

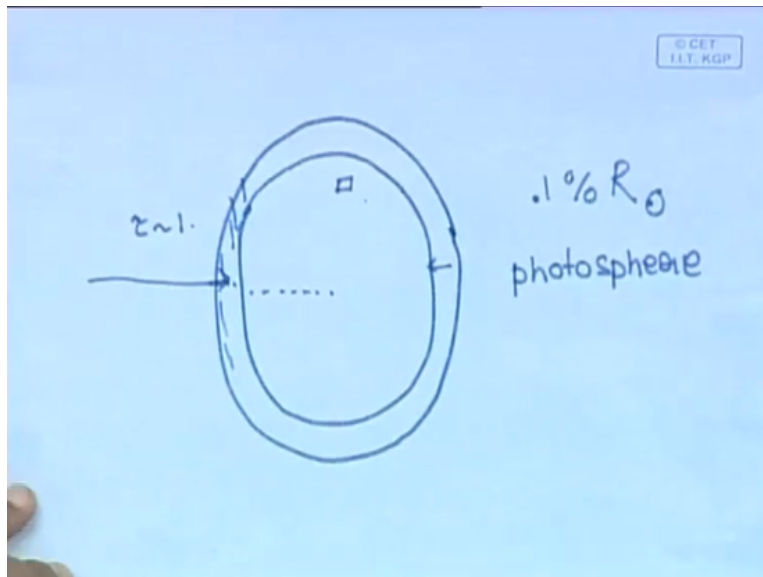
Astrophysics & Cosmology
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Lecture – 15
Stars: Magnitudes and the H-R Diagram

Welcome. In the last class, we were studying what we can possibly learn by looking at the electromagnetic radiation from the sun. And what we saw was that the electromagnetic radiation from the sun peaks at around 5800 at around no it has an effective temperature at 5800 Kelvin and if you look at the spectrum it peaks at around yellow and in the region around the peak where the bulk of the radiation energy is localized in frequency.

It is well-described by a Black-body Planckian spectrum with a temperature of around 5800 Kelvin. That was the first thing.

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The second thing that we learnt was that the radiation originates from a thin shell, well it originated from a surface it affectively originates from a surface where the optical depth reaches unity. So we are looking from here and the radiation originates from a surface of the sun inside the sun where the optical depth is of the order of unity. So it originates from a thin shell around this around this.

And the thickness of this is extremely small it is around 0.1% of the total radius of the sun, and this is called the Photosphere. So the Photosphere is where the optical depth reaches 1. So that radiation receives originates from here and there is evidence in the radiation that the temperature of the sun increases as you go inward. So the temperature increases as you go inside the sun. So there is evidence of the temperature increases as you go in.

These was the 3 things that we learnt in the last class. Today let us try to work out what the interior of the sun looks like. Okay. So we start off by considering this to be a spherical mass distribution some gas, okay which is in hydrostatic equilibrium. And we have written down the fluid equations if you remember and in the situation which you use to describe hydrostatic equilibrium the equations are quite simple so the equation there is only--

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$$-\frac{\vec{\nabla} P}{\rho} + \vec{f} = 0$$

$$\vec{f} = -\vec{\nabla} \phi ; \nabla^2 \phi = 4\pi G \rho$$

ρ is constant SUN.

$$\rho = \frac{M_{\odot}}{\frac{4\pi}{3} R_{\odot}^3} = 1.4 \text{ gm/cm}^3$$

Earth 5.5 gm/cm^3

So in hydrostatic equilibrium the pressure gradient divided by the density should be equal to rather this + the force per unit mass f , these 2 should be balanced. Okay. Now the situation that we are dealing with the force per unit mass on any element of this fluid is essentially the gravitational force. So we can represent gravitational force in terms of a potential; so this is $-\text{grad } \phi$ where the potential satisfies the Poisson equation.

So inside the sun the pressure gradient essentially balances the gravitational attraction of the material inside. So if you consider this element of the fluid the pressure gradient across this

balances the gravitational attraction of all the material inside this field. Okay, which is what is there in this equation, it is a self-gravitating system. Now in order to solve this equation we need to know the density is, we need some relation since there are 2 unknowns there is the pressure and there is the density.

We shall start off with a very simple assumption our assumptions here is that the density inside the sun is a constant. So we are going to make the simplifying assumption that ρ is constant. So let us estimate the value of the density of the-- the mean density of the sun. And we will assume that this value is there throughout. It is a rather simplifying assumption but we can make some progress with this.

So this value of the mean density is the mass of the sun which I have already told you the value divided by the volume of the sun which is $\frac{4}{3}\pi r^3$, and I have already given you these values; so we will assume that this density holds throughout the entire sun. Okay. And if you put in the values, so this is 2×10^30 kilos this is 6.96×10^8 meters, I have already told these values.

So if you put in all of these numbers the density the mean density of the sun comes out to be 1.4; let me express it in grams per cc, grams per centimetre cube. Okay. So the density of the sun we see is comparable to the density of water, right. For comparison let me also tell you the density of the earth, this is for the sun. The density of earth is around 5.5 grams per cc centimetre cube. Okay. So it is roughly 4 times the dense the earth. Okay.

So density of the sun, the mean density is comparable to that of water. And you might not think that this is some kind of a liquid but it is not so this is a mainly made of a hydrogen. And the temperature inside is sufficient temperature and the density inside is sufficient to ionize the hydrogen, so this is essentially a plasma where you have the charge the protons and the electrons free inside this.

The total charge is 0 but the protons and the electrons they are not bound to each other they are free. And they both behave like perfect gases. That is the situation. Okay. So we are going to

assume that the average density inside is this and it is a constant. So with this assumption let us go ahead and try to solve this equation. So solving this equation now is quite straightforward, we also have spherical symmetry, so we will just take multiply this over here.

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The image shows a handwritten derivation on a blue background. At the top right, there is a small logo for '© CET I.I.T. RGP'. The derivation starts with the vector equation $\vec{\nabla} P = -\rho \vec{\nabla} \phi \Rightarrow \nabla^2 P = -4\pi G \rho^2$. Below this, the equation is written in spherical coordinates as $\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dP}{dr} \right) = -4\pi G \rho^2$. This is then simplified to $r^2 \frac{dP}{dr} = -\frac{4\pi G \rho^2 r^3}{3}$. The final result is boxed: $P = -\frac{2\pi G \rho^2 r^2}{3} + K$. Below the box, it is noted that $P=0$ at $r=R_0$.

And what we have then is grad p =, so grad p = this-- if I put it here the minus sign s cancels out then I take it onto the right hand side, so this is = rho*grad phi with the minus sign. Further I could take a diversion of this and we are led to the equation that del squared p. Now we have assumed that rho is a constant so I can take it outside so I have del squared of phi so this is = -4 pi G rho square.

And we can write this in spherical polar coordinates so if I write it in spherical polar coordinates what I have is one/R squared, okay. So this is-- we have to now solve the equation. We are working spherical polar coordinates. So we integrate this equation from this inside towards out and if you integrate this what you get is this will be r squared here, 1/r squared and d/dr squared, so this is the Laplacian operator in spherical polar coordinates. Okay.

So what we can do now is we can multiply this with r square and integrate it ones so if I integrate it ones I will get r cube/3, so what this gives us finally then is r square dr, p = -4/3 pi G rho square r cube. Right. So what I have done is taken the r square there and integrated over r so that gives me r cube/3. Then this I can again divide by r square and so that will give me a factor of r

and then I integrate it so it will give me half r square, so finally the pressure = - 4/3 pi G rho sorry the r square and this will not be 4 it will be 2.

So it is - 2/3 pi G rho square, r square + a constant of integration K. Right, that is the pressure inside the sun in this simple model. Okay. Now let me ask the question, what is the pressure at the surface of the sun? We have to put some boundary condition, so we can measure things at the surface and let us ask the question what is the pressure at the surface of the sun. Now obviously the pressure outside is 0.

So the pressure at the surface of the sun, has to be 0 that sets the boundary condition. So at r = r nought p = 0 at r = the radius of the sun. And we use this to determine the constant k. So with this imposing this boundary condition what you have is that the pressure...

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$$P = \frac{2}{3} \pi G \left[\frac{M_{\odot}}{\frac{4}{3} \pi R_{\odot}^3} \right]^2 R_{\odot}^2 \left[1 - \frac{r^2}{R_{\odot}^2} \right]$$

$$P_c = \left(\frac{3}{8} \pi \right) \frac{G M_{\odot}^2}{R_{\odot}^4} = 1.36 \times 10^{14} \text{ N/m}^2$$

Earth. 10^5 N/m^2

$P = 2/3 \pi$, okay. Further, it is convenient to express this density in units of the total solar mass. Okay. So this then can be written as $2/3 \pi G M_{\text{sun}} / 4/3 \pi R_{\text{sun}}^3$ that is the density the square of this that is the density. So we have $2/3 \pi G \rho^2 R_{\text{sun}}^2$, $2/3 \pi$ squared and then we impose the boundary condition and we have term, so, right. So this is the pressure inside the sun.

So you can see that this all of these into one into constant of integration and this term into all of this is essentially just this. So we have written it like this, and you can it is now very clear that

when the radius r goes to the radius of the sun the pressure vanishes. Okay. Now we will use this to calculate the pressure at the centre of the sun. We want to calculate it is inside the sun so let us use this to estimate what the pressure at the centre is. Okay.

So the pressure at the centre at the core of the sun according to this then has a value let us see how much it comes out to be. So you have $2/3$ s here and in the denominator you have $16/9$, so a factor of 3 remains in the numerator, and here you have 16 and here you have 2 so factor of 8 remains here and you have a factor of π also in the denominator, so that takes care of all the numerical coefficients.

And then we have $G M^2 / r^6$ the mass of the sun squared/ r to the power of 6 so here we have r to the power of 6 the radius of the sun and here we have the radius of the sun square so this is R_{sun} to the power of 4 that is the pressure at the core of the sun using this simple the simple approximation assumption that the density is constant throughout. Okay. So we have an estimate for the pressure at the centre of the sun and it comes out.

It is expressed in terms of just the Newtonian the universal gravitational constant the mass of the sun and the radius of the sun, it depends on just 3 and there is a numerical coefficient outside. So you now put in the values we can put in the values of G the mass of the sun and the radius of the sun and if you put in the values the-- we get an estimate of the pressure at the centre of the sun and this comes out to be = 14 Newton's per meter square.

I will ask you to yourself check this put in the numbers and just calculate for yourself, check it for yourself. Okay. So this is an estimate of the pressure at the centre of the sun. Now I should point out that in this expression we have made some drastic assumptions and these drastic assumptions give us this result. Now in this result, this numerical factor depends crucially on the assumptions that you make.

If you had assumed some density profile then this numerical factor would have been different, okay. So a more realistic estimate will give us some other numerical factor over here but this part will more or less would be the same. Okay. So this is an order of magnitude estimate of the

pressure, some real-- so you should not take this very it is an order of magnitude estimate of the pressure. Say it will be within 10 or 100 times the actual pressure inside the sun.

But it gives us some feel for what the pressure. Okay. And this is the value that you get for comparison let us also estimate the pressure at the surface of the earth. Earth surface, we know the pressure is equivalent to 76 centimetres of mercury and we know the density of mercury so we can calculate the pressure at the surface of the earth and the value comes out to be of the order of the 10^5 Newton's per meter square.

That is the pressure on the surface of the earth. So you see there is considerably larger, many orders of magnitude larger than the pressure at the surface of the earth. It is an enormous pressure that you have at the centre of the sun. Basically the pressure is balancing is due to the entire weight the gravitational is balancing the gravitational of attraction of the entire mass that is above this.

Just like you calculate the pressure at the earth surface you take the air column above the earth and the weight of that is the pressure, okay similarly here you do exactly the same thing except that the gravitational acceleration also changes without the density could also possibly change with that we have not taken that into account over here. Okay. Now, next we would also like to estimate the temperature at the centre of the sun. Okay.

Now to estimate the temperature at the centre of the sun that is, so we will assume that the we have pure hydrogen, now we know that the sun is actually a combination of hydrogen and helium mainly; there are other elements but they contribute in significantly to the density. The density of hydrogen is also considerably larger than the density of helium so we shall for that— for the approximations here ignore helium and just proceed with hydrogen. Okay.

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$$p = n k_B T ; n = \frac{\rho}{m} = \sum \frac{\rho_i}{m_i}$$

$$n = \frac{\rho}{m_H \mu} \quad \mu - \text{mean atomic weight.}$$

$$\sum \frac{\rho_i}{m_i} = \frac{\sum \rho_i}{m_H \mu} \Rightarrow \mu = \frac{\sum n_i m_i}{m_H \sum n_i}$$

$$\mu = \frac{n_H m_H}{m_H 2 n_H} = \frac{1}{2}$$

So for any gas if it is an ideal gas then the pressure and the temperature are related as follows. It is-- the pressure is the number density of particles into the Boltzmann constant into the temperature. So if you know the pressure and if you know the number density you can determine the temperature. Okay. Now the number density is the mass density divided by the mass of individual particles. Okay.

And if I have many species of particles I have to then add up the contribution from each of these species. Now we write this number density of the particles in as the density of the gas divided by the mass of one hydrogen atom, so if it were purely hydrogen atoms this is the number density that you would have into this factor μ which is the mean atomic weight of the constituents, okay so μ is the mean atomic weight, right.

So essentially what we have done is we have written the sum $\rho_i / m_i = \frac{\rho}{m_H \mu}$. That is what we have done. The total-- if there are many components the sum of each the density of all the components is the total density ρ . And the number density we have expressed like this, so it is the total density divided by the mass of hydrogen atom divided by the mean atomic weight, okay.

So we have just expressed this term like this, right. So from here we see that μ , $\mu =$, now the mean density is the number density of each species into the mass of each species that is ρ ,

divided by this is the number density of each species the sum over that so it is-- sorry this is n_i , right.

So it is essentially the number density of each species into the mass of each species divided by the sum of the number densities of all the species into the mass of the hydrogen atom that is what you mean by the mean atomic weight, right, in units of the mass weight of the hydrogen atom. Okay, so this is the-- this is how you calculate the pressure. And here we are assuming that we have only hydrogen and the hydrogen is ionized, okay.

The temperature and the pressure are adequate to ionize the hydrogen, so we are assuming that the hydrogen is ionized, okay. So if the hydrogen is ionized then we have 2 species we have the protons and we have the electrons. Okay. The mass of the electrons we know is negligible compare to the mass of the proton, okay. But the proton is an electron contributed to the same amount to the number density. Right.

So from here, we can say that μ , so in the numerator we have the contribution only from the protons because the mass of the electrons is negligible. Okay. In the denominator, we have the contribution from both the protons and the electrons. So the numerator is essentially n_i where n_i is the number density of say protons into the mass of the hydrogen atom n_H let us say into m_H , the denominator is m_H and here the electron and the proton density are same.

So we have $2n_H$ so this comes out to be = half. Okay, so for ionized hydrogen this is half. If you put in the fraction of helium, then it will get somewhat modified. Okay. The mean atomic weight for ionized hydrogen is half so we can use this. So all that you have to do at the temperature is you have to—

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$$T = \frac{m_H \mu P}{\rho k_B} = 5.8 \times 10^6 \text{ K.}$$

$T =$, so the temperature = the $m_H \mu P$ and you have to divide by $\rho \cdot k_B$ let give this the temperature, the pressure that you have calculated to calculate the temperature inside the sun and we can use this to calculate the temperature at the centre of the sun. So let us take this and estimate the temperature at the centre of the sun. So you put in the values of, so we know the density we have just estimated it 1.4 grams per cc.

You know Boltzmann constant, you know the hydrogen mass and hydrogen atom and here you put half then all of these factors and you get the temperature the temperature comes out to be = if you use this the temperature comes out to be = $5.8 \cdot 10$ to the power 6 Kelvin somewhere over here. Okay. So again do not take this number very seriously the main point is that the temperature also will scale proportional to these factors.

And it will have some numerical factor which could be half by a few say one order or so. Okay. So the temperature at the centre of the sun from what we have estimated is somewhere of the order of tens of millions Kelvin. It is extremely hot which is adequate to ionize all the hydrogen so what I have told you is also validated here so inside the sun the hydrogen is ionized the temperature is of the order of the 10s of millions Kelvin.

And it falls off to around 5800 at the photosphere. Okay, so the temperature is around few millions tens of millions of Kelvin here and falls off to around 5800 Kelvin at the photosphere.

This is the temperature gradient as you come outside. Okay with this simple estimate this is what we arrive at. Okay. Now, the radiation also then; let us see what kind of radiation do we expect at the centre of the sun.

So if you have tens of millions of Kelvin what kind of radiation do you expect at the centre of the sun. Obviously, it is not going to be in the visible, right. At such high temperatures X-ray is the most dominant kind of radiation. So at the centre of this sun you predominantly have X-ray. Okay. So at the centre you have predominantly X-ray and then this light propagates outward and by the time you are at the photosphere.

You have lowered the temperature of the material and the radiation and you have a temperature of around 5800 Kelvin. Okay. So, now we have also studied radiative transfer, so let us now take a look at the propagation of this radiation from the centre of the sun to the outside that is the next thing that we are going to look at. Roughly let us get the rough picture of how the radiation comes from the centre of the sun to the outside. Okay.

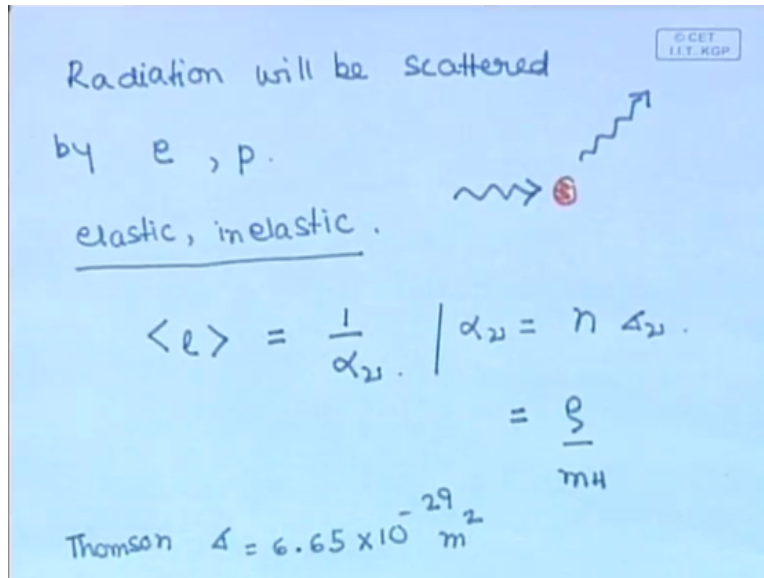
So that is the next question that we are going to look at. And if you looking at the propagation of your radiation, now this radiation is going to scattered with so matter and the matter that we have here is essentially protons and electrons. These are charge particles so electromagnetic radiation interacts with charge particles if you know that so they are going-- there is going to be scattering of the radiation from these charge particles. Okay.

And, this scattering in general there will be energy transferred in this scattering; scattering could be elastic or inelastic. Now here we know that the radiation loses energy as it propagates through this medium, so we know that the radiation the scattering is going to be in general both elastic and inelastic. We also know this from other consideration, X-ray when you scattered of electrons you know that there is going to be Compton scattering and which in elastic.

It can lose energy to the electron. You could also have inverse Compton scattering, okay. So there is energy transfer and between the electrons and the radiation and the matter as the

radiation propagates it losses energy transfers energy exchange energy with the matter and slowly comes out. Okay. So we will have both elastic.

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So the radiation will scatter will be scattered by the electrons and by the protons, right by the ions both. The electron is obviously going to contribute more than the ions but both of these affects are going to be there. And these will be are both elastic and inelastic. Both of these will be there. Now, we know, so let us try to estimate the mean free path of the radiation inside the sun. Okay.

And we know how to estimate the mean free path we have discussed this quite some time ago when we were discussing radiative transfer. So the mean free path which we can denote by L that is the mean distance between which the photon propagates freely before it encounters a scattering before it encounters before it interacts with the matter right is decided by where the optical depth becomes unity inside it.

So what we have to do is consider a photon which starts off from somewhere and see where the optical depth becomes unity when the optical depth becomes unity means it has interacted ones, high probability interacted ones. So this is essentially the condition that the length into the absorption coefficient should be unity or this mean free path we have seen is one by the absorption coefficient inside. Right, we have seen this.

And the absorption coefficient we know is the number density of particles into the scattering cross-section of each particle. Right. So let us consider for making a simple estimate let us just consider one process the Thomson scattering, Elastic scattering. So for elastic scattering, we know the Thomson scattering cross-section we have discussed this already when we were looking at the Eddington limit, this is in the problem.

So the absorption coefficient in the inside the sun we can estimate to be the density inside the sun which we have already estimated divided by the mass of the hydrogen atom, this will give us the electron density, we are assuming that the electrons primarily scattered the radiation through Thomson scattering. In Thomson scattering what happens, let me draw a picture. So in Thomson scattering this is my electron, the radiation incident on this is scattered into some other direction.

And you can treat the electrons as oscillating because of the incident electromagnetic radiation electric field and then calculate the dipole radiation from that oscillation, that is comes from scattering. Okay. And Thomson scattering we can assume for our purposes it senses the incident radiation into some other direction its frequency does not change. And the scattering cross-section of Thomson scattering.

We have already discussed mention this earlier it has a value for the Thomson scattering, and it has value σ which is independent of frequency and the value is 6.65×10^{-29} meter square. We have already discussed this earlier. Okay this is Thomson scattering. So the electrons inside the sun will scatter the radiation and the number density of electrons is the number density of hydrogen atoms same.

Because it is in ionized simply ionized so it is the density divided by the mass of hydrogen atom into the value of σ , this will give me the absorption coefficient and one by this absorption coefficient will give me the mean free path. If you take these numbers, we have already estimated this, this we know and this we know so you take these numbers put them in here. The photon from inside has to propagate over a distance which is 10^8 meters, 6×10^8 meters to get out.

And then finally it comes to us. It can only propagate around a centimetre before it gets scattered by Thomson, by electrons. And as a consequence of this scattering the electron that is propagating outwards will now be after scattering will now be propagating in some other random direction and so the picture is that the electron that the photons which come out get kicked back and forth randomly many times okay at intervals at these intervals at roughly 1 centimetre intervals and we have to keep on getting kicked around like this.

And they may get slowly out by such repeated kicks random kicks. Okay that is the picture. So we will now try to estimate some other-- let us try to make a more quantitative description of this. Okay. Now, we have studied radiative transfers, so let us see what happen if you try to use radiative transfer to describe this process of scattering, okay.

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isotropic, elastic.

$\alpha_{21} = n \alpha_{21}$

$$\frac{dI_{21}}{dt} = -I_{21} + S_{21}$$

$$j_{21} = \alpha_{21} \int \frac{I_{21}(\hat{n})}{4\pi} d\Omega_n$$

$$S_{21} = \frac{j_{21}}{\alpha_{21}} = \int \frac{I_{21}(\hat{n})}{4\pi} d\Omega$$

So we are going to assume that the scattering is Isotropic that is what we are going to assume. Okay, it is Isotropic and it is, so it sends off the incident radiation so we are modelling scattering as a very simple thing elastic and Isotropic. So the scattering is we model as follows; there is some incident radiation what the scattering does it is that is sends it off in some random direction the frequency remaining the same. Okay, that is our model simple model.

The real scattering process could be more complicated it could be an isotropic. The probability of scattered radiation going in different direction could be direction dependent. It could also be inelastic. But let us just make this simple kind of thing. Okay. For this simple kind of a – with this scattering we have already seen the absorption coefficient α_ν is the number density \times σ_ν . Okay.

And then we can write down the radiative transfer equation it is $dI_\nu/d\tau = -I_\nu + S_\nu$. Now, we know how to calculate τ we just did it. What we have to now look at is the source function. Now here in scattering the scattered radiation-- radiation that is scattered from this direction will be a source in some other direction because the radiation that comes in goes out in some other direction. Okay.

So let us ask what is the emissivity, emissivity coefficient, emission coefficient J_ν . So the emission coefficient we can calculate that is the radiation emitted per unit volume in a certain direction. Okay. So the incident radiation is I_ν and radiation from all directions will be observed equally so I have to integrate over all solid angles and it will be going out equally in different solid angle.

So I have to divide by 4π and multiply this with n into with basically α_ν . So the emission coefficient is the absorption coefficient into just this factor, right. The radiation that is incident is scattered into different directions that is all. So if you put in the fact-- the emission coefficient and the absorption coefficient are related like this. So the incident radiation from all directions the average over that and then it gets randomly distributed over 4π .

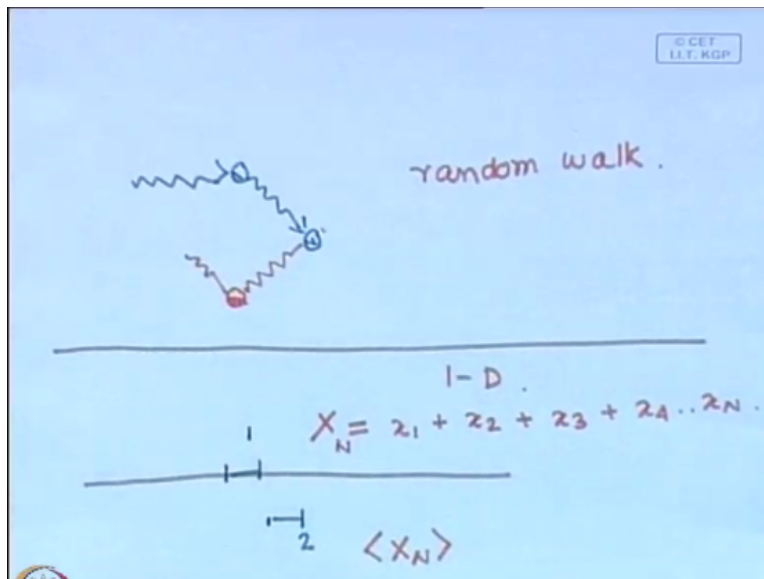
So from here it is clear that the source function which is J_ν/α_ν this is = the integral $I_\nu/4\pi$. So in the presence of scattering the radiative transfer equation essentially looks like this, right. So where is-- what is the difficulty with such an equation. Let us consider I am trying to follow the evolution of the light along a particular ray and I am looking at it at this point here at this value of τ .

Now if I want to study what happens when I go from here to here specific intensity from all directions over here, because all of these can get scattered into this direction. So I cannot solve my radiative transfer equation separately for one ray I have to simultaneously solve it for all different rays at all different locations which is-- so this is an integral differential equation and it is extremely difficult, okay. Integro-differential equation, okay this extremely difficult to solve.

And in journal there are no I mean there are no general solutions available like we had when there was no scattering. In this case with scattering which is you have to make approximation and then proceed. Okay, so it is very difficult problem we shall not go into this. We shall analyse the whole issue in a kind of a simple approach the approach that we shall make is as follows, we shall model this whole thing the whole propagation of this radiation as a random walk, okay.

So the incident just looks at any one photon.

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This is a photon like this it gets scattered over here and after the scattering it goes in some random direction and then again it scattered over here and it goes in some random direction let us say like this and again it gets scattered over here and it goes in some random direction could be here, okay. And in this process the photons move around that is the motion, so it is called a random walk.

So at every step at every scattering event the direction gets shifted to a random, random which was in direction. Okay this is called a random walk. Well, you can think of it as a drunker who sets out from the pub and wants to go home and his steps are all random. Okay, there is a probability that he will reach home but it is a difficult thing. It is a difficult task. So the photon here also starts off from the centre of the sun and it gets kicked around random.

There is a chance that one of the photon will escape and come out. Okay. So we would like to move out and understand how the photon propagates under such a thing. To understand this to analyze this let us consider a random walk in one dimension. So there is a 1-dimension x , we call it x -axis and my photon starts off from here. In one dimension, it is quite simple it can either go to the left or to the right.

So let us say that it goes to one step to the left and in my next step; so this is the first step, step one. Okay. Then, in my second step again it can go either to the left or to the right, so it could let us say it go to the right one, this is step 2. Okay. And each step it randomly decides whether it is go to the left or to the right. So, what we will do is let us consider the position of the photon after N such steps, okay.

So we will call this position of the photon after N steps and this will be a sum of small x_i that is the 1st step + the position after the 2nd step 3rd step + the 4th step etc. all the way to x_N . And these x_n 's are random numbers they could be either be $+1$ or -1 . Okay. Or they could be + some random numbers or - some random number. So if you now ask the question what is the mean, so random numbers can be generated.

I could have many realizations of this that at each step I generate a random number and determine the new position. I can do this process many times and ask what is the expected position of the photons after N steps. Okay. Now each of these are random numbers so the average which are equal probability of being positive or negative so the average of x_1 average of x_2 are all 0. Okay, so the mean position displacement is 0.

But, if you ask the question what is the mean squared displacement.

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$$\langle X_N^2 \rangle = \langle x_1^2 \rangle + \langle x_2^2 \rangle + \dots + 2 \langle x_1 x_2 \rangle + \dots$$
$$\hookrightarrow \langle x_1 \rangle \langle x_2 \rangle = 0$$
$$\langle X_N^2 \rangle = N \langle l^2 \rangle$$
$$\left[\frac{R_0}{\langle l \rangle} \right]^2 = N$$

So let us look at X_N squared, so this will have x_1 squared the mean of this + x_2 squared the mean of this and N such terms + there will be $2 x_1, x_2$ the cross terms. Okay. Now, we know that if you have 2 random variables which are independent then the mean value of the product is the product of the mean values. Okay, so this term is essentially the same as x_1, x_2 and it is 0. So it is only these N terms over here which contribute there are N such terms. Okay.

And the magnitude, so this is the mean square mean square displacement in for one step and this is going to be the mean free path of the photon, right. So we know that the mean square displacement after N steps is going to be the number of steps into the-- okay so this is the mean free path so this can I can write it as this the mean free path squared. Okay. That is the mean free path squared. That is the typical distance between each scattering. Okay.

So now you see that the photon, so to cover a distance okay so let us see to cover a distance R sun, the photon will need R sun by the mean free path the square of this-- these many steps. Right, for the photon to come out from the centre of the sun in steps of the mean free path random steps of that size the photon will need these many steps. Okay.

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$$\begin{aligned}
 T &= \frac{\langle \ell \rangle}{c} \frac{R_{\odot}^2}{\langle \ell \rangle^2} \\
 &= \left(\frac{R_{\odot}}{c} \right) \frac{R_{\odot}}{\langle \ell \rangle} \\
 &= 25 \frac{R_{\odot}}{\langle \ell \rangle} \sim 7600 \text{ YRS.} \\
 &30,000 \text{ YRS.}
 \end{aligned}$$

And the time the photon takes for each step, so we want to estimate the time the photon takes to come out from the centre of the sun, okay the time for each-- this is going to be the time for each step the time for each step is the mean free path divided by C, this into the number of steps which is-- will give the time the photon takes to come out. Okay, the photon does a random walk inside the sun and finally it at sometime it will be out in the course of this random walk.

So this is the time that it will take. It has to take these many steps and this is the time for each step which we can write as the, this is the time the photon would have taken if it travels freely from the centre of the sun out. And that time we know, the radius of the sun is roughly 2 light seconds. Okay. The radius of the sun is roughly 2 light seconds so this will be 2 seconds into the ratio of R sun by the mean free path. Okay.

So let me remind you what we are discussing, if the photon propagates freely from the centre of the sun outwards it will take barely 2 seconds to come out. But it cannot propagate freely it has to it gets scattered after travelling roughly every half centimetre half a centimetre, or whatever 1.7 centimetres. It changes direction. As a consequence, it takes much longer and the amount by which it gets increased is basically the ratio of the distance it has to cover to the mean free path.

So if you now put in these numbers we have estimated we know both these numbers, if you put it in these numbers this comes out to be somewhere around 7600 years. So it takes much longer for

the photon to come out because it has to slowly diffuse out from the centre through repeated scattering. Okay. Let me also remind you that the estimation that we have made is an underestimate because I have underestimated this scattering cross-section.

A more realistic estimate is 4 times this and it will be around 30,000 years. So if I take the right scattering cross-section and the densities the time it takes the mean free path actually goes down it is around half a centimetre and the time it takes around 4 times so it is around 30,000 years; this is the time the photon takes to come out. Okay. So let me briefly remind you what discussed today.

Today, we have learnt that the centre of the sun is an extremely hot its tens of millions Kelvin, and the pressure is also extremely high, the whole thing is ionized gas and from the centre you expect X-ray and things like that, X-ray is essentially and these X-ray get scattered repeatedly inside the sun and it slowly diffuses out that is how the radiation from the centre of the sun reaches us. Okay.

Now, an important question that we have not addressed is what is the source of this energy? And this is some that we have to look at we shall go into this in the due course as we progress.