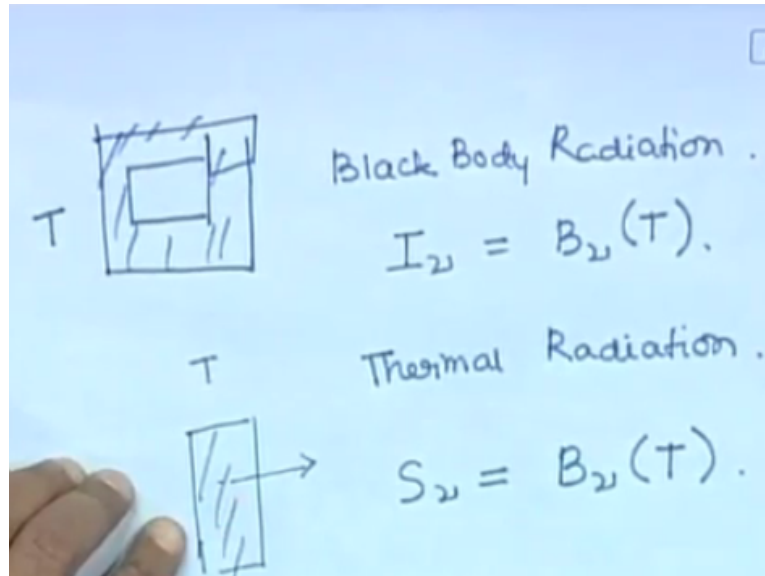


Astrophysics & Cosmology
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Lecture – 12
Thermal Radiation

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Welcome, we have been discussing thermal radiation and let me briefly recapitulate right at the start what we had discussed in the last class. Thermal radiation is radiation that is emitted by material, which is in thermal equilibrium and if we have radiation that is enclosed in a cavity like this and the whole enclosure and the radiation inside comes to thermal equilibrium at a temperature T , then the radiation that is inside this cavity is referred to as blackbody radiation.

And we had seen in the last class that this radiation; the spectrum of this radiation the specific intensity is just a function of temperature and this function is called the Planck function. We had also seen that if we have some material which is at a temperature T the material itself is in equilibrium, then the source function for this material, which is in thermal equilibrium is the Planck function.

So, these are 2 different things, so this is called a thermal radiation that originates from such a material is called thermal equilibrium is called thermal radiation it need not be; the radiation itself need not be in equilibrium with the material, so the specific intensity of the radiation

could in principle be anything. If the specific intensity of the radiation comes to equilibrium with the material then the specific intensity becomes the Planck function.

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Properties.

$TdS = dQ = dU + PdV$

$u = uV$

$p = \frac{1}{3}u(T).$

$dS = \frac{1}{T} \left[v du + u dV + \frac{4}{3}u dV \right]$

$dS(v,T) = \frac{1}{T} \left[v \frac{du}{dT} dT + \frac{4}{3}u dV \right]$

Otherwise, just the source function is the Planck function this is called thermal radiation, this is called black body radiation, okay then having discussed this, we went ahead and we calculated the entropy; how the entropy on this depends on the temperature and the volume.

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$\left(\frac{\partial S}{\partial T}\right)_v = \frac{v}{T} \frac{du}{dT}$ $\left(\frac{\partial S}{\partial V}\right)_T = \frac{4u}{3T}$

$\frac{\partial^2 S}{\partial T^2 \partial V} = \frac{1}{T} \frac{du}{dT} = -\frac{4}{3} \frac{u}{T^2} + \frac{4}{3T} \frac{du}{dT}$

$\frac{du}{dT} = \frac{4u}{T} \quad | \quad U \propto T^4$

Stefan Boltzmann's Law.

And based on this, we finally reached the conclusion that the energy density of this blackbody radiation is proportional to T to the power 4.


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$$u = a_B T^4$$

$$B_{\nu} = \frac{c u}{4\pi}$$

$$F = \pi B = \frac{c u}{4}$$

$$F = \sigma T^4 \quad | \quad \sigma = \frac{c a_B}{4}$$

$$S = \frac{4}{3} u V \propto T^3$$


And this constant of proportionality, we called it a_B the Stefan constant. Further, we also saw that the Planck function this we had seen earlier that the specific intensity is related to this multiplied by c and divided by 4π , you get the specific intensity, so if you integrate this over frequency the Planck function over frequency then that is $= c \cdot$ the energy density by 4π .

The flux; the brightness, this is the brightness is $c \cdot$ the energy density by 4π , which also scales as T to the power 4 and the flux from any surface of a black body cavity, we saw is; also scaled as T to the power 4 and this constant is called the Stefan Boltzmann constant σ and it is related to this Stefan constant like this, so it is $c \cdot$ the Stefan constant/4. We also calculated the entropy.

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Adiabatic Expansion.

$$T^3 V = \text{constant} \quad | \quad p = \frac{u}{3} = a_B T^4$$

$$p^{3/4} V = \text{const}$$

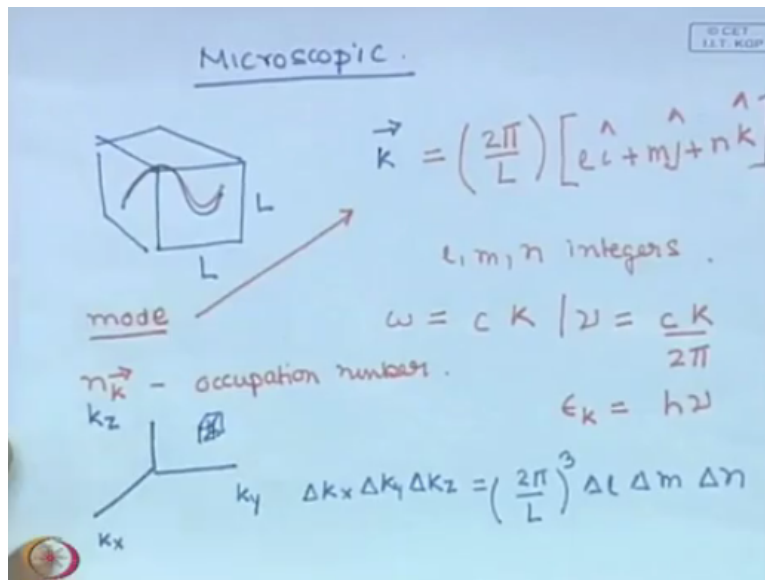
$$p V^{4/3} = \text{constant}$$

$$p V^\gamma = \text{const}$$

adiabatic index $\gamma = 4/3$

And the entropy we saw is proportional to $T^{\frac{4}{3}}$ and it is $= \frac{4}{3}$ the energy density by the temperature into the volume of the system, so the entropy density is $\frac{4}{3}$ energy density/ T . After that, I told you okay, we also looked at what happens when you have adiabatic expansion of radiation and we saw that for radiation which is made to expand or contract adiabatically; PV to the power $\frac{4}{3}$ is a constant.

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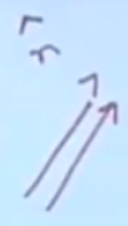


So, radiation behaves like an adiabatic medium with adiabatic index $\frac{4}{3}$ okay. Having done all of this, I told you that from my macroscopic thermodynamic considerations, you cannot proceed much further to determine these constants; the Stefan's constant or the Stefan Boltzmann constant you need to look at the microscopic nature of this. So, we looked at; that is what we were looking at and in the last class; at the end of the last class this is where we had reached.

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$$u_{\nu}(\Omega) = 2 \frac{h \nu^3}{c^3} n_{\vec{k}}$$

$$I_{\nu} = 2 \frac{h \nu^3}{c^2} n_{\vec{k}}$$

$$I_{\nu}(\hat{n}) \Big|_{\vec{k}} = \left(\frac{2\pi h \nu}{c} \right) \hat{n}$$


So, we had calculated a relation between the specific intensity, remember the specific intensity I_{ν} is for raised in a certain direction, so this is a unit vector along the direction of the rays. Let me draw a picture, so these are my rays that okay, so they along some unit vector \hat{n} and they will be in a spread of solid angles. So, the specific intensity at the frequency ν in this particular direction of the unit vector \hat{n} is related to the occupation number of the photons.

This n over here is the photon occupation number and photon occupation number of a particular mode, the mode being the vector; wave vector \vec{k} . How is this mode related to this frequency and the direction in which the wave is propagating the ray is propagating the wave vector \vec{k} , so the direction of this it is obvious is going to be same as the unit vector \hat{n} that is the direction in which the light ray is propagating.

And the magnitude is going to be 2π ω and ω we know is; so $\omega = c \cdot k$ right, so we can write this as $2\pi c \cdot k$, so ω $2\pi \omega$ and ω is c into, so we can; so this is going to be right, ω . So, this is the modulus of k the modulus of k is ω/c and ω is $2\pi \nu$, so we can write this as $2\pi \nu/c$, right, that is a modulus of k , the magnitude into the direction of the vector.

So, we have a relation relating the specific intensity in this direction to the occupation number of a mode \vec{k} with these factors outside. These factors one of them comes from the energy of the photons and others come from just counting considerations $h \nu$ comes from the energy of the photon and others come from just counting considerations and factor 2 comes here because light has 2 polarizations that we are assuming that the light is unpolarized.

So, both are equally present okay, so this is a very important relation and it has got several applications and we shall see these as we go along this course right now it is a very general relation between the specific intensity and the occupation number, the modes of the photon okay. Right now, we are interested in a situation where the photons in this cavity are in thermal equilibrium with the walls of the cavity.

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Thermal Equilibrium

Bose Statistics T $\mu = 0$

$$n_{\vec{k}} = \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

$$B_{\nu}(T) = \frac{2h^3\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

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So, they are in thermal equilibrium at a temperature T . Now, photons we know follow the; there are 2 statistics fundamental statistics; one is the Bose statistics the other is the Fermi statistics. Now, photons are; what are photons? They have spin one, so they are bosons right, so photons follow the Bose statistics. So, the occupation number n , so if the photons are in thermal equilibrium that is what we are assuming, right.

If they are not in thermal equilibrium, the occupation number could be anything but if they are in thermal equilibrium then the occupation number is given by the Bose statistics and there are 2 parameters here; 1 is the temperature of the system and the other is the chemical potential μ , which so there is a system which is bosons in equilibrium, so to completely specify the system you need to tell the temperature and the chemical potential, right.

The Bose distribution has 2 unknown parameters; the temperature and the chemical distribution; chemical potential Now, photons are not conserved that is something I should have mentioned right in the beginning also, so you can use these thermodynamic considerations to

you to calculate the energy and then everything else because photons can be destroyed and created okay.

So, the number of photons essentially adjusts itself to the temperature depending on the temperature, the number of photons will go up or down, okay. So, for things, for particles, bosons that are not conserved whose number is not conserved okay, there is no conserved quantity associated with a photon, there is no charge there is no mass, mass is not conserved any way but there is no charge associated with the photon.

So, these are massless uncharged particles, so the chemical potential they have no charge whatsoever no kind of charge, so the chemical potential is 0 and then you can straight away write down the occupation number. In thermal equilibrium, this is isotropic and it is one by the Bose distribution and with 0 chemical potentials it is one by exponential their energy by KT and the energy we know is $h\nu / KT - 1$, that is the Bose distribution.

So, the wave number is converted to frequency we have just seen how to do that and you put it in here and this tells you how many photons there are in this mode okay. So, once you assume that it is in thermal equilibrium it is; you can straight away just plug it in here and you get the Planck function okay, so just plug it in here if so these photons are in thermal equilibrium with at temperature T with some material, right.

Photons do not interact with each other, so they cannot come to thermal equilibrium by themselves, you required the cavity or some material which will bring it to thermal equilibrium. Once they come to thermal equilibrium, their occupation number is given by this Bose distribution and we can straight away write down the Planck function which is $2 h \nu^3 / c^2$ into the Bose distribution was occupation number.

So, we have to $2h$; h here is the Planck constant okay bear this in mind and K we shall use K_B here to distinguish it from the wave number, so K_B is the Boltzmann constant, h is the Planck constant and K_B is the Boltzmann constant, so the Planck function is $2 h \nu^3 / c^2$ and here we have $1 / \exp(h\nu / K_B T) - 1$, okay, so that is the Planck function that is the specific intensity of blackbody radiation, this is K_B of blackbody radiation okay.

So, let us now look at the behaviour of the Planck function both as a function of frequency and as a function of temperature, so let us now spend little time just looking at this. So, the spectrum of blackbody radiation how it depends as a function of frequency at a given temperature or if you fix the frequency how does it vary with temperature. Let us look at this okay.

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Rayleigh Jeans Limit

$$\frac{h\nu}{k_B T} \ll 1.$$

$$n_{\vec{k}} = \frac{k_B T}{h\nu} \gg 1.$$

$$B_{\nu}(T) = 2 \frac{\nu^2}{c^2} k_B T = \frac{2 k_B T}{\lambda^2}$$

So, first we will consider a certain limit; the limit being the situation where $h\nu / k_B T$ the Boltzmann constant * T is much less than one, so this is called the Rayleigh jeans limit okay, so we will first study the Planck spectrum in the Rayleigh jeans limit, what we mean is that $h\nu / k_B T$ is much less than 1, so the temperature of your system define the frequency and that frequency is basically $k_B T$ by the Planck constant.

If the frequency that you are looking at is much more than that is much smaller than that okay, so the frequency that you are looking at is much smaller than that, let us see what happens. So, if $h\nu / k_B T$ is much smaller than 1, then we can expand this in a Taylor series and this will become $1 + h\nu / k_B T - 1$, so 1 cancels out, so the occupation number in this limit, $n_{\vec{k}}$; let us just look at the occupation number that also is interesting is $k_B T / h\nu$ which is much > 1 .

Because that is the inverse of this, so each mode is occupied by a large number of photons that is the limit that we are looking; at each mode is occupied by a large number of photons it is very densely populated okay, that is the limit that we are looking at and in this limit the Planck function, so this whole factor is now $k_B T / h\nu$, so one factor of $h\nu$ cancels out and what

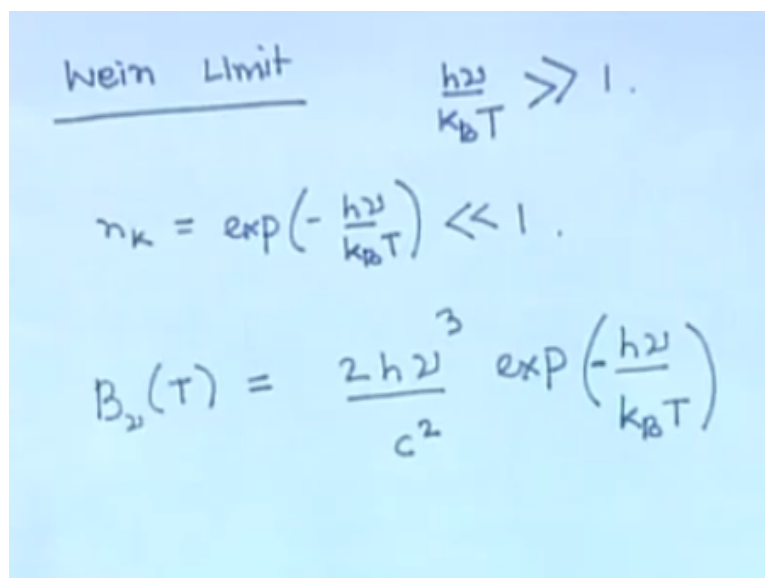
we have is; $2 \nu^2 / c^2$ the Boltzmann constant into the temperature of the radiation which can also be written in a form that is quite easy to remember $2 k_B T / \lambda^2$ okay.

So, in the Rayleigh jeans limit, we see that the Planck function takes a relatively simple form, if at a fixed temperature you look at the specific intensity then the specific intensity or the amount of energy increases proportional to the square of the frequency that is the first thing or it goes down as $1/\lambda^2$, so the higher the frequency the more the energy contained in this radiation.

This works as long as you are in this regime okay and it is linearly proportional to the temperature, so if you increase the temperature make it twice the energy contained will also just double okay and the other interesting feature to note is that the Planck constant does not appear anywhere over here. It is purely classical in that you do not need any quantum mechanics to understand this okay, which is again can be understood intuitively from the fact that you have many photons in each mode.

So the photon nature, the discrete nature does not manifest itself, you have a large number of photons in each mode and you can think of the electromagnetic wave is actually a wave the photon nature does not manifest itself okay and much of radio astronomy works in this regime in the Rayleigh jeans regime, so this is very useful if you are doing radio astronomy for example radio astronomers always use this.

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Wein Limit $\frac{h\nu}{k_B T} \gg 1.$

$n_k = \exp\left(-\frac{h\nu}{k_B T}\right) \ll 1.$

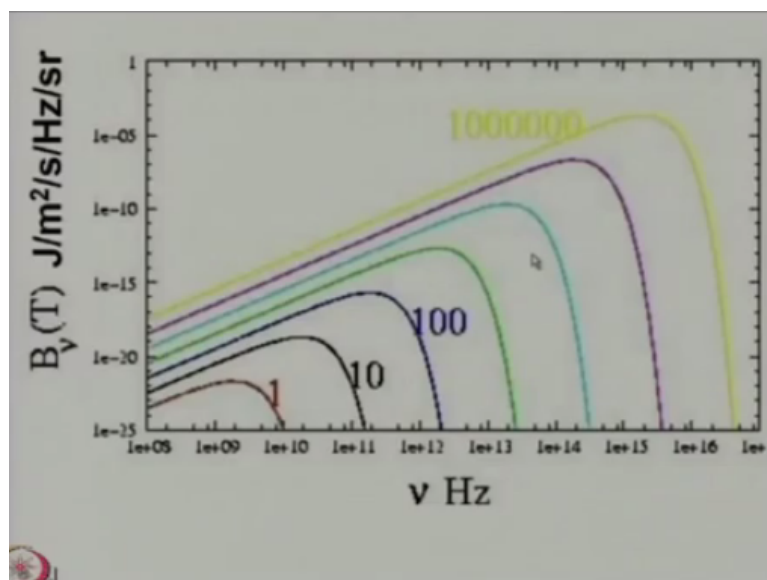
$B_\nu(T) = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{k_B T}\right)$

They do not have to bother about, in most situations they did not bother about the entire Planck spectrum, this is adequate. Now, there is the other limit that is the Wien limit or the Wiens spectrum and here it is assumed that $h \nu / k_B T$ is much > 1 , so if $h \nu / k_B T$ is much > 1 , the occupation number is essentially exponential - $h \nu / k_B T$ because this number is a very large number much > 1 , okay.

So, the occupation number and you see that this is obviously much less than 1, so the photons in each mode there are very few photons much less than 1 okay, that this is an average occupation number, so you expect to find typically less than 1 photon in each mode okay and in this regime the particle nature of the electromagnetic radiation actually starts to manifests itself it is in this regime okay.

And in this regime, the Planck spectrum now becomes $2h \nu^3 / c^2 \exp(-h \nu / k_B T)$, so there is an exponential cut off, this function falls off rapidly because of this exponential, so at a fixed temperature if you increase the frequency, this function falls off rapidly with temperature okay. So, let me show you a graph showing the; these are the 2 limiting situations.

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Let me show you a curve this picture over here shows you the behaviour of the Planck function this shows you the Planck function for different values of the temperature. So, let us just focus on any one of them, okay for first so let us just consider anyone of them, let us consider say this one over here okay this is for 1 kelvin, 10 kelvin, 100, 1000 all the way to a million. So, if you look at any one of them you see at low frequencies, this is a log-log scale.

So, both of these are intervals are logarithmically spaced, so this is proportional to ν^2 and then it increases and then there is a maximum somewhere and beyond which it falls off exponentially; the exponential cuts it off okay, so that is the first feature that we see of the Planck spectrum. So, at low frequencies, it increases as the frequency square and then there is a maximum somewhere and then beyond that it falls off.

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$$\text{Fix } \nu$$

$$B_\nu(T)$$

$$\frac{dB_\nu(T)}{dT} > 0$$

$$\text{Maxima. Fix } T$$

$$\frac{dB_\nu}{d\nu} = 0$$

$$\frac{\nu_{\text{max}}}{T} = 5.88 \times 10^{10} \frac{\text{Hz}}{\text{K}}$$

Wien Displacement Law.

This is a characteristic frequency and obviously the characteristic frequency is divided; decided by $KB*T$, it depends on the temperature. The higher the temperature the larger the characteristic frequency okay, so this is something that, okay before we come to this; let us ask the question what happens if I fix the temperature, so if I fix the temperature and look at a fixed frequency.

So, at sorry; if I fix ν and look at B_ν as a function of T , okay at a fixed frequency let us say I have look at this point over here, which is 10^{10} Hertz and vary the temperature increase the temperature now it turns out that ΔB , you can do this and check for yourself it is straight forward I am not going through the exercise, you just differentiate it with respect to time the expression that we have; with respect to temperature.

And you will find that this is positive okay, so it means that if I increase the temperature the value of the specific intensity is at a fixed frequency the value of the specific intensity is going to increase okay, so that is what we have here. So, if I fix the frequency and look at the curves for different temperature they all lie on top of one another okay. So, as the temperature increases the value of the specific intensity goes up okay.

So, these curves never intersect right because at a higher temperature the curve is always above the one at a lower temperature, so these curves never intersect, so if I know if I observe some radiation and I can measure; if I know it is a black body and if I can measure just anyone point on this curve, I know that the temperature immediately, it is unique okay. So, if from, if I know that it is a blackbody spectrum and if I can measure just one frequency then the temperature is uniquely determined.

Because the curves do not intersect okay, so if I am just one measurement over here, so I measure, if I can determine the entire temperature provided I have some a priori information or if I assume that it is a blackbody radiation okay, this is the first thing. Second thing is; you can determine where the maxima occur okay. In frequency, at a fixed temperature, fix T and ask the question del B where is this 0?

This will give you a the value of the maxima okay and we see that if you do this; if you do this exercise or if you look at the curve over here, so if you look at the curves over here, you will find that ν_{\max}/T and this is a constant okay and the value of this constant it is $5.88 \cdot 10$, so hertz/kelvin okay. So, this tells us that which you can determine this again I am not going through the algebra.

But you can determine this it is quite straight forward and exercise you have to just differentiate the Planck spectrum with respect to frequency and then find set it = 0, so you have to just differentiate this with respect to ν , so there is ν here and ν here and then set it = 0. If you do this exercise what you find is; that the frequency where you have maxima scales proportional to the temperature and the constant is given over here.

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$$B_{\lambda}(T) d\lambda = B_{\nu}(T) d\nu$$

$$= B_{\nu}(T) \left| \frac{d\nu}{d\lambda} \right| d\lambda$$

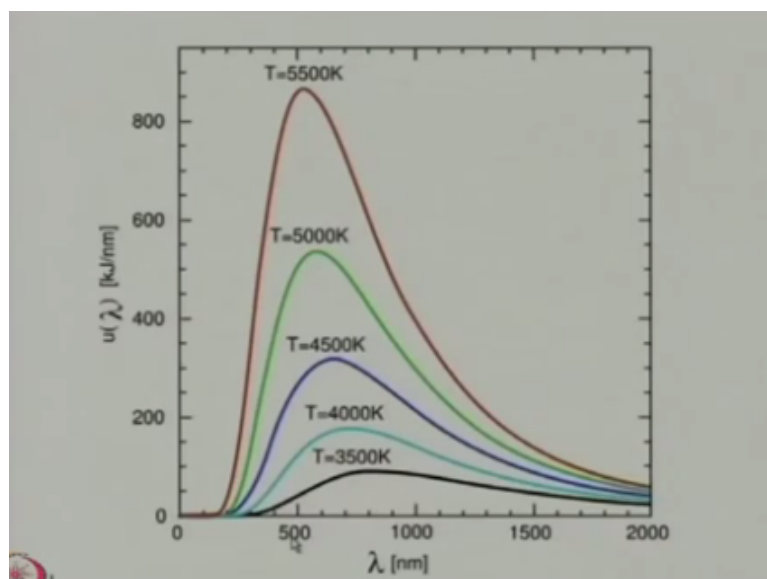
$$B_{\lambda} = \frac{c}{\lambda^2} B_{\nu}$$

$$\lambda_{\text{max}} T = 0.29 \text{ cm K.}$$

This is called Wien displacement law okay. Now, here we have been discussing the specific intensity which is the energy contained in a ray per frequency interval. Now, people also can often discuss another quantity which is the frequency the energy contained per wavelength interval okay, which is B_{λ} the Planck function as a function of λ okay, it is not just the Planck function as a function of λ .

So, $B_{\lambda} d\lambda = B_{\nu} d\nu$, the same energy is contained in the corresponding energy wavelength interval and we know that, so we know that we can write this as the mod of $d\nu d\lambda$, so which essentially tells us that $B_{\lambda} = B_{\nu}$ and this if you differentiate ν/λ , you will get c/λ^2 okay. So, these are 2 different quantities not only do you write it as a function of λ .

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But it also tells you the energy arriving in that interval of wave length in an interval of the unit wave length interval, whereas the specific intensity that we have been dealing with tells us the energy per unit frequency interval. Now, these curves let me just show you a picture of these curves and these curves obviously look different. So, this picture shows you B_{λ} okay effectively, forget about the scale over here this shows you the behaviour of B_{λ} .

Here again, you have a maxima and as you increase the temperature the maxima shifts to a smaller wavelength okay, so you can differentiate this also with respect to λ and set it = 0 and you have then, you will get a relation that λ_{max} that is the wavelength where B_{λ} is a maximum into the temperature T , this is $= 0.29$ centimetre kelvin okay, this is also known as; this also is known as the Wien displacement law.

So, you can use either of them either of them both of them refer to Wien displacement law and this tells us where B_{λ} has a maximum and the one that we considered earlier tells us where the B_{ν} has a maxima. Now, you cannot convert λ_{max} and λ_{ν} , you cannot relate them λ and ν_{max} by just a factor of c because they are the maxima of 2 different functions okay they are slightly different okay.

So, the key point here is that as you increase the temperature; as you increase the temperature the maxima shifts to higher and higher frequencies which ever you look at high, shifts to higher and higher frequencies or smaller and smaller wavelengths okay and just as an example look at these curves look at these curves then we see that at around 1 kelvin or 10 Kelvin somewhere over here the maxima are all in the radio microwave region okay.

Whereas for a few thousand over here it is in the optical visual range or ultraviolet visual infrared and if you are looking at 1 million or so it is in the x ray okay that is how it keeps as you increase the temperature the spectrum; the peak of the spectrum shifts and the bulk of the radiation is also shifting to at higher and higher frequency range okay. So, that is a brief discussion of the nature of the shape of the Planck spectrum.

Now let us ask slightly different question, what is the total energy density in the Planck spectrum? Right, so our aim was after all to calculate that constant a_B or σ from these microscopic considerations okay. So, let us ask the question, what is the total energy density in

this blackbody radiation? And to do this what are the things that we have to do? So, the total energy density we have seen; we have to take the Planck spectrum, let me go through this.

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$$u_{\nu} = \int \frac{B_{\nu}(T)}{c} d\Omega$$

$$u_{\nu} = \frac{8\pi h^3 \nu^3}{c^3} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

$$x = \frac{h\nu}{k_B T}$$

$$u = \int u_{\nu} d\nu = \frac{8\pi k_B^4 T^4}{c^3 h^3} \left(\int \frac{x^3 dx}{e^x - 1} \right)$$

So, you have to take the Planck spectrum and you have to; so we know that the energy contained in a particular set of rays in a particular direction this; in a particular direction, this is just this quantity into rather this quantity divided by c and if you want the total energy density then you have to also integrate over solid angle and you have to also integrate over frequency okay.

So, this size; the frequency interval we shall do next okay so, this gives us the total energy density of the blackbody radiation of ray, contained in rays in all direction and this d Omega integral can just be taken outside, so this is =, how much is this let me write down, so this is = h pi sorry no no; 8 4pi; this will give us 4 pi, so I have 8pi, I am just multiplying this with 4pi and here I have h nu cube/c cube * 1/exponential h nu/ KT - 1.

So, this is the energy density at a frequency nu in the blackbody spectrum and to calculate the energy; total energy density integrated over all frequencies, I have to integrate this over all frequencies okay. To do that; it is convenient to introduce a new variable, let us call that variable x such that x = H nu/the Boltzmann constant * T, right which makes this whole thing dimensionless, so let us do that.

So, to do that; if you do that then this integral this becomes, the quantity that is integrated is x cube dx e to the power x -1, right that is what is it to be integrated and the rest are just constants

outside, so let me write down those constants. I have 8π and we shall have k_B , so we have to multiply with k_B to the power 4 and T to the power 4, right, so that we have because we have 4 exponents over here, we had the n ones, ν cube and $d\nu$ over here.

So, I want to convert them into x , so I will have this factor K after multiply this with $k_B T$ to the power 4, so I have this and in the denominator I will have c^3 and I will have h to the power of 3, 1 power of h is cancelled out with this okay and this integral; we know this integral has a value π to the power by $4/15$, okay this integral has a value π to the power $4/15$, okay. So, now you see the energy density is some constants into T to the power 4.

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$$a_B = \frac{8 \pi^5 k_B^4}{15 c^3 h^3} = 7.56 \times 10^{-16} \text{ J/m}^3/\text{K}^4$$

$$\Delta = \frac{c}{4} a_B = 5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4$$

And these constants we had given the name the Stefan constant, so we can straight away identify what the Stefan constant is; the Stefan constant a_B ; the Stefan constant a_B , let us identify what it is, so I have 8 and I have in the numerator $8\pi^5 k_B^4$ to the power $4/15 c^3$ and h^3 that is the Stefan constant and this has a value. Now, you put in; all of these are all fundamental constants.

This is the Boltzmann constant, this is a Planck constant, this is the speed of light and π , so you put in all the values and what you get that this is $= 7.56 \times 10^{-16}$ joules per meter cube/ kelvin to the power 4, right. So, we have worked out the Stefan constant and we had seen earlier that the Stefan Boltzmann constant σ , the Stefan Boltzmann constant is c times; $c/4$ times the Stefan constant okay.

And this has a value which is $= 5.67 \cdot 10^{-8}$ watts per meter square that is the flux per kelvin to the power 4 okay, so we have calculated the 2 constants that we had arrived at just from purely thermodynamic and macroscopic considerations. So, this tells us the total energy density in blackbody radiation, this tells us the flux that is emitted from the surface of a blackbody, it emits for this watts per meter square kelvin to the power 4 is multiplied by the temperature okay.

So, you will get it in watts per meter squared that is the flux okay, so I hope all of this is clear. So, this kind of brings to an end our discussion of blackbody radiation okay, now in astrophysics, we rarely we do encounter blackbody radiation but the blackbody radiation also serves as a very useful tool in quantifying as a kind of model which we use to quantify any other radiation okay.

So, there are various kinds of effective temperatures so we; it is often convenient say, we receive the radiation from some source from some Astrophysical source, now we would like to associate a temperature with that source based on the radiation that we receive after all the radiation itself we would like to use it to infer something about the source, so we use the radiation that we receive to infer certain temperature of the source okay.

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Effective Temperature
 related to blackbody spectrum
 Brightness Temperature .

$$I_{\nu} = B_{\nu}(T_b)$$
 RJ Limit
$$T_b = \frac{c^2 I_{\nu}}{2 \nu^2}$$

And this is called an effective temperature for the source, the real temperature of the source we do not know but this gives us some handle on what we call the effective temperature of the source and these effective temperatures okay, these are related to the blackbody spectrum and

there are different effective temperature select which are used in astrophysics, let me go through this, so one of them is called the brightness temperature.

Now, what is the brightness temperature? Let us just see, so we are receiving suppose we have measured the brightness of the radiation from a source by brightness we quantify the brightness of the radiation using specific intensity right, so suppose we have measured the specific intensity of a source of the radiation from a source and now suppose we assume that the source is blackbody, if we assume that then we can say that if it were a blackbody then this should be equal to the Planck function at some temperature T .

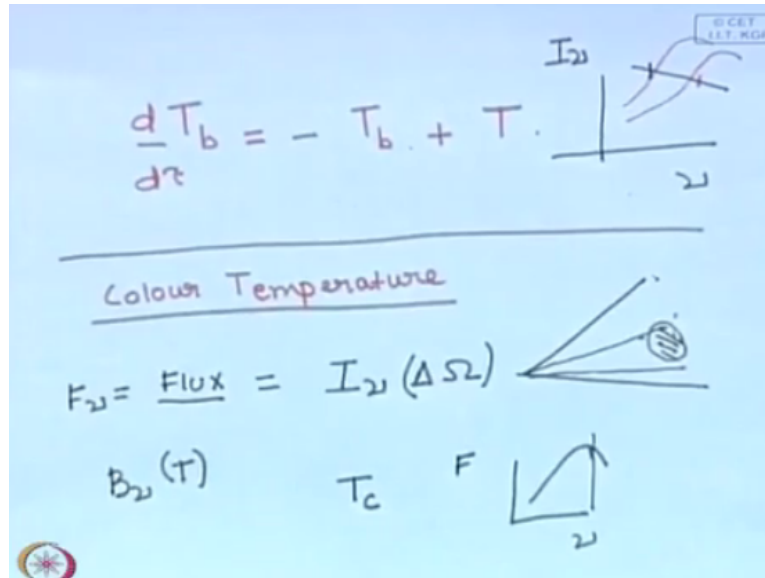
And I have measured the specific intensity at some frequency from some source okay, so I have measured just one value of ν , I have measured the specific intensity at a particular frequency, if I assume that it is a blackbody then that direction tells me the temperature of the blackbody this is called this temperature which I associated with the radiation is called the brightness temperature of that radiation okay, it quantifies the brightness of the radiation.

And in the Rayleigh jeans limit this is rather simple, so if I take; in the Rayleigh jeans which is very useful in radio astronomy this is quite simple, so in the Rayleigh jeans limit and this is going to be; you take the specific intensity and just multiply it with c square and divided by 2ν square it will give you the brightness temperature right, so if it were a planked spectrum the temperature would have been this much.

In the Rayleigh jeans limit, the temperature would have been this okay, so this is another way of just quantifying the specific intensity and it tells you the brightness of the radiation this is called it associates a temperature which quite often is easier to interpret we have a better feeling for it okay, so it comes out in the units of kelvin the specific intensity is in units of joules per meter square per hertz per second first irradiance, this quantity is something easier to interpret.

It comes out in the units of kelvin and it is a measure of the brightness of the radiation okay, it is called the brightness temperature and it is extremely simple; the relation is extremely simple in the Rayleigh jeans limit, otherwise you have to put in the Planck formula and determine this okay and it often gives you a good idea of what is going on in the radio for in radio astronomy where this is extensively used.

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Now, the radiative transfer equation in the Rayleigh jeans limit, the radiative transfer equation also takes on a very simple form, so in the Rayleigh jeans limit, we can instead of using specific intensity we can just use a brightness temperature, so the radiative transfer equation becomes $d/d \tau$ of the brightness temperature is equal to the specific intensity is now replaced by brightness temperature.

There are all these multiplicative factors, which will appear throughout this is = minus T_b plus the temperature of the source, the source function gets just replaced by the temperature of the material through which the radiation is passing okay, so the radiative transfer equation itself becomes much simpler in the; if you use the brightness temperature in the Rayleigh jeans limit okay.

And what happens now? So if the medium is optically thick we know what happens the specific intensity becomes equal to the source function right that is what we have seen if the material is optically thick whatever specific intensity whatever brightness comes on it what comes out is basically the source function if the material is optically thick, so here what happens whatever be the temperature; brightness temperature of the incident radiation the brightness temperature of the radiation that comes out is essentially the temperature of the material.

It is a blackbody with that temperature blackbody spectrum okay it is a temperature of the material okay, so this is one of the temperatures it is called the brightness temperature then we have another one which is called the colour temperature okay. Now, just before finishing this

brightness temperature see the brightness temperature is used in for any arbitrary radiation I just equated to the Planck spectrum at that value and find the temperature okay.

So, let us consider a hypothetical situation where I measure I_ν from some source which looks like this okay, this is obviously not a blackbody spectrum, so if I calculate the brightness temperature at this point what will I do? I will find which blackbody spectrum goes through this and I will associate that temperature, if I find the brightness temperature at this point again I will find which blackbody spectrum it will be a different one that goes through this and I will associate that temperature okay.

So, if I have something that is actually not a blackbody then different parts of the spectrum will have different brightness temperature I hope that is clear, if it is exactly a blackbody I will get the same brightness temperature throughout okay. There is something called the colour temperature, let me now tell you what the colour temperature is okay which again is quite useful sometimes, sometimes I cannot measure the specific intensity.

There are situations, where I have no idea of what the specific intensity is okay; I can only measure the flux okay. There are situations, let us consider this situation suppose I have a source over here and my telescope the instrument that I am looking at has an angular resolution which is larger than this okay, now what is the specific intensity? The specific intensity is the radiation coming per solid angle; I do not know the solid angle subtended by the source.

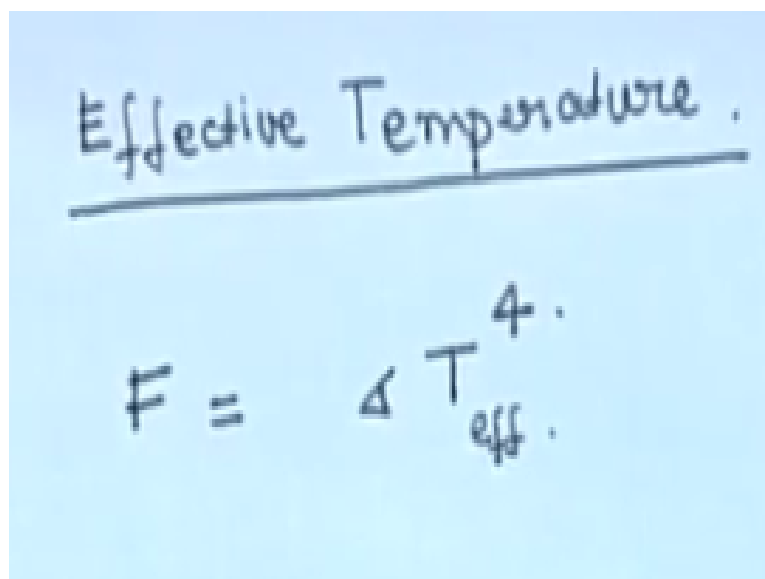
Because the source is smaller than the smallest angle I can measure, so I can only measure the flux the total radiation coming from this to determine the specific intensity, I have to actually write this, I have to divide by the solid angle subtended by the source and that will be some number okay but I cannot do this because I cannot determine this, it is smaller than the smallest angle I can measure with my telescope which happens.

For example, 2 stars I cannot determine the diameter angular diameter of stars it is much smaller than the resolution, the smallest angle that can be measured by most telescopes. So, I can measure only the flux which is the integral of the specific intensity which is the specific intensity into the solid angle of the source, I do not know what this is, so it will remain there as an unknown. Let us assume that the specific intensity is roughly constant over the source okay.

So, I can have a measurement of the flux F_{ν} and I find that its shape is very similar to a blackbody but I do not know what the obviously cannot say it is equal to a blackbody because I do not know the scale, I cannot say that it is = this B_{ν} because it is; I do not know the scale that is the solid angle involved okay, so what I will do is; I will just take the shape of the flux as a function of frequency and fit it to; see which blackbody spectrum the shape best matches okay.

And then determine; use that to determine a temperature called the colour temperature okay, so possibility is I could see that the flux has a peak and we know that the position of the peak is unique for a blackbody right; uniquely determined by the temperature. So, if I can measure where at what frequency the flux is maximum, I would then get an idea of the temperature and that would be one estimate of the colour temperature okay.

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Effective Temperature.

$$F = \sigma T_{\text{eff}}^4.$$

So, this is what the colour temperature is all about. We have a third kind of thing which is a third kind of definition effective temperature which is also used and this is called the effective temperature. So, suppose I have a source for which I can measure the total flux that is being emitted okay total flux that is being emitted, so if I can measure the total flux that is being emitted by per solid; from that surface of the source integrated over all frequencies okay.

So, this is the flux emitted per area of that source okay and we know that if it is a blackbody this will be σT to the power 4, so if I can measure the flux that comes from per unit area of that source and for example; the sun, I know the area of the sun, I know its radius, I know the,

so I can determine the area and if I know how much energy is being emitted per unit area of the sun how much flux, so then I can equate it to σT^4 .

And this is what is called the effective temperature, this again is perfectly correct only for a blackbody but in astrophysics, we applied we have to associate; we would like to associate a temperature with the radiation, so if I apply to some other source then this the temperature that we estimate using this is going to be called the effective temperature. So, if I tell you that the effective temperature of the sun has some value.

Basically, we have estimated the flux that is emitted by unit area of the sun and just fitted it to this Stefan Boltzmann law; put it in the Stefan Boltzmann law. Now, the temperature, the surface of the Sun may not be that it will be that if the Sun were exactly a black body, if not it will be something different but we have been able to; we have used this to characterize the radiation from the surface of the sun okay.

So, let me bring today's discussion to a close over here. Today we learned the nature of the Planck spectrum or the blackbody spectrum, how it is related to the Bose distribution and from there we derived the Stefan-Boltzmann constant and we then looked at various effective temperatures that are defined it used in; often used in astronomy and physics.