Astrophysics & Cosmology
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Lecture - 13
Thermal Radiation and the Sun

Welcome. Today, we shall discuss the interaction of radiation with a system that has 2 energy levels and we label these 2 energy levels as 1 and 2.

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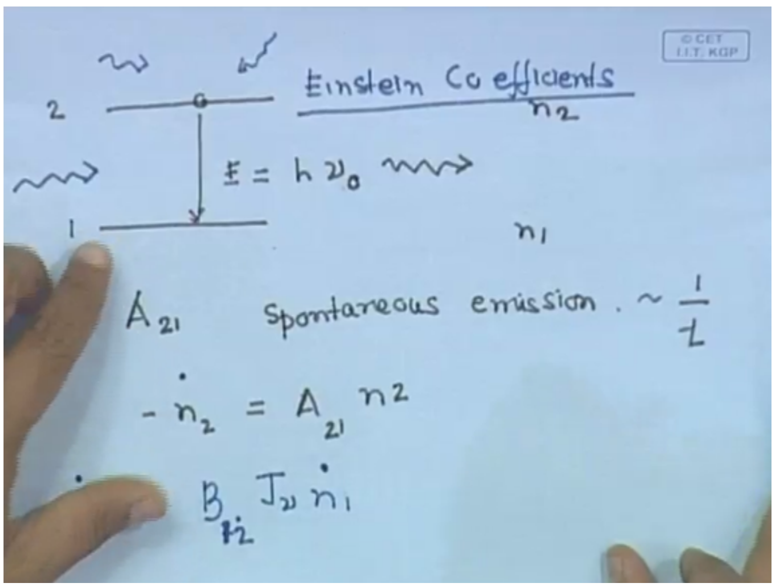

And if my system under goes a transition from 2 to 1 then the energy difference comes out in the form of radiation at a frequency so this energy difference E nu0. So, the radiation as a result of such a transition comes out at the frequency nu0. So, you could think of this as an atomic transition so the electron one of them this could be the ground state of the electron. This could be an excited state.

And this could be the transition between these 2 and we shall denote the number density of particles of atoms or whatever system in the ground state using n1. And the number density of particles in the excited state using n2. Now, we have different mechanism by which the system can go from here to here or back. So, if I take the system and have some of the particles in the excited state there is a probability that they will each come down the ground state.

At the rate at which this comes down is governed by rate coefficient A21. This is the rate of spontaneous emission. And this is a dimension of one by time and it is the order of the one by life time of this excited state. So, if I have a number density of particles in the excited state as a 2 then the rate at which it comes down to the ground state is A21*n2 and this is = -n2 dot which is also = n1.

The rate at which the number density of particle in this state goes up. So, this is the first process that could occur that is spontaneous emission of radiation by the excitation from the excited state to the ground state. Now, in the presence of incident radiation so if there is some external radiation incident on this system. This could also then cause the system to go from the ground state to the excited state.

So, this is the excitation of the system and this is given by what is called b coefficient. The b coefficient tells us that if there is radiation incident on this with this frequency nu0 so this is the – for any frequency or say it is obviously the frequency around nu0 which is going to frequency nu0 which is going to cause this excitation. So, if there is radiation incident on this with angle average specific intensity.

So J is the specific intensity, the brightness of the radiation, average over all angles. So, we are assuming that this excitation can be brought about by radiation coming in any direction on this. So, the angle average specific intensity into the number density of particles in the ground state into B12. This is the rate at which the particles in the presence of an external radiation field the particles go from the ground state to the excited state. So, this is going to be a minor n2 dot.

So, we have these 2 processes and these 2 coefficients A and B are referred to as Einstein coefficients. These are called Einstein coefficients and the equilibrium. So, suppose this radiation there is an external radiation and we have some atoms and we are looking at one excited state of the atom and the ground state. So, we want this to be in equilibrium so at equilibrium the rate at which the excitations occur due to the external radiation should be balanced by the spontaneous emission.

So, this is the requirement. Now, typically the absorption will occur in a spread of frequencies around nu0. So, the line has a profile so that is something called the line profile.

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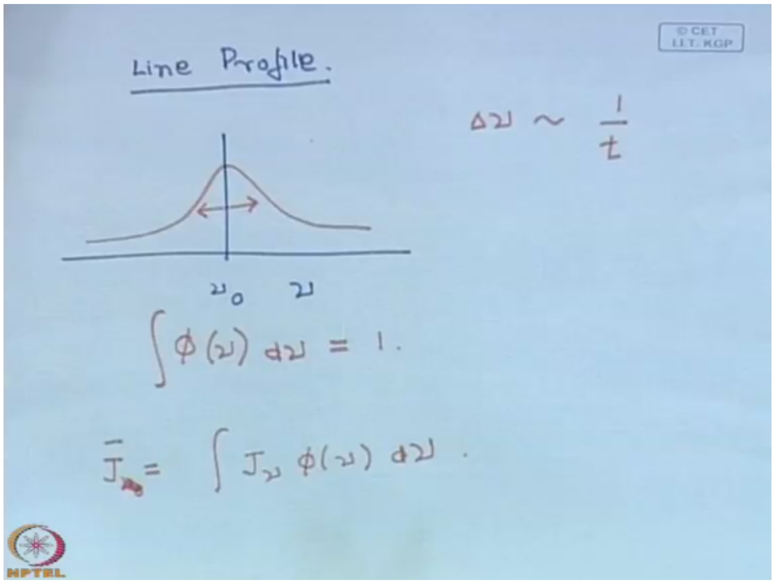

So typically my atom or my system will absorb radiation in some spread of frequencies around nu0 where nu0 is the characteristic frequency of this transition. And this spread of frequencies is what I have shown over here. There will be finite spread of frequencies in which radiation will be absorbed there will also be a finite spread of frequencies in which the radiation when this spontaneous emission occurs is emitted and this is called the width of the transition.

The natural broadening, natural width of this transition is of the order of 1 by the life time of this 2 state system. And we have something called the line profile function which is defined so that it is the fraction of the energy that comes out in each of these frequent intervals and it is defined in such a way so that this integral is = 1. So, when you have radiation of different frequencies impinging on this system.

Then we have to actually look at the average of the specific intensity not only over angles but also over frequencies and the frequency nu0. And this is J nu phi nu, you do not need this anymore d nu. So, it is the angle average specific intensity, also averaged over the line of profile. So, the maximum contribution comes at exactly nu0 but there are radiations and other things can also contribute to the excitation of the system.

And that whole average has gone into this J bar. So, we should actually replace J bar over here and that is the rate at which the system gets excited. There are transition so the excited state. So, if we consider a situation like this.

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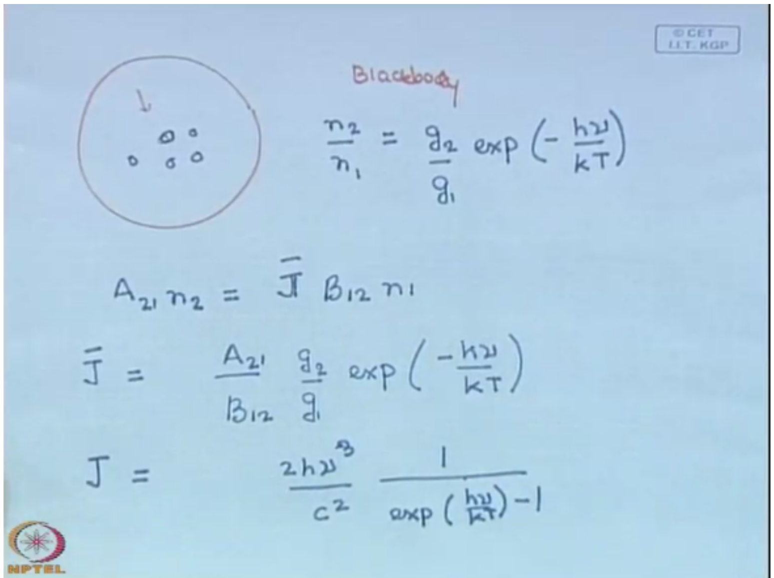

Another input that we need is in equilibrium so let us see we have a black body cavity let us say and so there is this black body radiation inside this black body cavity and we have these atoms also these 2 states system also inside this cavity and we wait till we are in thermal equilibrium. Now we know from thermodynamics from statistical consider mechanics that a 2 state system like this if it is in thermal equilibrium at a temperature t.

Then n2/n1 is going to be = g2/g1 exponential - h nu/K T this is the Boltzmann's factor. So, this is know from statistical mechanics that if you have a 2 level system in equilibrium at temperature T then the ratio of number of particles in the excited state to the ground state will be given by g2/g1 exponential - the energy difference by KT where g2 and g1 are the degeneracies of these 2 levels.

So there may be more than one quantum state that corresponds to the ground state that is quantified by g1. Similarly, there may be more than one quantum state that has the same energy level as the excited state. All of them correspond to the excited state and the number is given by g2. This is what is called the degeneracy. So, this gives me the number density the ratio of the number density in these 2 states.

Now, in equilibrium we know that this ratio should be given by this. Further in equilibrium we also expect that the rate at which the absorption take place should also be equal to the rate at which the emission takes place. So, it tells us that in equilibrium the emission the spontaneous emission rate which is A21*n2 that is the rate at which you have the spontaneous emission the particles go from 2 to 1 should be = J bar B12*n1.

So, this tells us that J bar should be = A21/B12 and then we have the ratio n2/n1 which in equilibrium we know is = g2/g1 exponential - h nu/KT. But we know that the black body radiation has a J bar so the frequency that we are into looking at is somewhere centered at nu0 and at that frequency we know that the black body radiation J =, Now obviously you cannot have this equal to this.

Because these Einstein coefficient just depend on the atom. They are not function of temperature neither these functions of temperature. So, the temperature dependence of this and the temperature dependence of this are quite different. So you cannot have this equal to this. So, there is some difficulty over here. If you have just these 2 processes you cannot have the system coming to equilibrium.

It is not possible. So, it was Einstein who noticed this and who resolved this problem what Einstein did, was Einstein postulated that there should be another process and this process is stimulated emission. So, in addition to spontaneous emission you if you have another process called stimulated emission.

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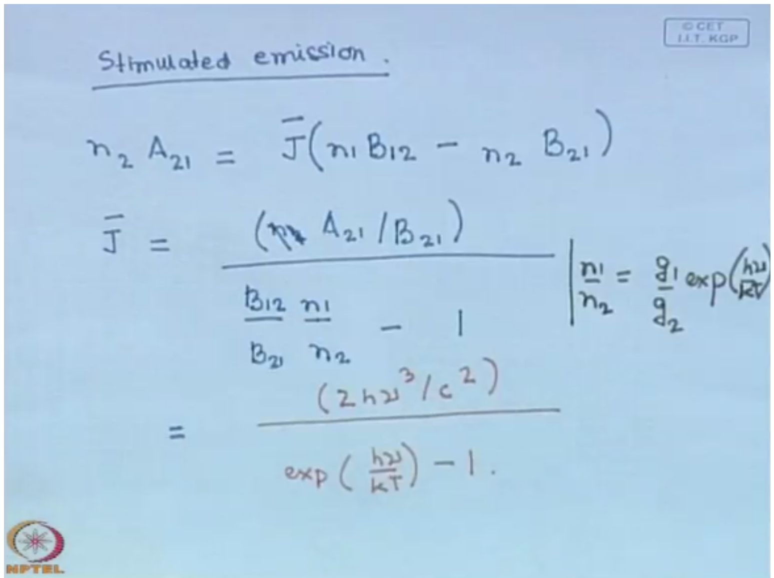

It is then possible to have a consistent where the system can be in equilibrium. So, what is this stimulated emission? So, the stimulated emission is as follows. When I have a system like this, these 2 level systems and there is some radiation incident of it. We assume that the radiation will induce particles to get excited. Now in addition to this we have to also admit the possibility that radiation can also cause the system to de-excite.

And this is what is referred to as stimulated emission. So, now if you take into account stimulated emission as well the rate at which you have the particles going from the –in the absence of any radiation what so ever the rate at which the de-excitation occurs that is the spontaneous emission is n2 A21 and in the presence of external radiation it can cause the transition to the excited state from the ground state to the excited state.

And that is given by J bar that is the average brightness into n1 B12 that is the rate at which you have the excitations and what Einstein proposed is you should also include a possibility that the incident radiation will also cause some de-excitation and the rate of that is given by J bar means specific brightness into the number density of particles in the excited state into another Einstein coefficient B21.

Now, if you include for this stimulated emission process so the B21 is the Einstein coefficient that tells us the rate at which simulated emission occurs. The information that this equation does not tell us is that the stimulated emission is in the same phase as the incident radiation and this is a very important property. So, the simulated photon that is emitted as a consequence of this incident radiation is in the same phase and at the same phase and frequency as the incident photon.

This is a very important property which we shall briefly discuss towards the end of this class later on. Now, if you allow for this possibility then in equilibrium we have J bar = –So, I am dividing throughout by n2 this will not be there. n2*B21 so I will have this ratio in the numerator. So, I am dividing this whole equation by this term and then I am going to this term on to the left hand side.

So, I have J bar = A21 divided by B21 and 2 cancels out and then I have over here B12/B21*n1/n2 - 1. We should also compare this with the Planck's spectrum. So, in equilibrium we expect this to be the Planck's spectrum. So, let us compare it if you compare it with the Planck's spectrum then we have 2 h nu cube/c square and in equilibrium the ratio of these particles n1/n2 is given by n1/n2 is g1/g2 exponential h nu/KT.

So, now you see if you put this in here it is these 2 can be same provided these Einstein coefficients satisfies certain conditions. So, let me write down these conditions provided.

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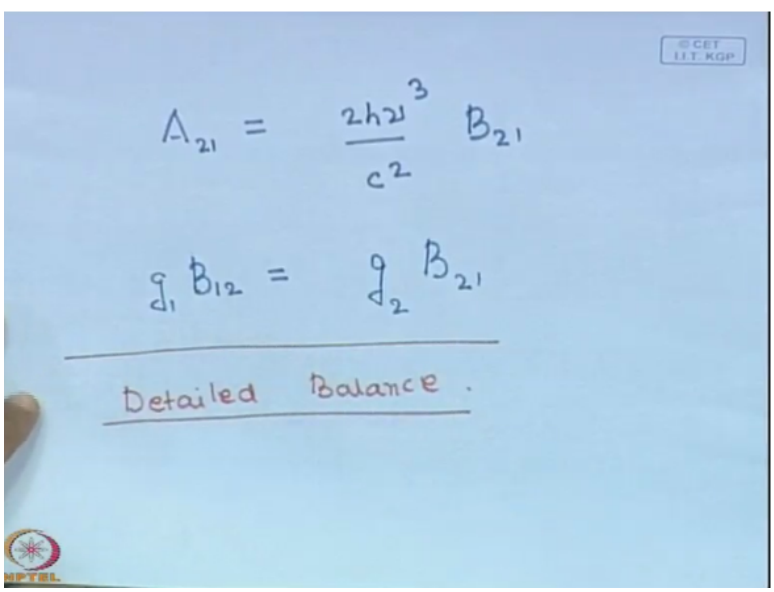

A21 = 2 h nu cube/c square B21 and we would like B12*g1 to be = B21*g2 then all that you have left over here is exponential h nu/KT. So, what we see is that thermal equilibrium is possible provided you have one more process allowed that is stimulated emission and further the different Einstein coefficient are not independent they are related through the relation given over here any one of them is adequate to determine all 3 of them.

And you cannot determine these values of A’s and B’s from these classical consideration you can determine them only from quantum mechanics and these values can be determined through quantum mechanics. So, the method that was used over here is a very important technique what we learned from this? This technique is called that of microscope detail balance. So, this technique that we use is called the method of detailed balance.

So, what does it do it assumes that in thermal equilibrium various process has to be balanced. The rate at which excitation occurs has to be balanced by the rate at which the excitation occurs. So, in thermal equilibrium we determine certain relations between the coefficients that determine these rates. And these coefficients we believe depend only on the properties of the system they have nothing to do with thermodynamics.

So, if you can determine certain relations between these coefficients in thermal equilibrium we expect these relations to hold in any general situation because these coefficients themselves have nothing to do with thermodynamic equilibrium. So, for example here the Einstein coefficients are depended on the properties of my system. The requirement that my 2 level system can come to equilibrium with black body radiation tell us that there has to be another process called stimulated emission.

And the different Einstein coefficients have to be related like this. All the 3 processes have to be related like this. So, this is something very useful and very new that Einstein discovered the fact that you can have stimulated emission. Let us first calculate what the emission and the absorption coefficient are. So, in terms of the Einstein coefficient, the system that we are dealing with we have a medium over here.

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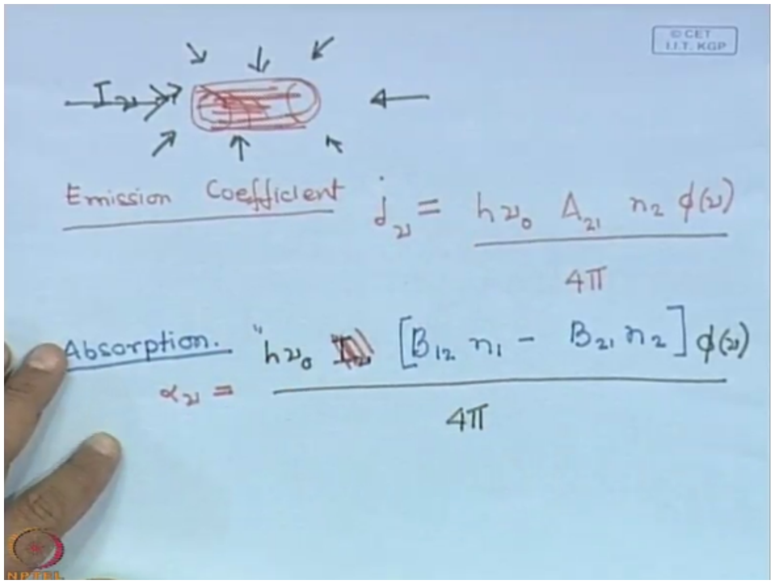

And this medium is made up of 2 level systems could be atoms so we could have a medium say gas filling up this region or something like that any 2 level system which is described in terms of these Einstein coefficient. Now, we would like to determine the emission coefficient and the absorption coefficient which we have defined earlier for radiative transfer in terms of these Einstein coefficients. So, let us first look at the emission coefficient J nu.

So, let us ask the question how much energy is emitted per second, per unit volume in the frequency interval d nu in the solid angle interval d omega that is j nu, emission coefficient. And we show that if you have a number density n2 then the rate at which the spontaneous emission occurs is n2*A21 and the energy carried in each emission is h nu0 and this energy gets distributed over different frequencies as phi nu and it gets distributed uniformly solid angle 4 pi.

So, this is the emission coefficient in terms of the Einstein coefficient. Since the stimulated emission depends on the incident radiation we shall club it together with the absorption process instead of treating it as part of the emission process. The spontaneous emission is independent of the incident radiation which is also the property of the emission coefficient. So, let us now look at now the absorption coefficient.

So, here again the number density of particles in they fix in the ground state is n1. So, the rate at which we have the excitations to the excited state, the excitations here. The rate at which energy is absorbed or the number of transitions that occur per unite second is this in to B12 - B21*n2. Now let us assume that the specific intensity that is incident I nu is the same from all directions.

Let us assume that the specific intensity that is I nu is incident the same from all directions. So, then J is the same as I. J is the angle average specific intensity, j is the same as I so we can write this as I nu the energy that is absorbed in each such transition is h nu0 and the fraction that is absorbed in the frequency interval d nu around nu is put in the factor of phi nu and this is what get absorbed from 4 pi (()) (29:23) from all directions.

So I have to divide it by a factor of 4 pi to take into account the absorption from a particular ray. From here you can identify that the absorption coefficient is essentially this. So, we have that absorption coefficient and the emission coefficient in terms of the Einstein coefficient. Now, let us next write down the radiative transfer equation in terms of these. In general, the radiative transfer equation is dI nu d tau = - I nu+S.

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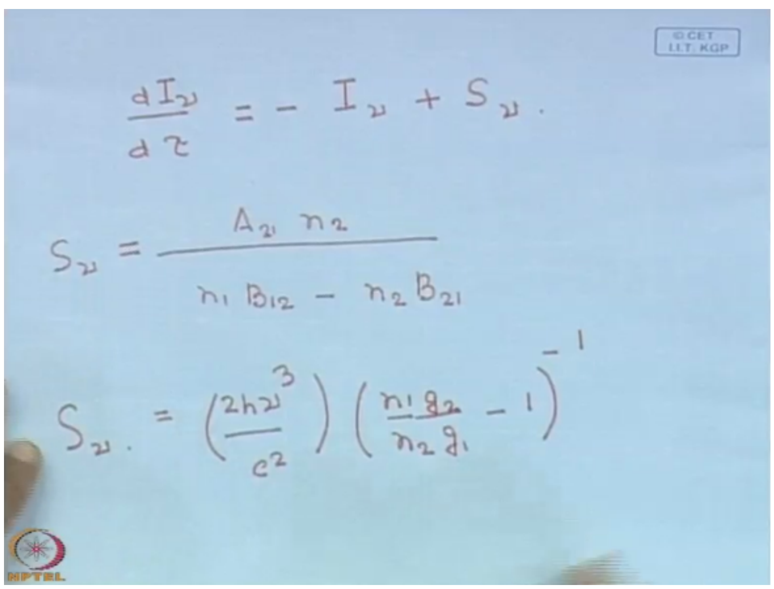

Where S nu is the source function tau is the length into the absorption coefficient which we have already calculated. The thing that remains to be calculated is this source function. Let us look at the source function next for this 2 level system. The source function is the ration is the absorbed emission coefficient to the absorption coefficient that ratio you can calculate. We can identify from this and it is equal to, again let us divide everything by n2*B21.

So that is the number density of particles in the excited state into the Einstein coefficient for stimulated emission if I do that in the numerator I have the ratio A21/B21 which we have seen that this ratio is 2h nu cube/C square. So, this is = 2 h nu cube/c square into what remains is n1/n2*B12/B21 and B12/B21 is g2/g1 the ratio of the degeneracies. So, that is my source function.

So, let us look at the radiative transfer equation. So, the radiative transfer equation is given over here and the important thing over here is the source function. Now, there are 2 possible situations that you could have the first one is thermal emission.

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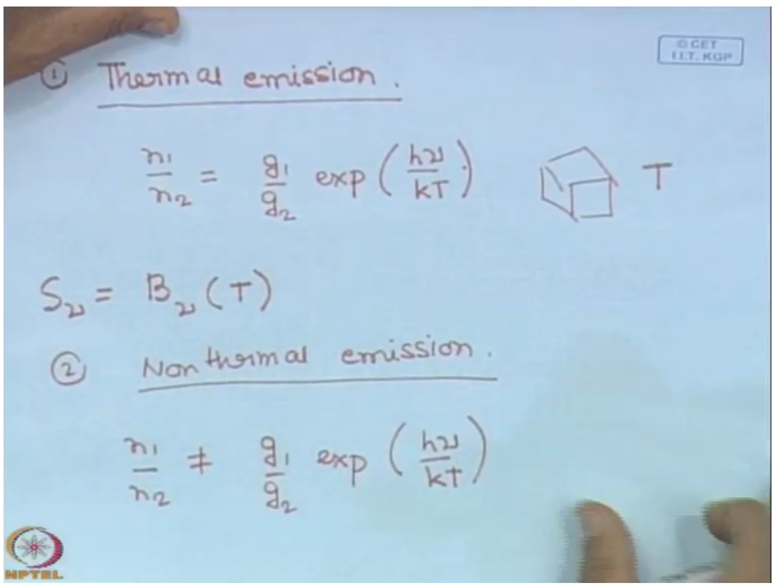

In thermal emission the ratio, the 2 levels that we are dealing with are in thermal equilibrium and we have seen that when the 2 levels are in thermal equilibrium n1/n2 = g1/g2 exponential h nu/KT. So, in thermal equilibrium this is some material, some atoms let us say containing some atoms I am really interested in 2 levels of this atom. So, if this in thermal equilibrium at a temperature T then the ratio n1/n2 is given by this.

And the source function is then the Planck function corresponding to the temperature of the medium. If you have thermal equilibrium, thermal emission then the source function that occurs over here is the Planck function and given sufficient optical depth the radiation reaches the

Planck function itself. If you do not have adequate optical depth then it is some other value which is somewhere in between this incident specific brightness and the Planck function.

Now, the second possibility is none thermal emission here this refers to any situation where n1/n2 != g1/g2 exponentially does not have the Boltzmann's formula and there are a large variety of situation where you can have populations 2 levels which are out of thermal equilibrium. There are a variety of populations. A specific case of this situation which is of particular interest is where you have something called population inversion.

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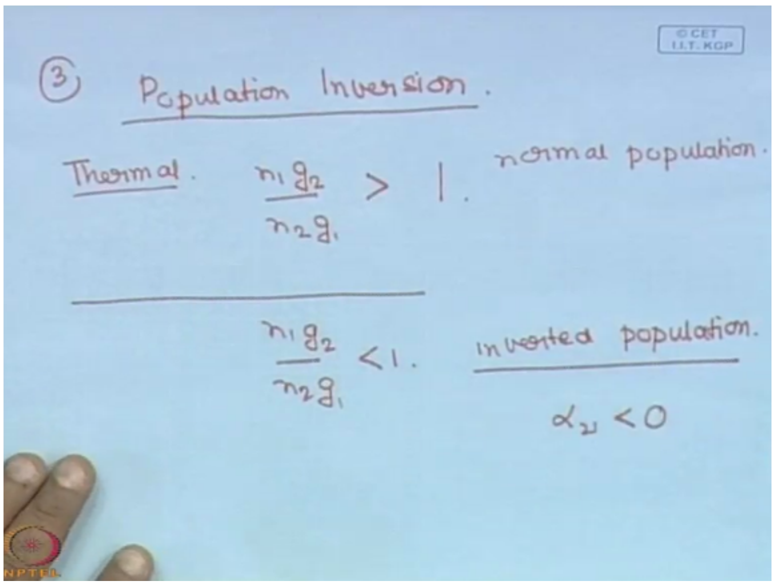

Now in thermal equilibrium the ratio n1 g2/n2/g1 this number is always >1. So, in thermal equilibrium is always >1. It is quite obvious from here exponential of a positive number is always > 1 so the ratio n1 g2/n2 g1 is always >1 and if this ratio is >1 the source function is necessarily positive which means that the absorption is in excess of the stimulated emission. When I have some incident radiation the absorption of that radiation occurs at a rate faster than the radiation is able to stimulate emission and the source function is positive.

And you have the normal situation in these materials this is called a normal population. And you could in a variety of none thermal emission where this does not hold in a variety in a large number of such situations also you typically have this condition being satisfied. So the population is said to be normal. But there are situations where you have what is called an inverted population in situation where the population is inverted you have n1 g2/n2 g1<1.

Essentially, in thermal equilibrium the lower energy state has a larger population then the upper energy states. So, there are more particles in thermal equilibrium in typical none thermal situations. Also there are more particles in the lower end of these states, less in the upper energy state. So, if you have incident radiation the absorption where particles go from here to here is more, the emission is less.

And the net effect is that these systems the 2 level system takes radiation, absorbs energy from the incident radiations or it is in equilibrium with the incident radiation. But when you have population inversion. So population inversion is the situation where there are more particles in the excited state less in the ground state and if I put in some radiation then these stimulated emission exceed the absorption.

And in this situation the absorption coefficient and the source function both of them become negative. So, in such a situation the absorption coefficient alpha and the source function, the source function becomes negative and the absorption coefficient also becomes negative. So, the absorption coefficient becomes negative this term exceed this term and my absorption coefficient becomes negative.

So, my optical depth actually decreases as I go further and further along my system and we know that if I have this is called an inverted population here alpha<0 and we know that this is my medium.

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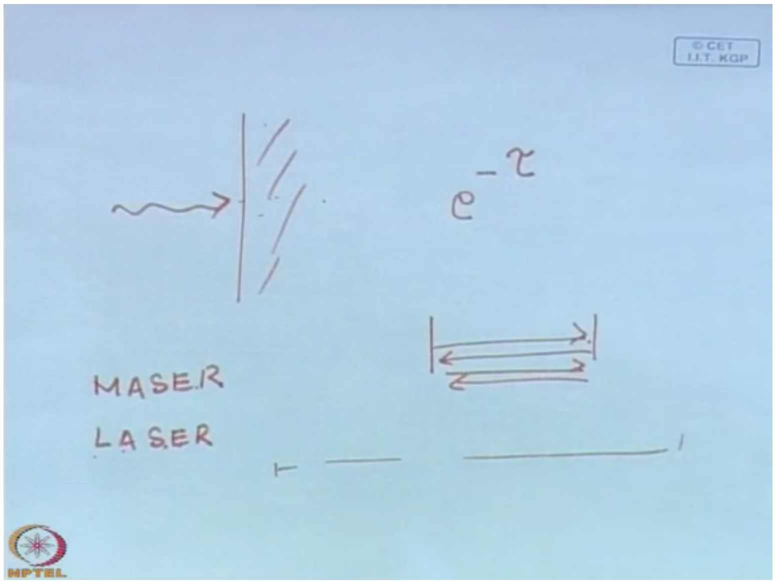

This is some radiation incident on the medium we know that inside the brightness falls off as e to the power - tau. But if it's so happens that alpha is negative then the optical depth actually is negative inside tau is negative inside. So you have an amplification of the radiation inside this medium and it is this which is called maser or laser. So, this is microwave amplification by stimulated emission of radiation or light amplification by the stimulated emission of radiation depends on which wavelength the whole thing is occurring.

So, the net effect over here is that whatever incident radiation that you have gets amplified when it goes through the medium. So, if you have population inversion then whatever radiation you have incident on that gets amplified when it goes through the medium. And the fact that the photons that are emitted are in the same phase and have the same frequencies also very important then you have an extremely coherent radiation that comes out.

So this is the basic the whole of lasers are essentially based on this extra thing that Einstein introduced the possibility of stimulated emission which is bound to occur in nature. And there are astro physical situations where you have inverted populations and you find substantial amounts of (()) (42:11) activity. In terrestrial situations the laser that we have you require a cavity and the light is made to traverse the cavity large number of times.

So that you can have a substantial amount of amplification but in astrophysical situations the length scale are so large that a single traversal can produce a substantial amount of amplification. So, this is as far as we are going to discuss about these Einstein coefficients and some very important applications. So if you are going to discuss the propagation of interaction, the interaction of radiation with atoms or molecules where there are specific frequencies at which transition occurs in all these situations.

You have to apply the Einstein coefficients one of them will be known and with that known value we can calculate the rest of the properties like the absorption coefficient. The emission coefficient and there are various possibilities that could then occur in such a medium and we shall go into the details as we go along this course, various applications. Now, for the rest of today's class I am going to discuss 2 problems in radiative transfer. The first problem, so let me discuss now 2 problems.

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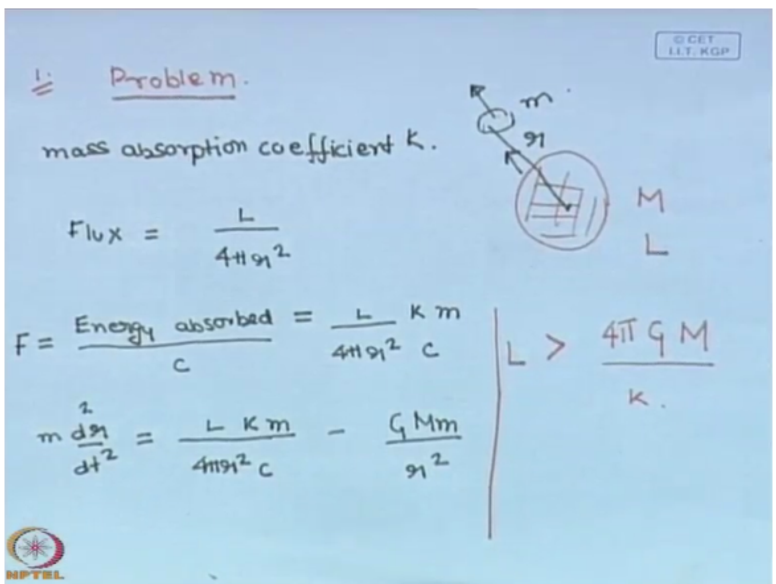

So, we are going to discuss problem number one. The first problem is as follows. We have a luminous source of mass M and luminosity L and we would like to see determine what is the condition that a gas cloud let us say the gas cloud has mass m. So, there is a gas cloud over here and we would like to determine the condition that the radiation force due to the light coming out from this is sufficient to eject.

We want the gas cloud to be ejected so we would like to calculate the condition that the radiation force is adequate to eject this gas cloud. So, the information that we require is that the material of the gas cloud over here has mass absorption coefficient kappa. Now, I have not introduced the mass absorption coefficient earlier we have introduced the volume absorption coefficient alpha which tell us the radiation that is the fraction of the incident radiation that is energy that is absorbed per unit volume here we shall work in terms of the mass absorption coefficients.

So, that tells us the same thing per unit mass. So, let us calculate the radiation we have to calculate the radiation force exerted by this massive object luminous object on this gas cloud. So, the question is how do we calculate the radiation force on this. So, let me outline how to proceed. Let us first calculate the energy flux at the location of the cloud. Let us assume the cloud is a distance r away.

So, let us calculate the flux the radiation flux over here. The radiation flux is going to be L/4 pi r square that is the amount of radiation energy incident on this gas cloud from this luminous object. Now, let us ask the question what is the energy absorbed by this gas cloud per unit time so that is the energy absorbed. So, not all of this energy is going to be absorbed it is this flux L/4 pi r square into the mass absorption coefficient kappa*m the mass of this gas cloud which is going to be absorbed per unit time and we know that the (()) (47:57) energy.

The momentum rate, the rate at which momentum is absorbed by this so we have to divide this by c and this will give us the force because that is the rate at which momentum is absorbed and the rate of transfer of momentum is the force. So, we have calculated the total force exerted by the radiation on this gas cloud. Let us now use this to calculate in the equation of motion what is gas cloud?

The equation of motion of this gas cloud is m d square r d t square = so there is an outward force and the outward force is L/4 pi r square c kappa m. There is also an inward force which is the gravitational attraction due to this massive object and that inward force is - G capital M, a small m r square. So, the condition for ejection is that the outward force should exceed the inward force so let us find that condition.

We can find that condition by just equating these 2 and the luminosity should exceed some value so that this term is always is in excess of this term. So, the factor of r square cancels out and the mass of the gas cloud also cancels out and we are led to the condition that L should exceed. Let us see what it should exceed. It should exceed 4 pi*G into the mass of this gravitating object divided by kappa. So, this is the value of the luminosity.

If the luminosity is in excess of this value, the radiation coming out from this big mass can in principle eject such a gas cloud. This consideration gives rise to a very important concept the concept being called the Eddington.

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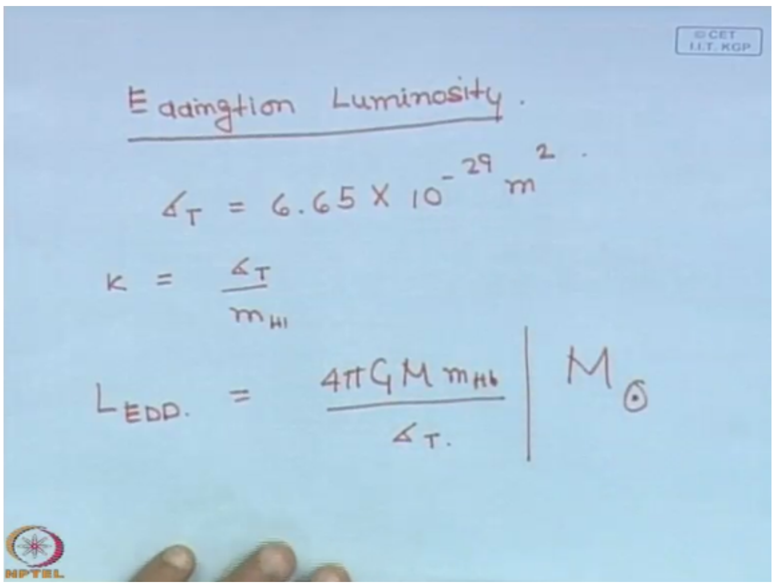

The concept was first introduced by Eddington famous astrophysicist Eddington. It’s called the Eddington Luminosity. The consideration here is quite simple suppose we consider a massive object like this and let us assume that the massive object is made of hydrogen and that hydrogen is ionized there is a lower limit to the mass absorption coefficient that come from the Thomson scattering.

So, any charge particle is going to scatter radiation through Thomson scattering and the cross section of Thomson scattering is sigma T and this has a value. The value is known, the value of 6.65*10 to the power - 29-meter square. For an individual hydrogen atom the electron essential is what does the Thomson scattering it has a value given over here. It is in meter square and so this is for an individual atom so the absorption coefficient per unit mass is going to be kappa = sigma T.

The Thomson scattering cross section by the mass of one hydrogen atom that is the lower limit to the value of Kappa mass absorption coefficient that you can have it can be more there could be other processes. It could be more then that but this much is going to be there. So, now we have a lower limit to the mass absorption coefficient. Now, suppose I have an object of a certain mass and it is extremely luminous.

And if it is so luminous it may the possibility may occur where it is so luminous that luminosity is adequate to blow off a part of the gas that makes up the object. So, then the object would become unstable and this upper limit to the luminosity is called the Eddington luminosity. So, if an object of mass m has a luminosity more than the Eddington luminosity the radiation will disrupt the object. It will force out.

It will push out parts of that object and the object will get disrupted. So, this is upper limit to the luminosity of an object of mass m and you can easily estimate what the value of this is by just putting in this Thomson scattering cross section and assuming its hydrogen atom the mass of a hydrogen atom. So, I leave it to you as an exercise to calculate the Eddington luminosity what you have to do is you have to just put in this here.

So what you have is Eddington luminosity is 4 pi G M the mass of an hydrogen atom m h divided by the Thomson scattering. And I request you to calculate the Eddington luminosity for a solar mass object the mass of the sun. We have discussed what the mass of the sun is. So, this is the first problem. We shall discuss other problems in radiative transfer as we go along the rest of the course.