, it now experiences the same the pressure P2; it was earlier experiencing the pressure P1. When the bubble is moved slightly it up it now experiences a pressure P2, it has a difference; it is a surrounding which has a different pressure P2; the temperature also lower then where it was earlier. Okay. And we will assume that it moves from here to here, sufficiently fast.

So that you can it does not exchange any heat with the surrounding. So we will assume that it moves from here to here sufficiently fast so that it does not exchange any heat so in this process where the bubble moves from here to here the exchange of heat delta Q is 0. So it is an adiabatic process. And since the bubble moves from a place where the pressure is higher to a where the pressure is lower right it has moved up the pressure is lower outside.

So it has moved to a region of lower pressure; th bubble is going to expand; so it is going to be adiabatic expansion. So, this motion is going to be adiabatic, the bubble is going to undergo adiabatic expansion. And as a consequence of this adiabatic expansion to another pressure P2 its temperature is going to fall it is going to be cooler it is going to be cooler then what it was earlier.

Now the question, the main question is it cooler is it cooler then the surrounding or is it hotter than the surrounding; that is the main question. Okay, the bubble is going to cool compare to T1 when I move it from here to here, okay. And it is going to come to another temperature let us call that temperature T2 prime and it will have a density rho to prime where these are the these pertain to the bubble.

So the surrounding has a temperature T2 density rho 2, the inside of the bubble it expands adiabatically; the outside temperature is determined by the radiation transfer; the bubble moves from here to here and expands adiabatically; its temperature will be different and its temperature is T2 prime and its density is rho 2 prime, okay. Now the question is what happens to the bubble when it comes here and what happens essentially depends on whether T2 prime.

So there are 2 possibilities. If T2 prime is < T2, so if the bubble when it moves here is cooler than the surrounding the density here is going to be more than the surrounding density. If the density of the bubble is more than the surrounding density, then the bounce force is going to act downwards and the bubble is going to come back to its equilibrium position and there will be no convection.

So in this position, if this condition is holds then there is no convection. Whereas, if the bubble when it moves from here to here is hotter than the surrounding medium then the density here in the bubble is going to be lowered and the bubble is going to experience and upward bounce force, so if T2 prime is > T2, we are going to have its going to experience an upward bounce force; it is going to move up.

And it will continue to move up and you will have convection setting in, okay so you will have convection. Okay. So let me remind you again the important criteria is, that if I move the bubble from here to here the temperature of the bubble should be higher than the temperature of the ambient medium surrounding when I move from here to here. This is the condition for convection in general. Okay.

And in this case the temperature different that profile when the temperature here is determined and here are both the surrounding temperature are both determined by the radiative transport of energy. Okay. So we have to, so to determine the condition for convection what we have to do is that we have to compare 2 things; we have to compare the rate-- so what are the 2 things that we have to compare; we have to compare the temperature gradient.

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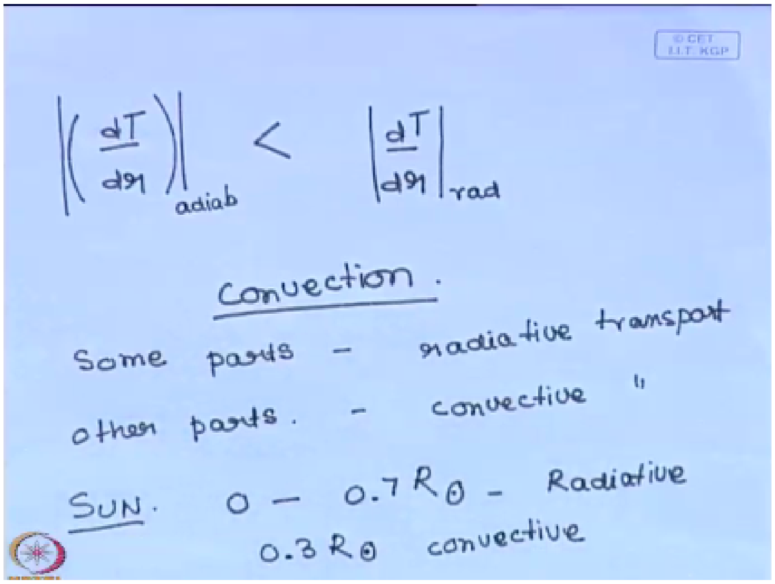

This is the rate at which the temperature changes if I move the bubble adiabatically. So that is the rate at which the temperature changes if we move the bubble adiabatically and we know that this is going to be a negative quantity the rate of change of temperature as it moves outwards is going to be negative, the temperature here is going to be < the temperature here, so we look at the mode of this.

And we will compare it to the rate at which the ambient temperature changes for the radiative energy transport, Right. So this term determines how much the temperature falls when I go from here to here. This term determines how much the temperature of the bubbles falls when I go from here to here. And we want the surrounding temperature to fall more than the temperature of the bubble then the fall in the temperature of the bubble, okay.

So the condition for convection is that this should be more than this. The, the temperature the magnitude of the temperature gradient due to the energy transport radiative energy transport should exceed the magnitude of the temperature gradient due to adiabatic expansion, okay. And if this condition is satisfied then convection will start, then you will have convection. Okay. Now let me remind you that the radiative; the temperature gradient due to energy transport due to through the radiation is dependent on the luminosity at that point. Okay.

And so it happens so what happens is that. If the luminosity at this point become so large that the required temperature gradient exceeds the adiabatic value, adiabatic temperature the temperature gradient due to adiabatic expansion then you have convection setting in. Okay. Convection is a more efficient energy transport mechanism. Okay. And if convection sets in then the-- it wins it takes over.

So once convection starts then it starts it becomes once this condition is satisfied the convection becomes the most dominant energy transport mechanism. Okay. Now when you solve the structure of a star then you have to solve all of those 3, 4 equations which I shown you just now. So if the luminosity somewhere become sufficiently large that this equation is this condition is satisfied you have convection, okay.

So when you solve the equation of the structure, equations for the structure of the star it usually happens that the condition for convection is valid in certain parts of this star only, okay, so in some part of the star, you will have radiative transport of energy, this condition will not be satisfied.

And in other parts of the same star you will have convective transport and you will know this only when you solve the equation which I just outlined showed you in the beginning of todays class. So for the SUN, let must just give you the numbers, for the SUN it turns out that from 0 to 0.7R sun; so 70% of the solar interior you have radiative transport of energy. And the remaining outer 0.30% of the star of the sun you have convective transport of energy. Okay. Now let us next, calculate this.

So when conjunction occurs the energy transport is through the convective through the motion of bubbles once this condition is satisfied the energy transport is through the motion of these hot bubbles, so let us now calculate the temperature gradient that is setup in such a situation. In such a situation usually, the temperature gradient reaches this value and its remains there. Okay. So conjunction is a very efficient energy transport mechanism, the moment you have convection the temperature gradient does not exceed this value. Okay.

So the bottom-line of this discussion is that when the temperature gradient due to radiative transport is < this value you have the—

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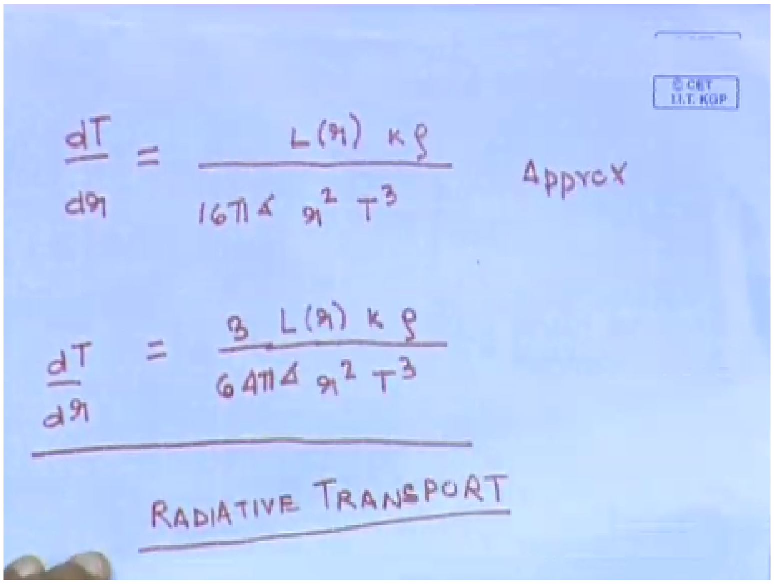

You have the temperature gradient given by this. In regions where the radiative temperature gradient exceeds the adiabatic value it remains at this value because convection is a very efficient transport mechanism it does not permit typically the energy the temperature gradient to exceed this value. Okay, so you have this value in most of the region where convective energy transport is occurring. Energy gets transported at rate which maintains the adiabatic temperature gradient.

Though in the outer parts of the sun you do have regions where the temperature gradient is higher than this and those are the regions where you have super adiabatic temperature gradients. But by enlarge this is value that you have, okay. So let us calculate this value.

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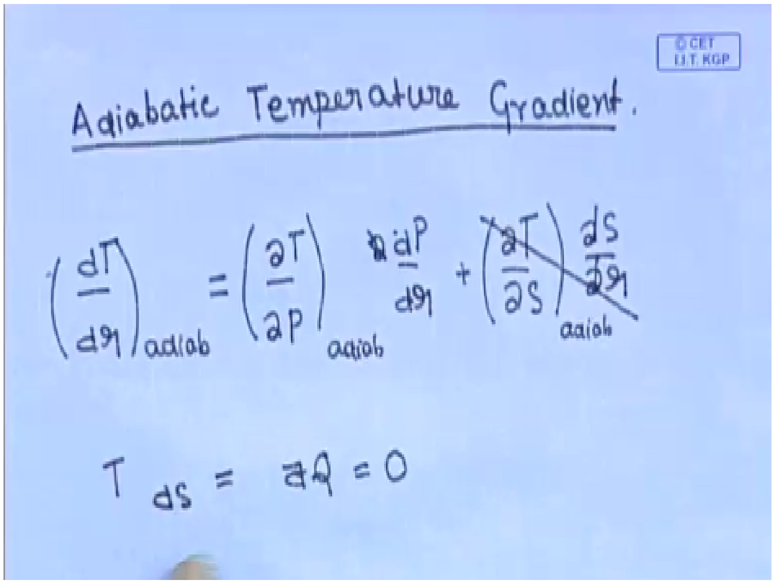

So we would like to calculate the Adiabatic Temperature Gradient. Okay.

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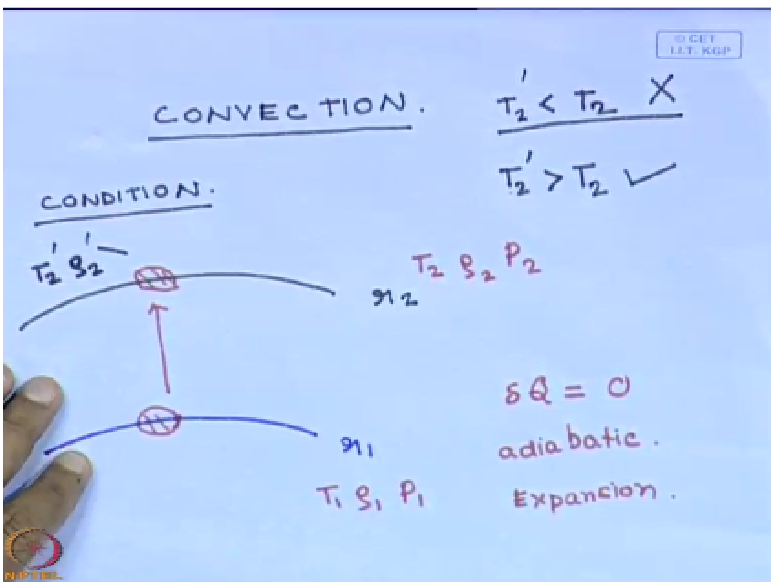

So, let us again consider this, this is the bubble that we are dealing with and we would like to calculate what happens to the temperature of this bubble if we move this up some distance delta R. It is that equation that we are looking at. So we would like to calculate, and the gas can be described the state of the gas can be described in terms of 2 state variables. Right, we know in thermodynamics.

So for this exercise this is convenient to think of 2 state variables as the pressure and the entropy, okay. So the temperature of the gas is now a function of the pressure and the entropy of this bubble of this volume of gas. Okay. So then this derivative can be written as del T del P adiabatic*del P dP dR + del T del S adiabatic* the del S del R; dS dR, okay Sn the pressure the entropy in the pressure state variables they vary with R.

Now if you have adiabatic expansion of this volume of gas then the heat transport the heat exchange is 0 and we know that the entropy change ds*Td the temperature gives us the heat exchange and we know that this is 0 in an adiabatic process, so in an adiabatic process this term does not contribute. And we have that the temperature gradient is equal to the change the derivative of the temperature with respect to the pressure into dP dR, the pressure gradient.

And the pressure gradient we have already; we already have in equation for the pressure gradient so let me show you that equation that was the hydrostatic equilibrium.

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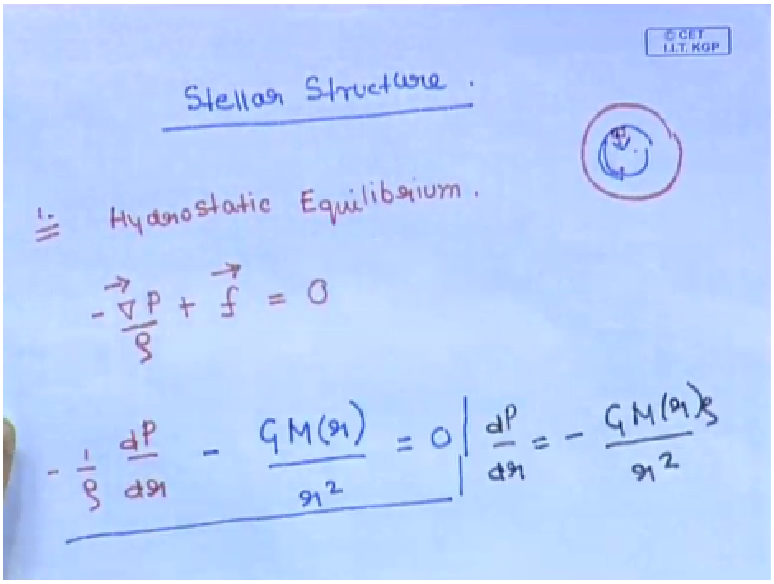

So this is the pressure gradient dP dR is the acceleration into the density. Gravitational acceleration of the matter inside into the density, so we can use this to calculate the pressure gradient. Let us now calculate the rate of change of temperature with respect to the pressure for an adiabatic transformation. We have already done this for a gas of photon let me now do it for a perfect gas of atoms, okay. So we know that it has an equation of state.

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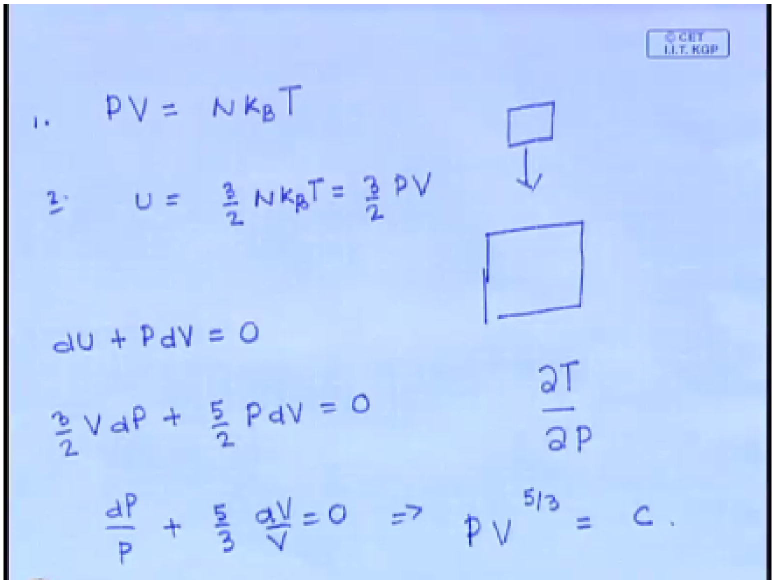

So let us consider a gas and this gas is made to expand adiabatically, we would like to calculate del T del P for such an adiabatic solution. And the state of the gas we know the gas satisfies an equation of state, that PV = NKBT, N is a number of particles; KB is the Boltzmann constant, T is the temperature, okay. And we also know that the internal energy of the gas U is 3/2 NKBT which we can also write as 3/2*PV. Okay.

Now for an adiabatic process applying the first law of thermodynamics dU + PdV is 0, which can now be written as 3/2 and here we will have 2 terms one will be PdV when I differentiate this and other one will be VdP. PdV term can be combined with this, so I will get, okay let me write down the 2 terms, one term will be dP and the other term will be + 5/2 PdV = 0. Right, because this will give us 2 terms when I put this here.

One of them, I can combine with this and I will get the factor of 5/2, okay. And here we can divide throughout by P*V and what we get is dP/P + 5/3 dV/V = 0 and this we know is log P, this is log V; so what this tells us is that PV to the power 5/3 is a constant. Okay. For in general, let me now just write down the general thing.

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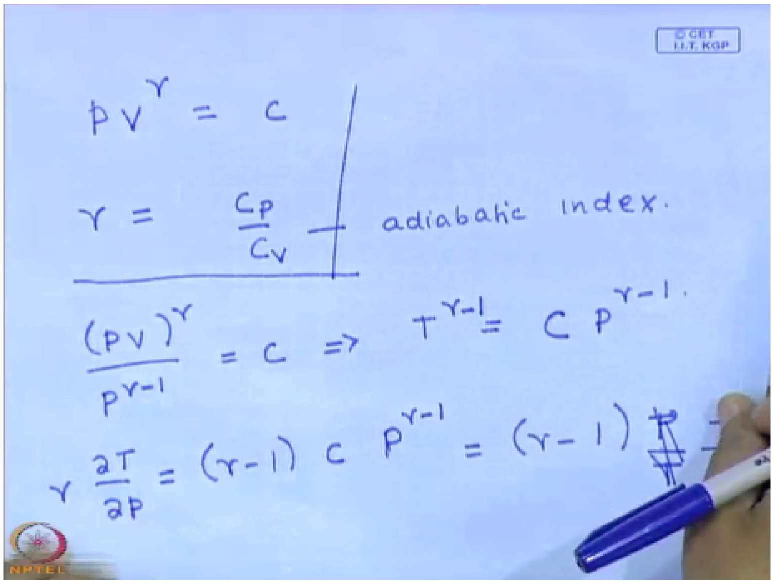

In general, PV to the power gamma is a constant where gamma is the ratio of the specific heat at constant pressure to the specific heat at constant volume. And for an ideal gas we have this. Okay mono atomic ideal gas, single particles. Okay. This is in general for a general adiabatic transformation. And here, we can this is called the adiabatic index gamma. Now we would like to evaluate del P del T and we have to use this.

So we know that PV = NKBT so I can write this as PV to the power gamma divided by P to the power gamma -1 = C which essentially tells us I can replace this by NKT and these factors of N and K are constant so I can absorb them inside this constant what it tells us is that T to the power gamma I can replace this with T to the power gamma = some constant let us say capital C*P to the power gamma - one.

And we can differentiate this now, so we want to we can differentiate this del T/del P and if I differentiate it what I will get is that del T/del P*gamma = gamma - one, C P gamma - 2, CP gamma -2, we can write, so we can write this as gamma -1*T to the power gamma -1, right there should be a T to the power gamma -1 here. So P to the power, this whole thing can be written as P/T. Sorry T/P. Right.

So P to the power gamma -1 we can write as P, so we can just eliminate this from here and you will get T/P. Okay.

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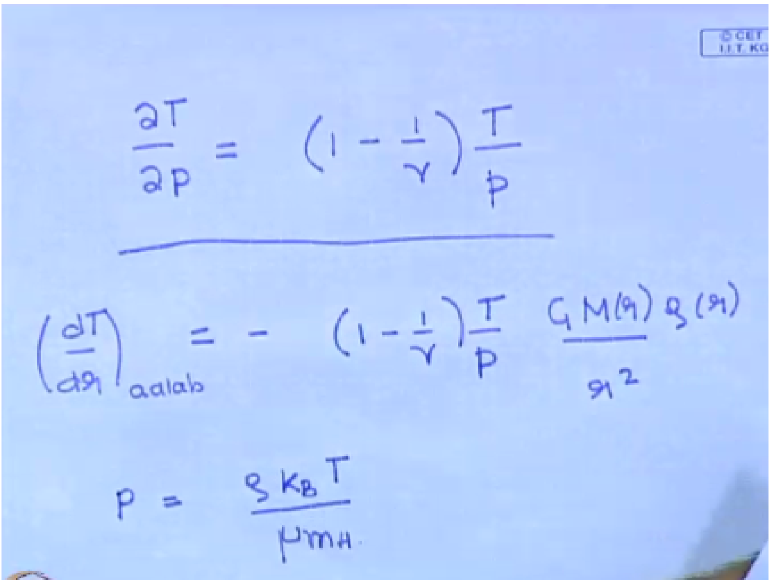

So what you have at the end of the day is that del T, del P =-- so we will 1-1/gamma*T/P. Okay, so that is for a adiabatic process. And we use this to calculate the adiabatic temperature gradient dT dR, so this is now =, so we have --- let me put in one by one okay we have one - 1/gamma T/P, so this is del T del P and then we have del P del R, del P del R is - GM(r)/R square*rho. Okay, that is the dP dR this term, hydrostatic equilibrium. Okay.

And we also know that, P = rho KBT/the average molecule atomic weight, atomic weight divided by the mass of one hydrogen atom. This is essentially the same thing, this is essentially the same thing here written in a different way that is all. Okay. So is the same thing as this, PV = NKBT written in a slightly way in terms of the mass density. And we can use this to replace P.

So here we have P rho, so here we also have T rho/T so we have T rho/P so then substitute in those factor in this equation what we get is the final equation.

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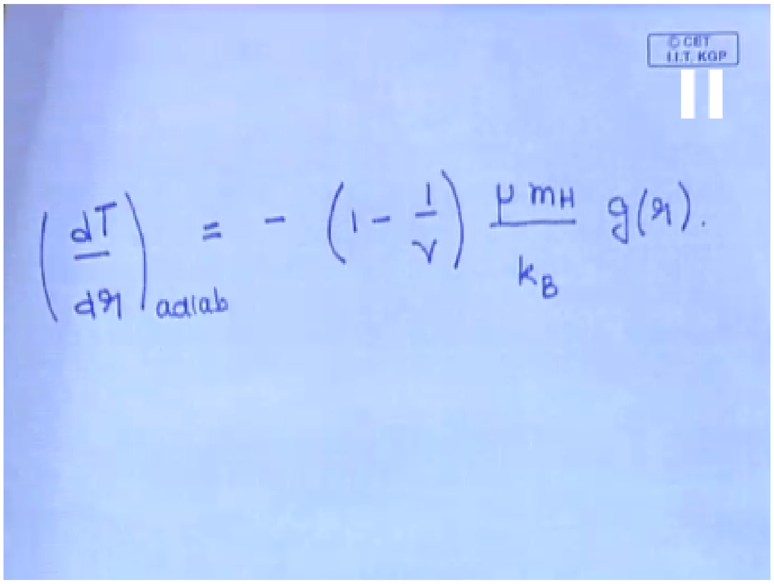

dT dR adiabatic = - one - 1/gamma and then we have so ho T and P comes here, so we have Mu mH/KB into the gravitational acceleration at the point, at the distance from the centre. Okay. So we have now obtained the 4 equations that have to be solved to determine the structure of a star and these 4 equations let me show them to you once more the equation that we have to simultaneously solve.

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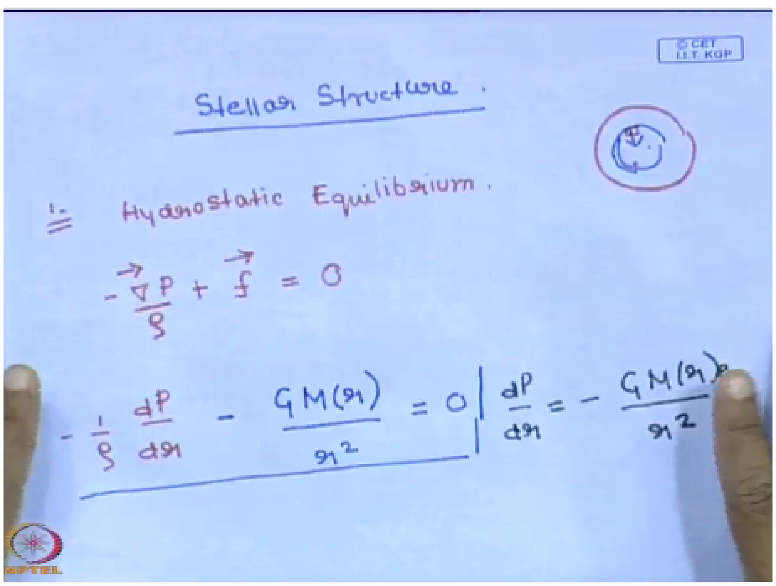

So the first equation is the equation of hydrostatic equilibrium. This gives the pressure as a function of r.

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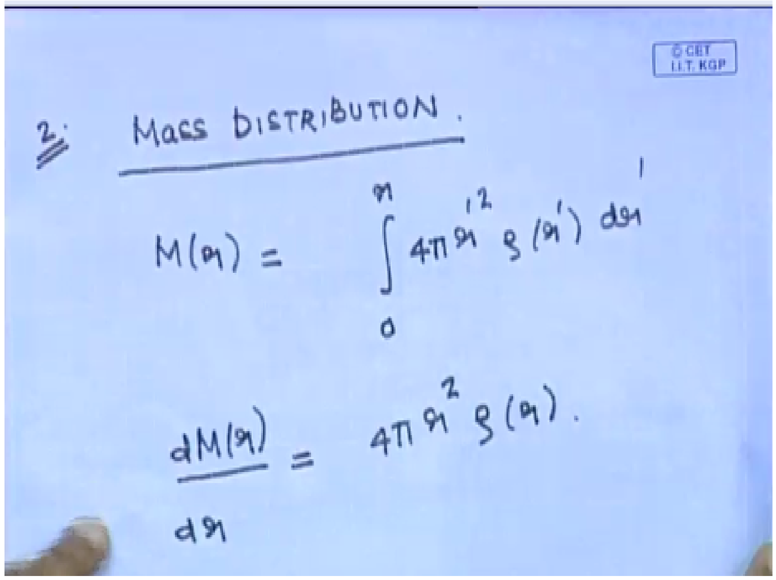

The second is the mass distribution.

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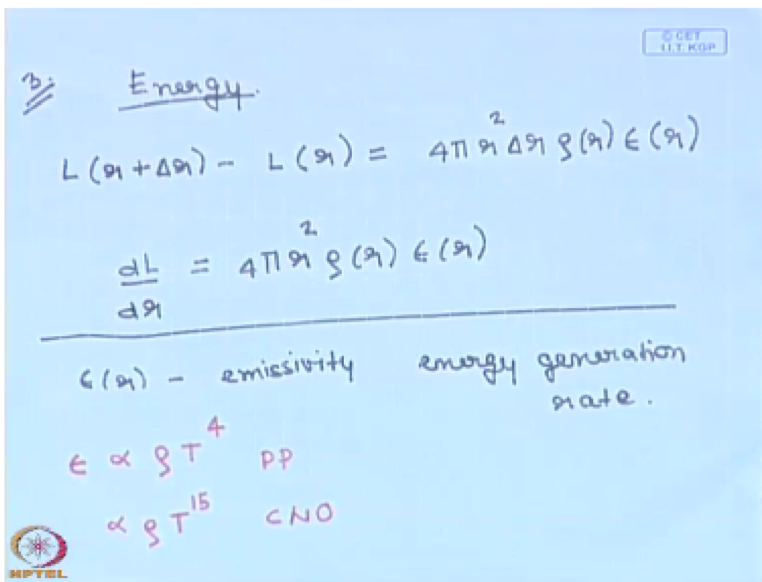

The third is the way the luminosity changes with distance.

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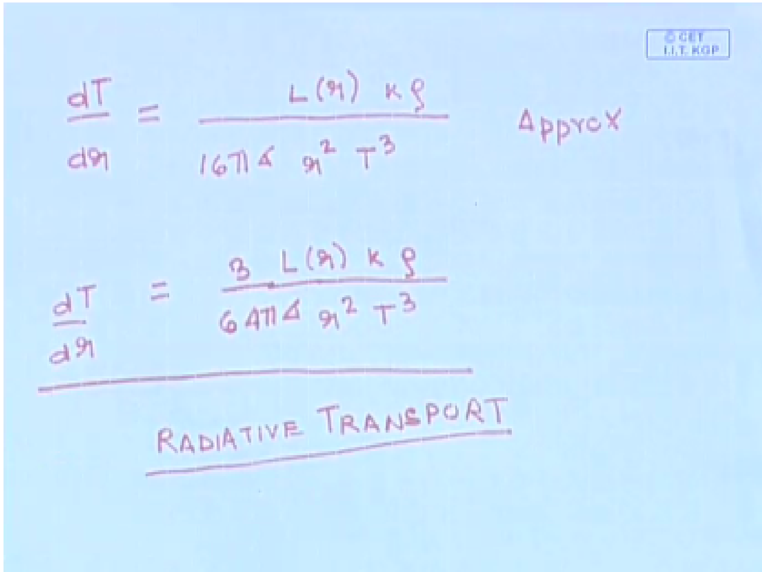

And the forth is the temperature gradient which is decided by the energy transport mechanism, if it is radiative transport you have this, if it is convective transport you have this. And which of them you have to use is determine is to be determine by seeing which one is, so large if this is smaller than this then you have to use this, the moment this becomes bigger than this, you have to use the adiabatic temperature gradient, and we know that convection had been set in. Okay.

So you have to simultaneously solve these 4 equations. In addition to this, equations there are certain secondary equations which are also required. So let me tell you what these secondary. So these 4 equations in themselves are not adequate.

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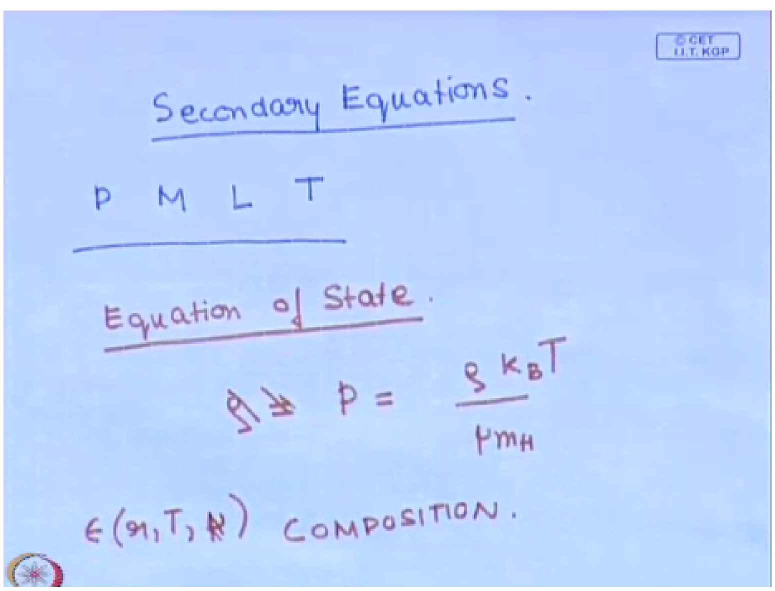

There are certain secondary equations that also have to be solved, okay. So if you look at this you have 4 unknown quantities, the 4 unknown quantities are the pressure the mass, all of these are functions of R then you have the luminosity and the temperature all of them vary with r. But there are other quantities that appear in the equations. So these quantities for example, if you look at the first equation, you have the density, right.

So you have to-- for the density one has to use the equation of state the equation of state are just wrote down which is the density = or P let me write it differently and P = Rho/Mu mH KBT you have to use this, right. So if you use this then the density is no longer in independent quantity it depends on the pressure and the temperature. You also need some other factors so let us look at those, so you need to know, okay.

Let us look at the mass profile, the mass profile does not involve anything new, it has a density which again you can express in terms of pressure and temperature. But the moment you look at the luminosity the luminosity has the, the energy generation rate or the emissivity. If that comes in when you want to calculate, let the luminosity, the rate at which the energy is being generated at different parts. So you need to be told the, the energy generation rate, Epsilon.

And this has to be calculated as a function you need to know this as a function of R, we know that this is the function and the temperature and also the chemical the composition, so there is a composition over here, right, not only on Mu but also on the composition, so it also depends on the composition. And it will change and this is through nuclear fusion at the rate at which the energy is being generated will be different.

If you change the composition, if you change the fraction of hydrogen we have seen that the energy generation rate changes, it goes as a square of the hydrogen fraction, we have seen this already, okay. So it depends on the compositions, so you need to also tell the composition as a function of the-- this position and okay at the position along the star and that will again change as the star keeps on burning more and more fuel. Okay.

Let us look at the next equation the next equation is the temperature gradient. So if you have radiative transport then you also need to know the opacity that is the absorption, how much radiation; what is the absorption cross-section. Okay. Or the absorption coefficient per unit volume, a unit mass or there is a opacity kappa okay. And kappa again so you need to told-- this kappa opacity.

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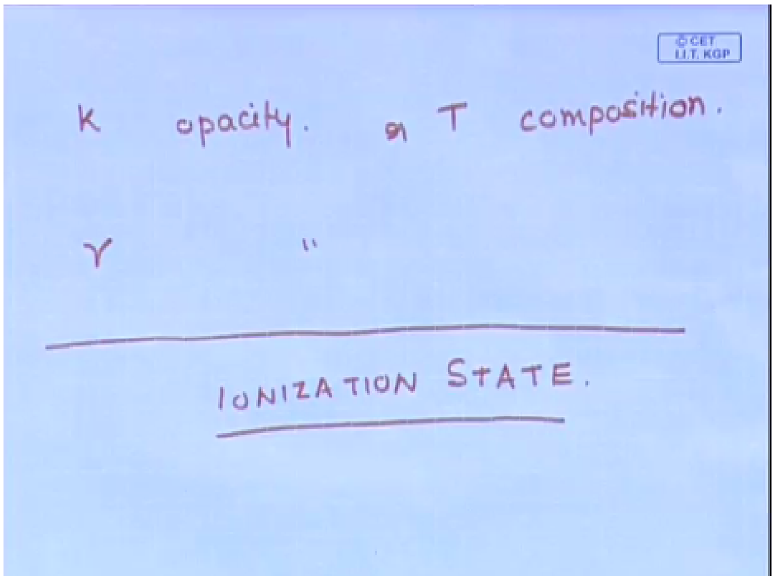

And this again will be function of r, T and the composition.

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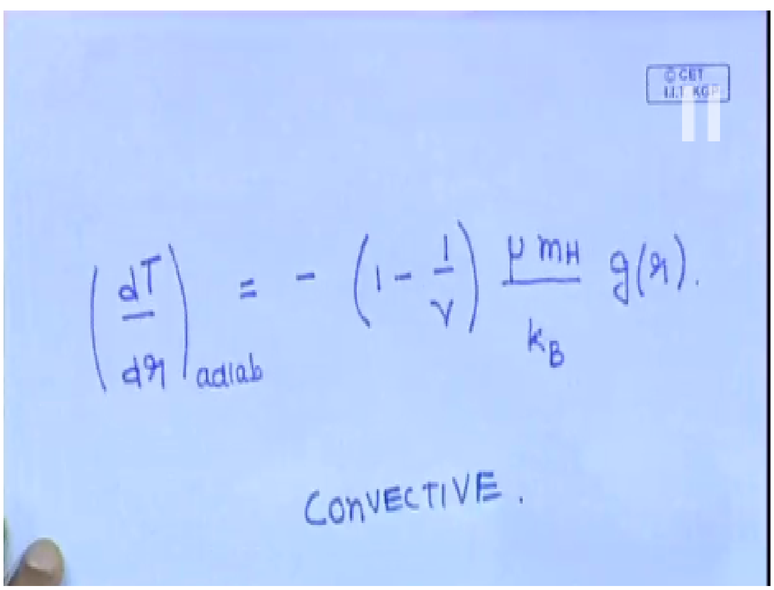

If you have convective transport and to determine which of these 2 is going to be affective you need to also know the gamma, the adiabatic index. Right, so one has to also calculate gamma, is that all factors that one has to simultaneously solve, that you need gamma and it could vary, the value of gamma could in principle vary from position to position. So this again has to be calculated and it will depend on the composition etcetera.

In addition to all of these, one also needs to know the ionization state of the gas. One has to solve for this along the star. It is not necessary that all the different components of a start will all be equally ionized at a given temperature and pressure, right. So one need to solve for the ionization state of the material inside the star simultaneously with all of these. Okay. And the ionization state for any species is tells us what is the ration of the fraction that is ionized to the neutral fraction, okay.

So you can use this we have to calculate essentially this. And this is given this has to be calculated using the Saha ionization equation. So what is the Saha Ionization equation for a gas at a certain temperature and pressure also given at a certain temperature and density you may say, it let us calculate what fraction of the gas is going to ionized.

And if there are several possible ionization states it tells us the fraction of the gas which is of the atoms which are going to be in each ionization state. So this is something that I am going to take up in the next class, the Saha Ionization Equation and let me stop todays class over here, tomorrow we are going to take up the Saha Ionization Equation and then how to proceed then what are the solutions that one obtains for this stellar structure.

Today, I have told you what the basic equation for determining the Stellar structure are, if you want to determine the state of a star you have to simultaneously solve all of these with the appropriate boundary conditions, the boundary conditions either the pressure should vanish at the surface, okay, that is the main boundary condition.