

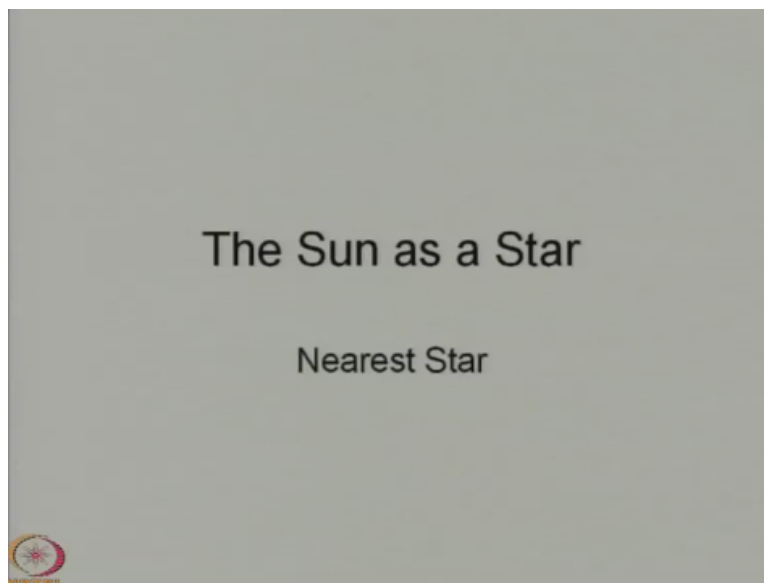
Astrophysics & Cosmology
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Lecture - 14
Virial Theorem and Its Application to Stars

Welcome, let me remind you that we started off this course by discussing our solar system. The planets, asteroids and various other objects that compose our solar system after which we shifted our attention to 2 important tools. One is the study of fluids, fluid mechanics and the other was the study of radiation. Because much of our information in astrophysics is obtained by studying the radiation coming from astrophysical sources.

We shall now move on and the topic that they are going to take up next is stars. And we are going to start off our discussion on stars by considering the star that is nearest to us that is the sun. So, today we are going to discuss certain properties of the sun.

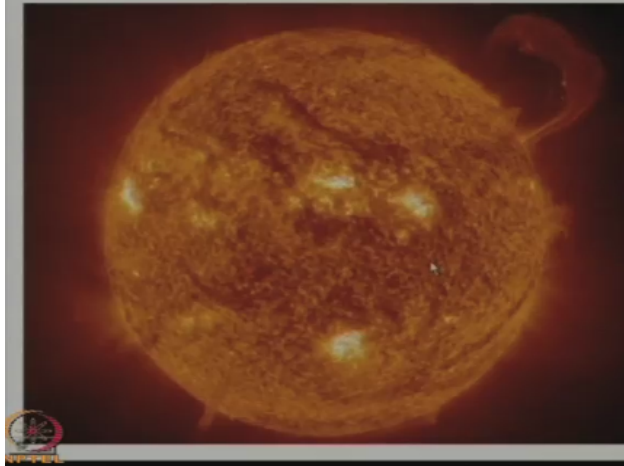
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The sun as a star that is the nearest to us.

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The Sun



This shows you a picture of the sun take on some particular day. Much of our information about the sun is obtained by studying the radiation coming from the sun to the surface of the earth. And this radiation is extremely important because it is essentially the source of energy on the bases of which the entire life is built up on earth. So, the life for example on earth is sustained by eating the nutrition that is obtained from the photosynthesis.

And much of our energy requirements also met by burning fossils fuel which were produced were converting the solar energy into other forms. So, the life on earth as we know it now is sustained by solar energy so the radiation coming from the sun is extremely important for our survival of life on earth. So, let us start off our discussion of this radiation and this radiation can be characterized by something that is called the solar constant.

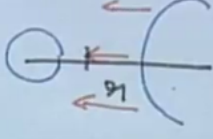
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Solar Constant.

$$f = 1.36 \times 10^3 \text{ W/m}^2$$

Determine $r = 1.15 \times 10^8 \text{ m}$.

$$L_0 = f 4\pi r^2 = 3.90 \times 10^{26} \text{ W}.$$


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So, this is the quantity that can be directly measured. So, let me just draw a cartoon picture here this is the earth and this is the sun. Obviously, it is not to scale and the radiation from the sun is incident on the earth and what the solar constant measures is the flux of this radiation outside the earth atmosphere. So, not all the radiation incident on the earth atmosphere enters the atmosphere so if you were to measure the radiation the flux of radiation coming on the surface of the earth.

It would be much less considerably less because part of the radiation gets reflected back. So, the solar constant is the radiation incident on the earth surface measured outside the earth atmosphere and this has a value 1.36×10^3 watts per meter square. So, this is the flux of solar radiation total solar radiation incident on per unit area of the surface of the earth. And you have to measure this outside the earth atmosphere.

And it is this energy which is incident on the earth which essentially keeps us alive. Keep the whole civilization going keep life on earth going. So, this is what is called the solar constant. Now, the distance from the earth to the sun, this distance between the earth and the sun let us call it r is known from a variety of observations and we have not discussed it. I leave it as an exercise for you to look up the literature.

And see how the value of this r the distance, the earth sun distance is determined. So, how is this determined exercise? That is the exercise which you have to do. Look up the literature and find

out how the distance the earth, sun distance can be measured. And it is well known now that this has a value 1.5×10^{11} meters. We have discussed this. This is what is called the astronomical unit.

So, the earth sun distance is also known and once you know this you can combine the flux the measured flux at the earth, sun distance and you can determine the luminosity of radiation coming out from the sun. So, the solar luminosity L_{sun} is the flux $\times 4 \pi r^2$. So, these 2 things are directly measurable and this can be determined from a variety of observations. So once you know these things you can determine the total radiation that comes out of the sun.

This is the solar luminosity and if you do this exercise then you will find that this has a value which is equal to that is the total solar luminosity. The total energy emitted by the sun in one unit second. So, the sun emits 3.9×10^{26} Jules in 1 second that is the wattage of the sun. It is a source of energy just like an incandescent lamp or bubble. It also emits out energy. Now if you know the energy emitted by the sun you can determine.

So, we know let us try to now quantify this energy source. And when we discussing the properties of radiation let us assume to start with that the sun is emitting black body radiation. So, let us try to make a modal for the sun where the sun is a sphere. So, this is a sphere, sun is like a sphere like this. This is what the sun as we see it. So, let us try to make a modal where sun is a sphere of radius R_{Sun} .

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$\theta (32') = R_{\odot} \times 2$
 $R_{\odot} = 6.96 \times 10^8 \text{ m.}$
 $4\pi R_{\odot}^2 T_e^4 = L_{\odot} \quad \left| \theta = 5.67 \times 10^{-8}$
 $T_e = 5800 \text{ K.}$

Which is emitting it is a source it is a black body source. Now the sun is seen to subtend an angle of 32 roughly arc minutes on the sky. This can again be directly measured you can determine the angle that the sun subtend on the sky. So, this is the angle that the sun if you take the image of sun the angle that it obtains on the sky is roughly around 32 arc minutes. So, if you know the distance to the sun which we already have discussed.

The product of these 2 if you convert this to radius will give you the radius the diameter of the sun. So, this will give you a diameter of the sun. Remember you have to convert this into radius. This is an arc minutes so you can easily check this for yourself so the sun has an angular diameter of 32 arc minutes this angle is 32 arc minutes and you multiplied with the distance and you will get the diameter of the sun which is twice the rate.

So you can use this to determine the radius of the sun. And the radius of the sun if you determine it has a value 6. So it takes roughly 2 seconds for light to travel from the center of the sun to the radius. So, we know the radius of the sun and now we are trying to model the sun as a black body which is a radiating energy so the amount of energy that is radiated by a black body like this we know it is the surface area $4\pi R_{\text{sun}}^2$ that is the surface area into a temperature.

So if you imagine that the source is a black body and determine the temperature of that black body then that temperature is what you call the effective temperature of the source. We have

already discussed this. That is how the effective temperature is defined. So this into the effective temperature to the power 4 is the solar luminosity. So, the effective temperature is the temperature that would be required by a body with the same surface area of the sun to radiate as much energy as the sun does.

So, you put in the values again here so you have to put in the values so the effective temperature here. If you calculate it so the sigma you have to put in the fact area into the temperature to the power 4*the Stefan–Boltzmann constant sigma. So sigma we have already calculated sigma for first principles when we were discussing the microscopic nature of black body radiation and sigma the Stefan–Boltzmann constant has a value 5.67×10^{-8} in SI units.

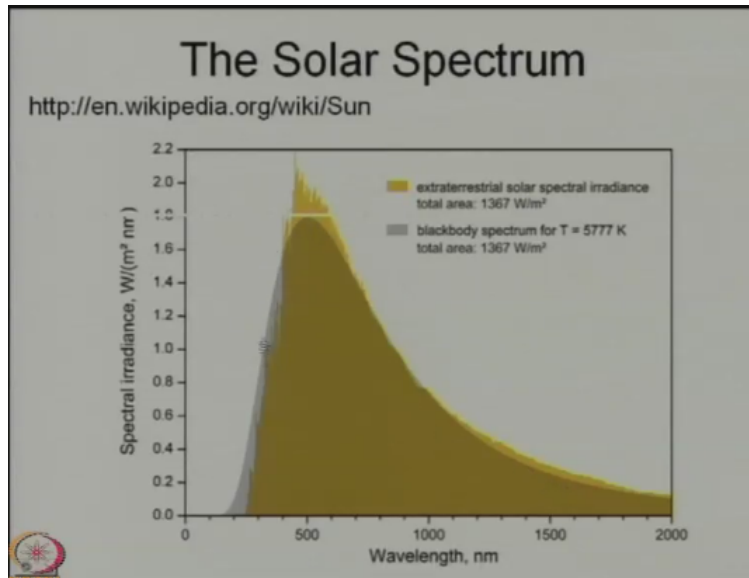
So, you have to get the units correct. So, using this you put in these value so we have already we have told you what sigma is you know what the radius of the sun is and you know what the luminosity of the sun is so you put in these values and you can determine the effective temperature of the sun and the effective temperature comes out to be around 5800 kelvin. So let me remind you what we have done.

We have assumed that the sun is a black body and using this we have estimated the temperature that the black body should have if the luminosity is to match the luminosity of the sun. If it really is a black body the spectrum is then fully determined. So, let us now ask the question how well does the actual measured spectrum of sun match that of a black body with this temperature. So, how good a picture do we get about the sun if we think of it as a black body.

Black body remember there radiation is in thermal equilibrium with matter that is what black body is. So, how good is it? How good a modal, how good a picture do we have about the sun? If we think of it as a black body with the temperature of 5800 kelvin. To do this what you have to do is you have to just compare the spectrum of the solar radiation which is incident on the earth. So you can measure the spectrum with the black body spectrum with this temperature.

So, this shows you the solar spectrum and let us take a look at this picture.

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So this is source from where this picture was taken. So the maroon thing over here is the measured solar spectrum and the gray thing over here is a black body curve, blank spectrum at 5777 kelvin. 5000 close to this 5800. 5777 kelvin and the part of the electromagnetic spectrum that is shown here essentially corresponds to optical this is somewhere here is the optical part the visual part.

One visual range and here you have the ultraviolet part and it extends into the infrared. So, there are a few interesting things that one should note here. The first thing that one should note is that the solar spectrum, the spectrum of the radiation from the sun peaks in the visible part of the spectrum. So, the bulk of the radiation from the sun comes in the visible part of the electromagnetic spectrum. The peak is around yellow which is why the sun appears yellow.

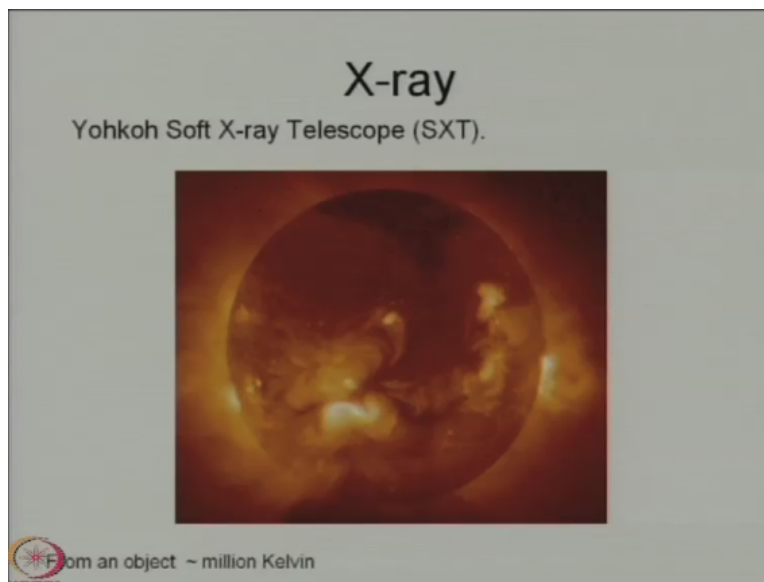
The sun to our eye appears yellow. The sun light appears yellow this is because the black body radiation and also the solar spectrum both peak in somewhere around yellow, around 5000. Now, the match as you can see is quite good in this region so in the visible the ultraviolet and in the infrared the solar spectrum is quite well fit by a black body at 5777 kelvin, 5 thousand 7 hundred and 77 kelvin.

So, it is a reasonable assumption to think of the sun as a black body with a temperature around 5800 kelvin quite a reasonable assumption. Because you see that in the black body curve gives a

reasonably good fit to the solar spectrum. And the bulk of radiation actually comes in this visible ultraviolet and infrared part. So, the bulk of the radiation coming from the sun can be well modeled using a Planck distribution spectrum.

But if you look at other wavelengths beyond this range which have not been shown here so if you go to shorter wavelengths you have the far ultraviolet and you have x-rays and then you have gamma rays. The fit there is fit observed spectrum there is not well fit by a black body spectrum at 5800 kelvin. So, let me show you a few pictures before we discuss it.

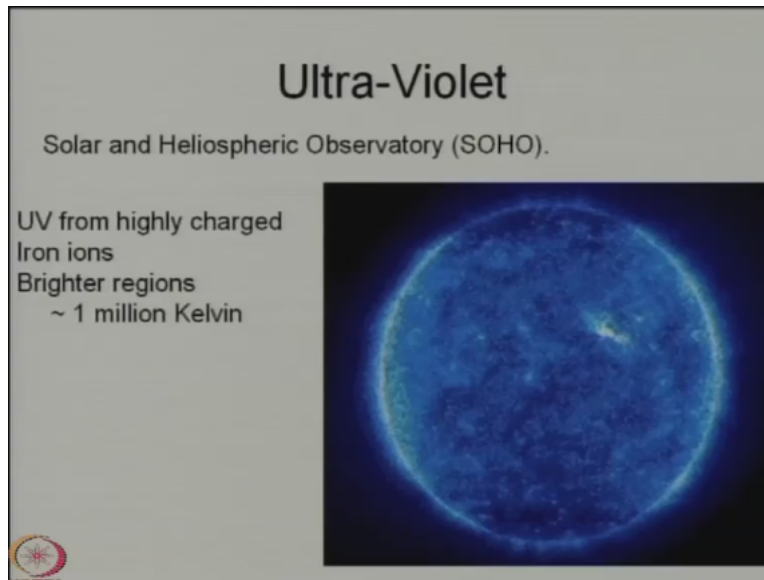
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So this is an x-ray picture of the sun. This picture has been taken by the Yohkoh Soft X-ray Telescope. This is an X-ray picture of the sun and the X-ray radiation coming from this if you compare it with black body spectrum it would correspond to something like a million Kelvin which is quite different from the temperature which characterizes the bulk of the radiation which is at 5800 kelvin.

So, the actual x-ray radiation from the sun is far in excess of what you would expect from a black body spectrum with this temperature. So, it is far in excess of what you would expect from a black body spectrum of this temperature. The x-ray also far ultraviolet.

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This is a ultraviolet image of the sun, this image is from the SOHO observatory. This is a space observatory and this image over here measures the ultra violet from the highly charged iron ions and the bright parts in this image would tell you that the temperature there is around a million kelvin which again is far in excess of 5800. "Professor - student conversation starts" In the previous thing and here it is from iron so you can ionize the iron.

So you can determine the conditions required to ionize and produce that much radiation from iron. No, it is a particular spectrum line for iron. So if you infer a temperature from this the temperature of the medium there would be around 1 million Kelvin. It is a particular line which is emitted by highly charged, highly ionized iron. Well there are methods of doing it. "Professor - student conversation ends".

The main point is that if you go to wavelengths which are beyond the ultraviolet or ultraviolet x-ray or even gamma rays you find that the radiation is far in excess of what you will expect from a black body curve but if you look at the contribution to the bulk of the radiation that is very small for those regions. The bulk of the radiation is over here and that is well characterized by a black body spectrum.

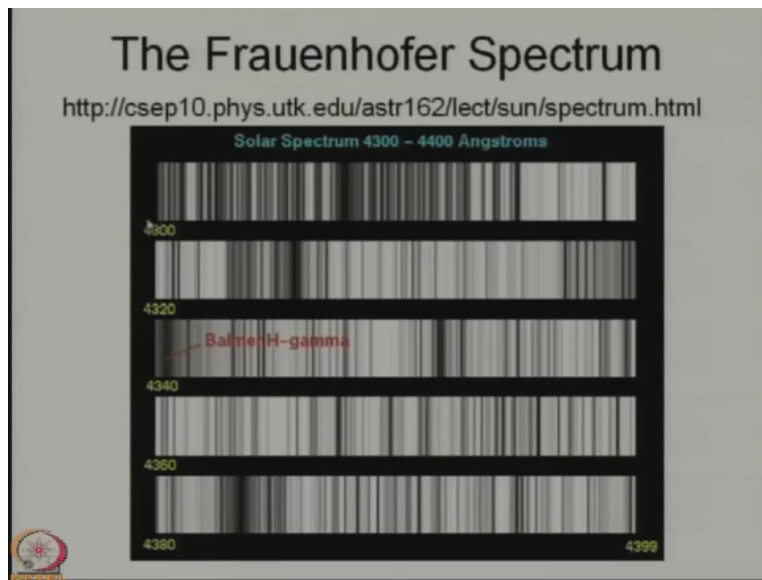
Similarly, if you look at the radio part of the spectrum that is the larger of wavelengths there again you have radiation in excess of what you expect from a black body curve. If you

extrapolate this black body curve and all of these things are associated so the amount of radiation in these ultraviolet rays or in the radio they are closely linked with activity on the sun. The sun is not a peaceful spherical object emitting radiation continuously.

There is all kind of activity on the surface of the earth. You can see some of it in this picture. So you can see that there is some material being ejected over here. This is a solar flare so there are birds and flairs and the radiation for example in radio or in other wavelengths it goes up when you have these flare activity increasing. So, that is the function of time also and you can even observe gamma rays related with these solar flares and burst.

So, the other parts of the spectrum they do not really are not well described by this particular black body that is the first thing.

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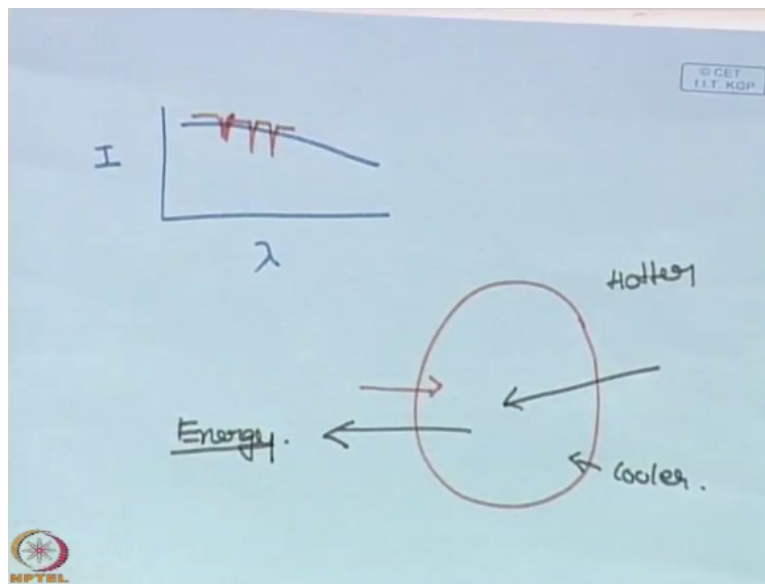
Now, even in the bulk of these spectrum so we have seen that around 4000 angstroms it is quite well described by a black body spectrum. But you take a closer look so this is a high resolution spectrum. So let us see how high resolution it is. So this from here to here this entire thing is 20 angstrom and from here to here again is 20 and 20 and then like that. So, it is a very high resolution spectrum. This entire range over here corresponds to 20 angstrom.

And if you look at it in a high resolution then you see that on top of this continuum so you have

this continuum radiation. This is what is the continuum radiation, it is continuous. It is not a discrete function there are no discrete ups and down. On top of this continuum if you look at it very closely you will find these absorption lines. You can find these absorption lines and these absorption lines correspond to different transition of atoms.

So here you have Balmer H gamma light you could have different kind of such lights. So, if you take a close look at the spectrum at a very high resolution you find that on the top of the continuum spectra so what we mean by this let me show it schematically. So, if you look at it at very high resolution or you could draw it as a function of frequency.

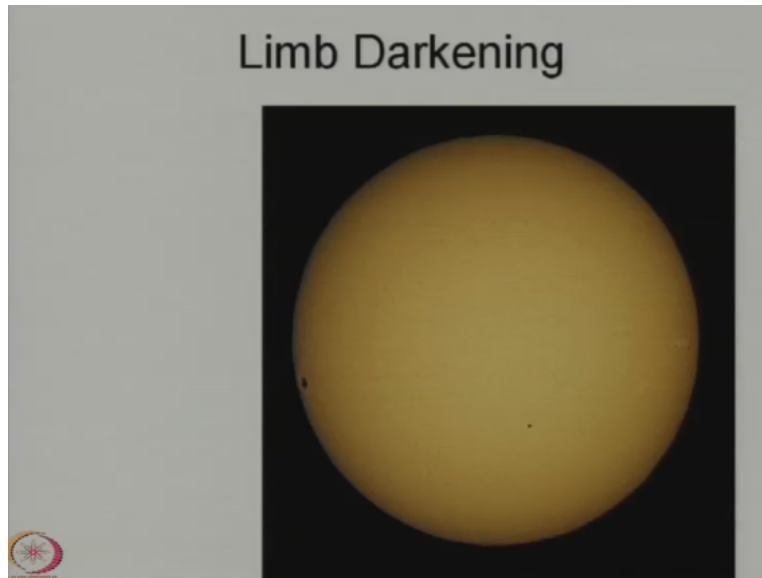
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Then this was some the continuum that you have on top of this you have these absorption line so they are actually the spectrum falls here then it goes down here. There are absorptions lines like this. So, there are all kinds of such absorption lines which you can see in these pictures. And these are often referred to as the Fraunhofer lines or the whole thing is referred to as Fraunhofer Spectrum.

This is what you see if you look at the solar spectrum in detail at high resolution. At low resolution it is quite well fitted by a black body spectrum. There is another evidence let me show you that and then we shall discuss the implications.

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And this evidence is limb darkening so this shows you a picture of the sun an optical picture of the sun. And if you look at this picture I am sure you can make out that it appears brighter in the center and it appears to get darker as you go away from center towards the edges. And in astronomical parlance the edges are referred to as limbs and this is the phenomena shown over here is referred to as limb darkening. So the outer regions get darkened.

So, the outer region of the sun appears to be darker. Now, if the sun for a black body of the same temperature then it should appear equally bright because the temperature is the same everywhere. So, this is something that seems to indicate that it is not exactly a perfect black body there are some deviations from that. These spectrum lines now there are atoms so this spectrum lines what do they tell us?

So, from these looking at the spectral lines we can see that there is material in the sun which is cooler than the rest of the material. Because if the entire material were at the same temperature then it would be all in thermal equilibrium and you would just get the black spectrum. So, it indicates that there is material in the sun which is cooler than some rest of the materials and if you have cooler material it will then absorb.

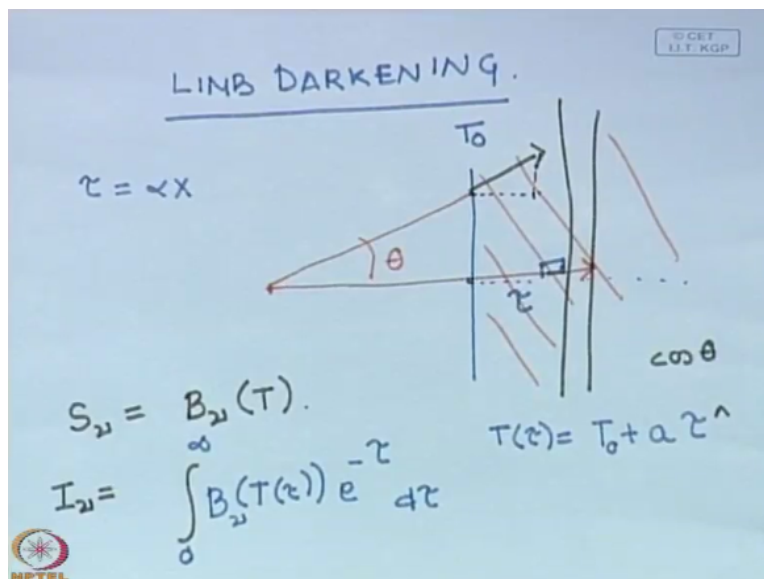
So, there are temperature differences that you are seeing essentially. So, there are temperature differences on the sun and the thing is that we would like to understand this. So, the question

arises does the temperature increase or decrease as you go into the sun? So, let me draw a picture so the question this is the sun and if we go into the sun does the temperature increase or decrease? That is the question that one would ask.

Now, we know that energy flows out from the sun that is the well-known fact. So we know that energy flows out radiation flows out and we also know the laws of thermodynamics that heat flows from hotter to cooler regions. So this would tell us clearly that it is hotter inside and it is cooler as you go out. So, that is the broad picture it is not that if the whole thing is precisely one black body with radiation and matter in thermal equilibrium at the same temperature.

There are temperature gradients we believe and it is hotter inside the sun. It is cooler outside. So, let us this picture to make some idea about the sun. So, the first thing that we would like to discuss is the phenomena of limb darkening. So, let us first take up for discussion the phenomena of limb darkening. And we shall make a very simple model for this which may not give you the entire picture but it will give you some idea of how one could go about.

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Modeling and understanding what is going on so the sun is spherical so we shall not go into this spherical geometry we shall make a very simple model. Our simple model is that we are looking at some plain parallel kind of a thing. So, we shall assume it to be plain and the temperature, on the right hand side we have the sun. So, this is the sun the material of the sun and we shall

assume that the temperature at the surface is T_0 and the temperature increases as we go inside.

It increases with distance inside the sun. Remember that when we discussed the propagation of radiation I told you that it is most convenient and natural to quantify length intervals using the optical depth. Optical depth was α the absorption coefficient into distance x , $\alpha \cdot x$. So, the distance inside we are going to quantify using the optical depth τ . That is the important thing for radiation so we shall assume that the radiation inside this increases and it is $T_0 + a \cdot \tau$.

So, we will assume that it increases linearly inside that is the simple model that we shall make. So, the optical depth is 0 over here and as you go inside the optical depth increases and let us say that it increases all the way to infinity or to a very large number as you go inside the sun. As this entire thing is in thermal medium so each part over here is a thermal emitter and at a temperature T which is given this. This is our model for the sun.

We shall forget about the fact that it is spherical, plain parallel. So, let us now ask the question we are looking from this direction, the observer is here and the observer is looking into the sun. So, actually it is the radiation from the sun coming to the observer so let us ask the question what is the radiation received by observer from different directions on the sun? So, we will call this angle θ . And this is how it increases with optical depth if we go straight inside the sun.

If you instead of going straight in, if you look at the light coming from this direction what would be the temperature as a function of the optical depth? So, optical depth is α into the distance but the temperature will increase only by this amount so for any angle θ the temperature is going to be multiplied by factor of $\cos \theta$. So, there will be a factor of $\cos \theta$ here if I am looking at some other angle.

This is if I go straight in but if I go at an angle the optical depth is going to increase but I am not going to go the same distance inside the material. So, the temperature increases like this. And I am now going at angle so the temperature increase is not going to be as much it will be less. Now, let me write down the general solution we have already solved the radiative transfer equation so we can straight away now write down the general solution in terms of the source

function inside this material.

So, let me write down the general solution and the source function inside this material is just the plank spectrum so the source function inside this material is the Plank spectrum at the temperature T so the material inside this is emitting thermally at a temperature T where T is increasing as we go in. The temperature increases as we go in that is the main thing. So, the source function also increases like this.

So, we can straight away write down the solution and the solution we know is I_ν at this point the solution is equal to –What is the solution? We have to take the contribution from each element over here and the radiation from here when it comes propagates to here will be attenuated by a factor E to the power $-\tau$ that is the main thing and you have to add up the contribution from all such elements as you go in.

So, this is going to be 0 to infinity and then you have the source function which here is B the Plank function which is the function of the temperature at that optical depth τ to the power $-\tau$. This is the solution so this is the specific intensity that is going to be measured over here. So, each small interval $d\tau$ is going to emit this much that is the source function and it is going to get attenuated by a factor e to power $-\tau$ inside.

And have to add up the contribution from all the way inside. So, the specific intensity over here in this situation. Now, we will simplify the problem to some extent so what we will do is we will assume that –we can describe it in the Rayleigh–Jeans approximation. In the Rayleigh–Jeans limit.

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$$B_{\nu}(\tau) = \frac{2 k_B T}{\lambda^2} ; I_{\nu} = \frac{2 k_B T_b}{\lambda^2}$$

$$T_b = \int_0^{\infty} [T_0 + a \tau] e^{-\tau} d\tau$$

$$\int_0^{\infty} e^{-\tau} d\tau = 1 = \int_0^{\infty} \tau e^{-\tau} d\tau$$

$$T_b(\cos \theta) = T_0 + a \cos \theta$$

So, in the Rayleigh–Jeans approximation we can write this as 2 Boltzmann constant/lambda square into a temperature. The relation between the source function and the temperature is quite simple. It is a linear relation so we will assume that we are working in this limit and the specific intensity then also is 2KB/lambda square into the brightness temperature which you would measure.

So, we are quantifying the intensity in terms of the terms of the temperature, the brightness temperature and in the Rayleigh–Jeans limits they are related like this. So, the more the brightness temperature the more the specific intensity or brightness. So, we can now write down this equation straight away in terms of the temperatures just get rid of the factor of 2 the Boltzmann constant/lambda square.

And we have that the brightness temperature over there is = 0 to infinity $T_0 + a \tau$ that is the temperature of the source at corresponding point $e^{-\tau} d\tau$. So, we have to do these integrals and both of these integrals we can easily check they give you a value 1. So, you have to integrate $e^{-\tau} d\tau$ from 0 to infinity and this has a value 1 and also $\int_0^{\infty} \tau e^{-\tau} d\tau = 1$.

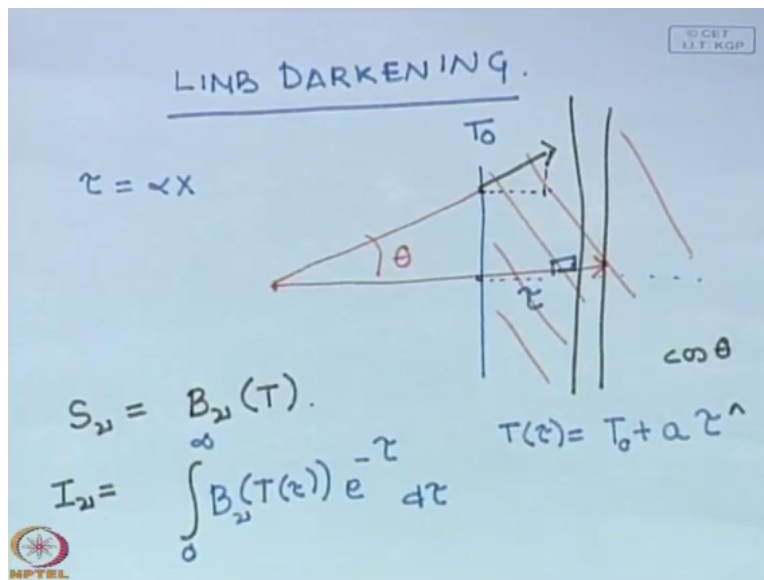
Both of these integrals have value 1. There should be a factor of $\cos \theta$ here. The relation between temperature and optical depth is different in different directions. So, what it tells us that

the brightness temperature T is a function of $\cos \theta$ and it is $= T_0 + a \cos \theta$. So, we have worked out the brightness, temperature profile that the observer over here would see if he were to look into the Sun under these assumptions.

So the assumption is that the temperature inside the sun falls off linearly as we go inside and the whole thing is plane and we have assumed we have worked in the Rayleigh–Jeans approximation. You can do the whole exercise more rigorously putting in the spherical geometry of the sun and working in the entire with correct Plank formula. But our aim just here is to get a qualitative idea rough quantitative and qualitative idea of what is going on.

So what are the messages that we can derived from this simple exercise? Let me enumerate them one by one. So the first message is let me draw a picture again so this is my medium.

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I am observing here and this is the angle theta and the temperature inside has a gradient like this. It is increasing linearly. So what do we see the first thing that we see is that the brightness temperature that will be observed is actually the temperature where the optical depth becomes = unit E . So, the brightness temperature corresponds to the temperature at the place where the optical depth becomes unit E .

So, the temperature inside is $T_0 + a \tau \cos \theta$ as you go inside the medium the optical depth

increases and the temperature of the radiation that you will see here is essentially the temperature at the place where the optical depth becomes unit E . So, when you observe the radiation from the sun we are observing the radiation essentially from the part of the sun's atmosphere where the optical depth is of the order E , is one.

If you go inside the sun where the optical depth is larger so photons that are emitted over here what happens to these photons? Optical depth here is large so the probability of a photon getting propagating is $e^{-\tau}$. So the photon that is emitted here gets absorbed by the time it comes to the surface. So, the photons that are emitted at high optical depths get absorbed inside the medium so they do not come straight to you.

The temperature here is higher but these photons that are emitted inside do not propagate to you they get reabsorbed inside and it was only the photons that are emitted around the region where the optical depth is 1 which can propagate freely more or less to you. So, when you are looking at a medium that has got temperature gradients and things like this you will see the radiation from the region where the optical depth is of order unit T . And you will be seeing the region.

So, this is where let us say the optical depth becomes unit T . So, let us say this is the point then you will see the radiation coming from this entire point where optical depth is less than or up to order unit T . This region is called the photosphere that is the part of the sun which you see from outside it is called the photosphere. It is all the way till where the optical depth becomes of order unit.

The second point is that the temperature of the radiation if you plot it as a function of θ when you look straight into the sun the temperature will have a value $T_0 + a$ when you look at a grazing angle it will fall. So the temperature profile this is $\pi/2$ the temperature profile will look something like this. That is just $\cos \theta$ basically $T_0 + \cos \theta$. So, as θ increases the value of $\cos \theta$ falls and the temperature is going to fall.

So, if you look in this direction you are going to see a smaller temperature as compared to this direction. And why is that so? That is so because optical depth 1 is reached at a much shallower

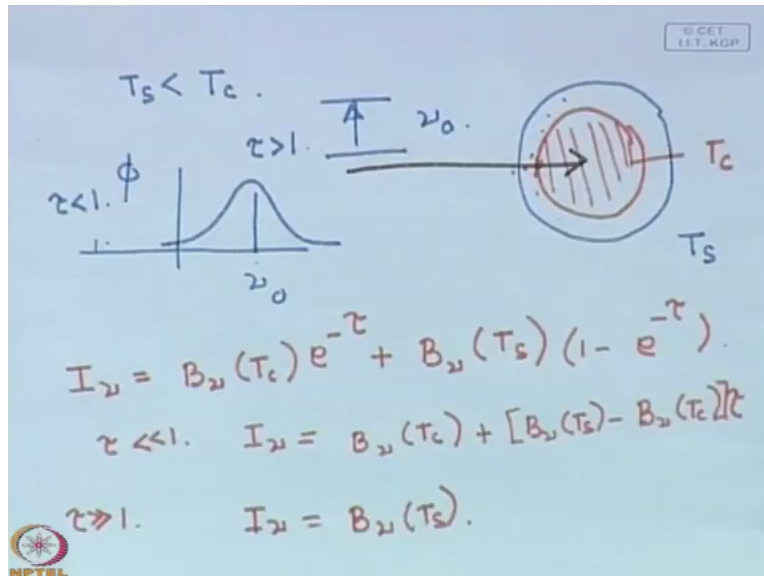
place where as here you are going straight in. So, optical depth 1 is reached much deeper and since the temperature increases inside so if you look straight in you are looking at a much hotter part if you look at an angle you are looking at a cooler part. I hope that picture is clear.

So when you look at an angle over here optical depth 1 will possible be reached over here where as in this direction it will be reached much deeper inside. So, when you look at an angle you are looking at a shallower part of the sun where it is also cooler. And as a consequence the radiation coming from here is also going to be less. So when you look at the sun. So, let us look at the picture of the sun.

When we look at the sun when we make an image of sun when we look straight in we are looking deeper into the sun and appears hotter. The sun is hotter as you go in. when you look straight in you are looking deeper so you are seeing a hotter part of the sun. If you look at an angle the optical depth becomes 1 at a much shallower place. So the temperature there is less as compared to what you would have probed here so it appears darker. The brightness is less.

The temperature is less so the specific intensity is also less. So, this is essentially direct evidence Limb darkening that the sun is actually hotter as you go in. Now, let us also look at the second thing so let us try to now model what causes those spectral lines, cooler lines that you see. Let us try to make a model for that let us try to get a brief understanding of what is going on over there. So, to do this let us make a model.

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So, when you look at the sun we have seen that you only see the region where the optical depth of the order unity. So, let us look and think of the sun as opaque sphere of some temperature T_c this is the core and this is where the optical depth is order unit T looks like this. You do not see inside this so it is opaque. So, we have an opaque sphere of some temperature of some temperature T_c .

And let us now consider an atmosphere this is the atmosphere of the sun nothing outside of it. The shell of gas at a temperature T_s and T_s we have seen will be less than T_c . This is our model the atmosphere the outer shell of the sun where the optical depth is less than 1 is at a lower temperature T_s as you propagate inside the optical depth increases and when it becomes 1 we think of it as an opaque shell, sphere inside it at a higher temperature T_c .

And let us assume that this shell is filled with a gas which has 2 energy levels and the transition between these 2 corresponds to a frequency ν_0 . And in our discussion of these 2 level transitions I have told you that there is this line profile ϕ_ν . The transition really does not occur at a single frequency. It will occur in a spread of frequencies so let us model this by the line profile ϕ_ν which looks something like this.

So, the absorption coefficient α is large when you are near ν_0 right for this material outside. But if you are away from ν_0 over here the absorption coefficient is extremely small

and the optical depth is proportional to the absorption coefficient. So if you are at a frequency over here somewhere over here the optical depth is more than 1. If you are at a frequency over here the optical depth is much less than 1.

This material does not absorb anything that the modal. That is the gas which fills the outer shell. It has 2 states and if you are at a frequency which corresponds to this difference somewhere near here then the absorption coefficient shoots up. And this response is described by this line profile. So, let us now consider a line of sight which goes like this through this. So, in general we can write down the solution we have already worked out this.

So if you have something like this so the solution the specific intensity outside we can write down straight away the specific intensity outside is going to be the specific intensity of this radiation which is $B_{\nu T_c}$ attenuated by this optical depth + the Planck function of the shell outside. These are all thermal the material here is in thermal equilibrium at T_s . The material here is in equilibrium at T_c so we are all emitting the source function and the intensity are all Planck spectrum.

This into $1 - e^{-\tau}$ to the power - tau. This we have already worked out. So, this is the background specific intensity it gets attenuated by this factor this is my source and the contribution from the source if it is at a uniform temperature is $1 - e^{-\tau}$. So what happens when you are looking far away over here so when you are looking where in the region where the optical depth is much less than 1 in that region I knew the specific intensity is essentially $B_{\nu T_c}$.

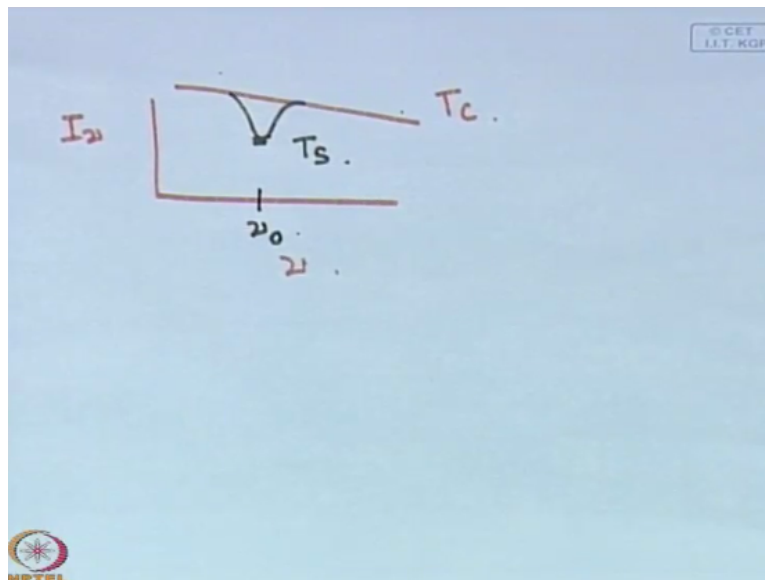
So, if I do a Taylor expansion of this now this will become $1 - \tau$ and this will become just tau. So, I have + into tau. Its optically thin this is a negative number so there will be a small decrement and as the tau tends to 0 this small decrement will be gone. So, I will essentially see the black body spectrum of this. Now let us consider the other limit where the optical depth is > 1 .

So, where the optical depth is > 1 in the limit where the optical depth is extremely large, let us just look at that limit. So, if the optical depth is much > 1 than I_{ν} will go over to the temperature, black body spectrum with temperature of the shell. And if you have the intermediate

region it will slowly go from this value to this value. As you approach this frequency the specific intensity will slowly go from this value, τ is very small so I have this.

As you approach larger and larger τ it will slowly go over to this value and then again it will come back to this value. So, let me draw a schematic diagram of what you expect.

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So if the shell were not present then I would have something like this continuum black body spectrum. But because of this so this is at a temperature T_{core} but because of the material in the shell which is at a temperature which is lower near ν_0 the specific intensity will go down and it will approach the value T_s the black body spectrum corresponding to T_s . Here it will be a black body spectrum corresponding to the core temperature near ν_0 .

The optical depth is extremely large it will approach a black body spectrum with temperature of the shell. So, it will appear they will be less bright and this is how we can understand these absorption lines that you see in the spectrum. So in the atmosphere of the sun the outer parts are cooler and you have these 2 atoms over there which have got 2 level transitions which cause these absorptions that you see.

So, in today's class we have discussed our picture of the sun and all other stars in general. So what we see is that the overall spectrum near the peak is well described by a black body. Away

from the peak, the black body does not give a good picture and the atmosphere you can think of it as temperature increasing as you go inwards. The part of the star or the sun that we probe is through our visual of electromagnetic observations is the region till where the optical depth become of order unity.

And the temperature increases as you go inside the star. So let us stop our discussion of the sun and the stars here for the day. We shall resume on this in the next class.