Astrophysics & Cosmology
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Lecture - 10
Radiative Transfer

Welcome, we have been discussing radiation and in the last class, we discussed how you could describe the energy that is being carried by a ray and I told you that this could be described this could be quantified in terms of the specific intensity or the brightness, which is denoted by I nu and this quantity is defined.

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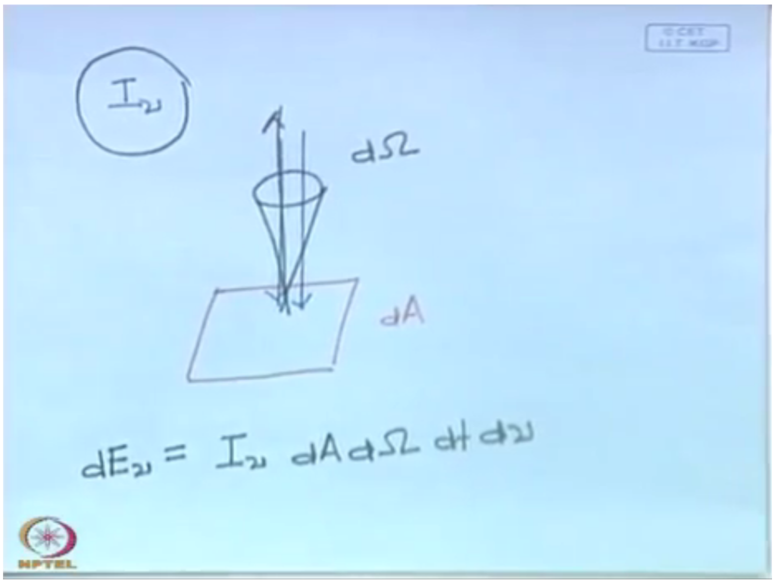

For a ray, we have a particular specific intensity and we have defined the specific intensity in such a way so that it tells us the following thing. If we place an area element of size dA normal to the direction of the ray and ask the question what is the energy coming into this area element from the solid angle interval d omega in the direction of the ray. So the energy incident in this ray is given by the specific intensity into the area element, into the solid angle, into the time interval, into the frequency interval.

So this specific intensity tells us the energy incident that is being transported by this ray, which is also the energy that is incident on this area element dA from the solid angle d omega so it is from a specific direction, very small range of directions in the frequency interval d nu and the time interval dt that is how the specific intensity is defined. So this is called the specific intensity or brightness.

And this is what we use to characterize to quantify radiation. On provided you are dealing with length scales, which are much larger compared to the wave length where we can think of light as rays and then in the last class, we studied certain properties of this specific intensity. I told you actually I showed you that it is conserved if light propagates in vacuum. So if light propagates freely in vacuum, the specific intensity does not change along the ray that we saw this in the last class.

And we saw various other things that you could calculate once you knew the specific intensity you could calculate the flux; you could calculate the pressure and various other things. In the last class, we had just briefly started discussing what happens when light propagates through a medium and that is what we are going to discuss in today's class right. So let me just briefly repeat what we had discussed right at the end of the last class.

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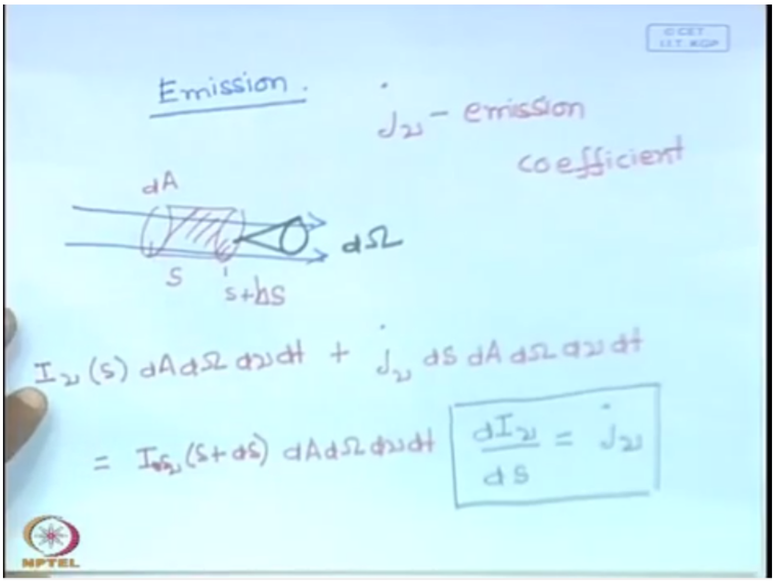

So we were considering emission, the emission of radiation. So our attention is on a particular ray and it is travelling in a medium so this is the medium and our ray is not in a single direction, it is in a small range of directions so in the solid angle interval d omega so this is what we are interested in, in the solid angle interval d omega, in a frequency interval d nu, in the time interval dt that is the energy that is incident.

So we are interested here in rays travelling in the direction of the solid angle interval d omega and the light here is propagating through a certain medium. So the incident energy we have already calculated this let us say that this has a volume. Let us first calculate the incident energy. Let us use S to label the points along the ray and this is S, this is s+delta S. This point is S, this point is S+delta S, dS you may say okay S+dS.

Now the incident energy on this surface, let us call the surface dA. We just saw how much the incident energy is, the incident energy is I nu and the point S*dA d omega d nu dt. Now we next define something call the emission coefficient J nu. So this medium through which the light is passing is also emitting radiation, so when the light comes out the energy that would come out would be the energy that is incident + whatever has been emitted by the medium in between.

Now the emission coefficient is defined in such a way so that J nu*the volume element. So here what is the volume? The volume here is dS*dA. This is the energy that is emitted out in the solid angle interval d omega, in the frequency interval d nu, in the time interval dt. So the emission coefficient tells us the energy that is emitted in a particular solid angle d omega.

In the time interval dt, in the frequency interval d nu/a volume dV where here dV is dA*dS that is the volume that the light is going through that is what gets added to this ray okay. So the light that comes out is now the specific intensity I nu at the point S+dS and everything else remains the same. So with this definition of the specific intensity of the emission coefficient it essentially tells us per unit volume, the energy emitted in that particular per unit frequency interval, per unit time, per unit volume, per unit solid angle okay.

So with this definition of the emission coefficient, the difference in the specific intensity in the length interval dS is J nu. So from this we can straight away say that dI nu dS = J nu. So when the ray travels with the distance dS through a medium with emission coefficient is J nu, this is the rate at which the specific intensity changes okay, this increases by an amount J nu dS okay.

So this is the emission so we model the emission process, here we are not going into the microscopic detail of how the emission occurs right. We are not going into the microscopic detail of how the emission occurs along the ray. We just quantifying whatever mechanism you have through this emission coefficient, which tells us that per unit volume that is the energy emitted in this solid angle d omega, in the frequency interval d nu, in the time interval dt which is of our interest okay.

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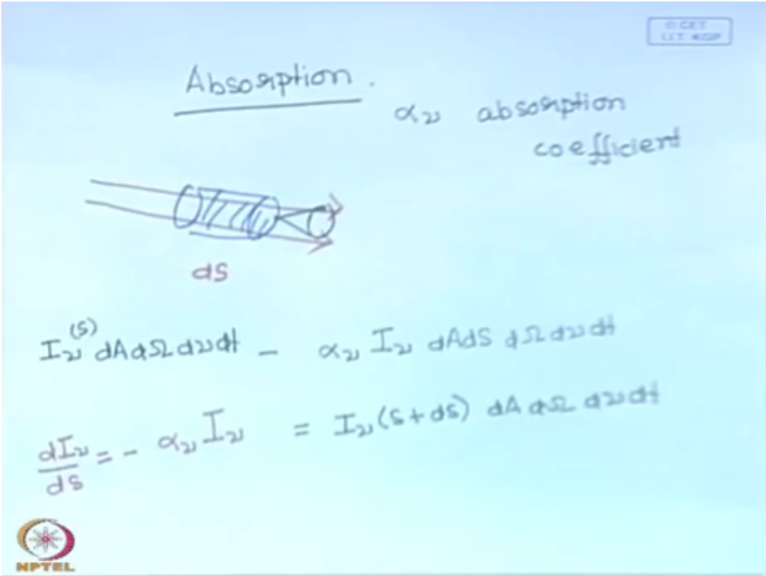

And as a consequence of that the specific intensity changes like this. Let us now quantify the absorption that is the other possible process so we are looking at absorption next okay. That is the next thing of our interest and here so we have the same thing, we have a ray which is propagating and it goes through a medium and the question is what happens to the specific intensity when the ray has travelled through a length dS through some medium, it will get absorbed.

In general, there will be emission and absorption both so how do you quantify the absorption? And for this there is something called alpha nu, which is the absorption coefficient okay. Here again let us go through the same exercise. So the incident energy per unit we are interested only in a certain solid angle so the incident energy is I nu*dA d omega d nu dt. Now the energy that gets absorbed here. This is at the point S okay so at the point S.

And the energy that gets absorbed is alpha nu times I nu*dA the volume, so the absorption the energy that is removed by a volume dV is alpha nu into the incident specific intensity into the volume. Again we have all those elements in the solid angle okay and this is equal to the specific intensity at the point S+dS. Again doing the same thing, so the absorption process now gives rise to a decrement in the specific intensity.

And the decrement we see is given by dI nu dS= -alpha nu I nu okay. So let us try to just get a picture of this of why we have written it in this way that it is proportional to the incident specific intensity into some absorption coefficient.

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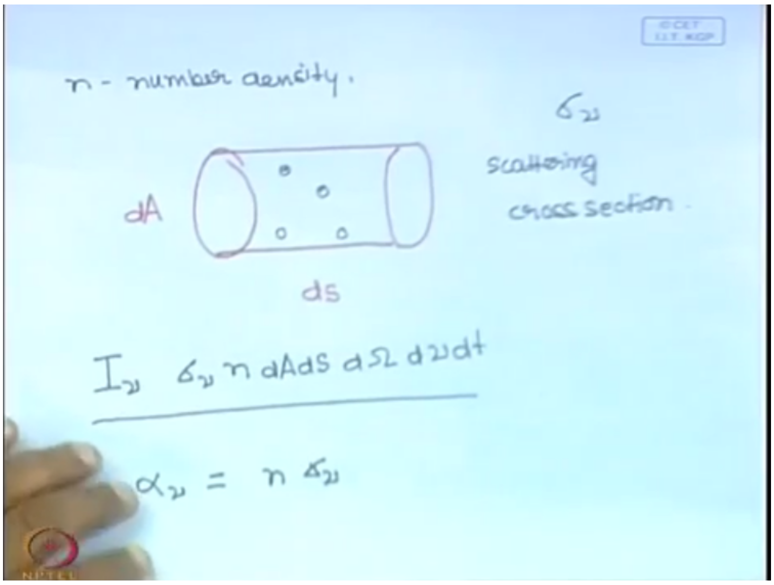

So consider a hypothetical situation where we have a volume. This is the volume that we were interested in, dA dS and in this volume, we have small absorbers distributed microscopic absorbers and each of these absorbers has a scattering cross section of sigma nu that is the scattering cross section, it is the area which each of these microscopic small absorbers that is the area through which it absorbs.

So the incident energy is let us just ask the question what is the total area that is actually absorbing, so these absorbers have a number density n okay. So let us assume that there are these small absorbers with number density n so n of these absorbers per unit volume and each one has a surface cross section area, a scattering cross section of sigma nu that is the area which it presents for absorption okay.

So let us ask the question what is the total area covered by my absorbers in this volume? To do that we have to take the number density and multiplied by the volume, so that is the total number of absorbers. Each absorber has an area sigma nu so that is the total area, which is presented by the small, small absorbers and the total energy that is absorbed now is going to be the I nu the incident specific intensity into this area into the solid angle into d nu dt right.

So this is the energy loss that is going to be caused by these small, small absorbers located over here, which with number density n and scattering cross section sigma nu right. So which is why we have written it which is one picture which kind of gives us a feeling for why we have written the absorption term as the incident specific intensity into some absorption coefficient.

And here we see that the absorption coefficient can also be written in terms of the scattering cross section of these individuals, microscopic scatters and it can be written like this where sigma nu is the scattering cross section, the area of each of these small, small absorbers and n is the number density okay. This is also quite useful sometimes when we know the scattering cross section of the individual microscopic absorbers.

So now we are in a position to combine both of these and write down the radiative transfer equation, which tells us what happens to the radiation as it propagates through a medium okay. So let us write down the radiative transfer equation now.

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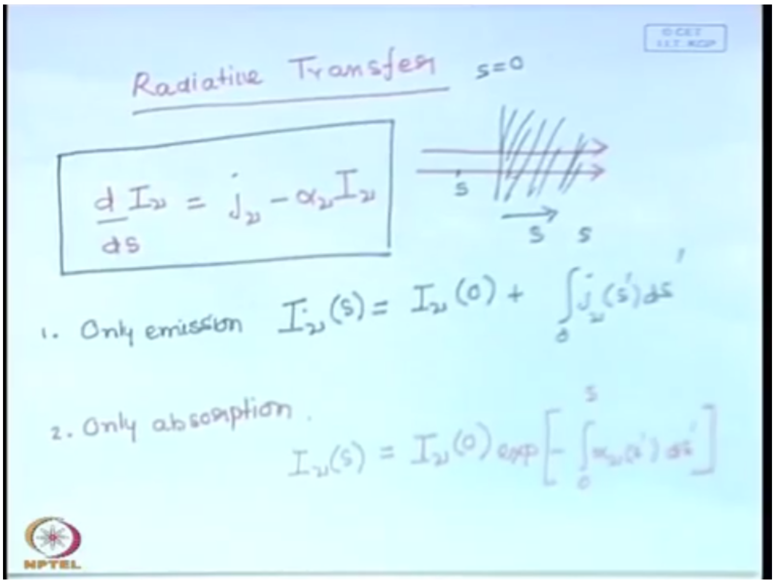

So this is called the transfer of energy through radiation, radiative transfer which is very important in astrophysics and the equation now as we are interested so this is the situation that we are considering is that we have a ray of light. We are interested in a particular ray of light, S is some distance parameter along the ray and the radiative transfer equation is the derivative of the specific intensity with respect to S.

So there will be an input of an energy due to emission and that is given by J nu and there is a removal of energy, which is given by - alpha nu I nu right. So this is the radiative transfer equation, which governs the propagation of radiation with which we can use to study how radiation propagates through any medium okay. So the rest of this class is largely going to be devoted to the discussion to the analysis of this equation right.

So let us start off by discussing 2 simple situations. The first situation is where we have only emission. So if you have only emission, so this is going through a medium so we have a medium over here right so as the light propagates through a medium let us suppose at the absorption is very small may be neglected and it is the emission that is important so the light is propagating through this medium, it is the emission that is important.

In that situation we can forget about this and then we have I nu this let us say is S=0 when it enters the medium okay and we are interested in I nu as we go along the ray so this is increasing S. So I nu at some point S is I nu S = 0+ the integral 0 to S. Now the emission coefficient can vary inside the medium, it need not be a constant so I have J nu S prime dS prime.

And if it is a constant then the problem is very simple, I have just got the emission coefficient into the length okay. So that is how the specific intensity increases okay so that is one of the simple situations. The other simple situation, which we encounter quite often is where you have only absorption and we encounter this very often. For example, light is propagating through a slab of glass or through some water or through the atmosphere when we look at the stars.

So in any such situation typically it is absorption which is important and not the emission. So in such a situation where you have only absorption, here again the solution is rather simple. So here we do not have the emission term so you can bring this I nu on to the left hand side and integrate this and we can straight away write down the solution without going through the mathematics the solution is I nu S. So S is some point inside the medium = I nu 0.

So when you bring the specific intensity on to this side you will get a log of the integral of alpha nu*S dS. So when you invert the log you get an exponential. So this is the incident specific intensity*exponential -0 to S alpha nu S prime dS prime right. So it decays exponentially if the absorption coefficient is a constant then it just decays exponentially with the increasing length - alpha nu*S okay.

So this is how the specific intensity changes with the increasing distance inside this. Now we are interested in a situation next where we have both emission and absorption. So the question is how to solve the equation, the radiative transfer equation. In such situations, it is very convenient to introduce something called the optical depth so let me introduce the optical depth.

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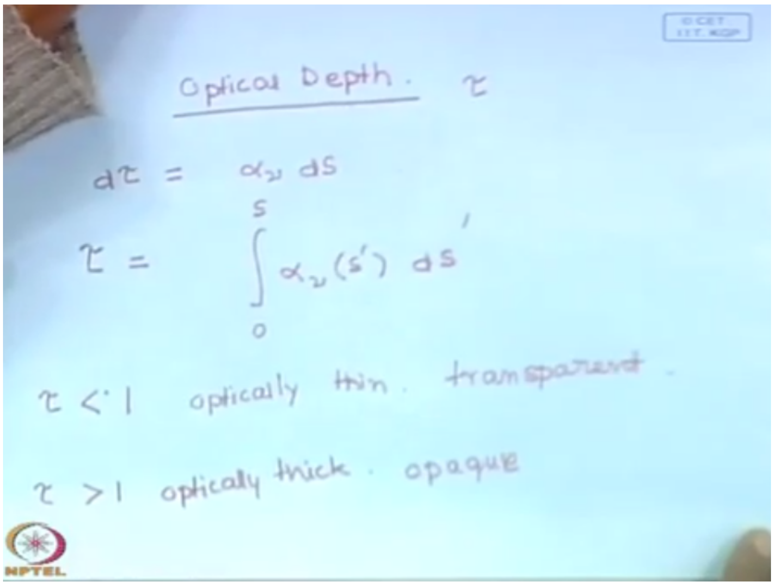

And we use the symbol tau to denote this quantity called the optical depth and this is very useful and very interesting quantity if you are doing radiative transfer okay. So this optical depth so the way we proceed is that we divide this equation with the absorption coefficient alpha nu okay. If I divide this equation with the absorption coefficient alpha nu then we have in the denominator over here we have d tau = alpha nu dS okay.

And this is called the tau is the optical density. So we can integrate this and determine tau so tau the optical depth it is a function of S obviously and this is 0 to S alpha nu S prime dS prime. This is the optical depth okay so we are interested in the propagation of this light into this medium S is 0 over here, S increases in side and the optical depth is the integral of the absorption coefficient into dS as you go inside the medium and you integrate till the position you want to calculate it at okay.

Now what does the optical depth tell us? Let us take a look at this okay. So to get a clear picture of this, medium is said to be optically thin okay. If tau < 1, a medium is said to be optically thin, so it is said to be optically thin. So let us see what it implies you see this term over here what happens to the specific intensity as you go inside the medium. It falls off exponentially right and it falls off exponentially as e to the power - the optical depth.

So if the optical depth is small say my medium is only this much and my optical depth is small so this is a small number so my specific intensity whatever goes in largely comes out. There is very little loss inside. If this number over here is quite small, is very small then the exponential is very close to 1 and whatever goes in roughly comes out. There is a very small attenuation inside or we can say that the chance of a photon, which enters this medium coming out without being absorbed is quite high.

The average photon does not interact with this medium, does not get absorbed by the medium, it just goes through and comes out okay so this is what is called optically thin. This is a medium that is optically thin and we would also call it transparent okay. So in such a medium, the typical photon just goes through the medium without interacting with it without getting absorbed okay.

So typical photon here just goes through the medium for an optically thin medium, it is transparent the typical photon goes through the medium without experiencing any absorption without interacting with the medium such a medium is said to be optically thin and this is measured by this quantity tau okay such a medium is said to be transparent and this is measured by this optical depth tau.

Tau > 1 it is optically thick so such a medium is said to be opaque what do we mean by this so this is the optical depth and as the radiation goes through the medium, it is specific intensity falls off exponentially and if this is a large number then it would fall off considerably, there would be a considerable drop inside. So the typical photon essentially cannot propagate through the medium without being absorbed it is guaranteed to get absorbed inside okay if tau > 1.

The typical photon in general gets absorbed in the medium okay so this is the significance of this quantity be optical depth, it gives us a picture, it essentially tells us the probability of a photon getting absorbed in the medium. If the optical depth is small, the probability is small. If the optical depth is large, the probability is essentially exponential of e to the power - tau is the probability okay.

So this quantifies whether a photon will go through the medium without getting absorbed or whether it will get absorbed in the medium okay and if it is much < 1 it is transparent. If it is of the order one or more if it is more than 1 it is opaque so this is the optical depth. So it is convenient to write down the radiative transfer equation in terms of the optical depth. We also define another quantity called the source function.

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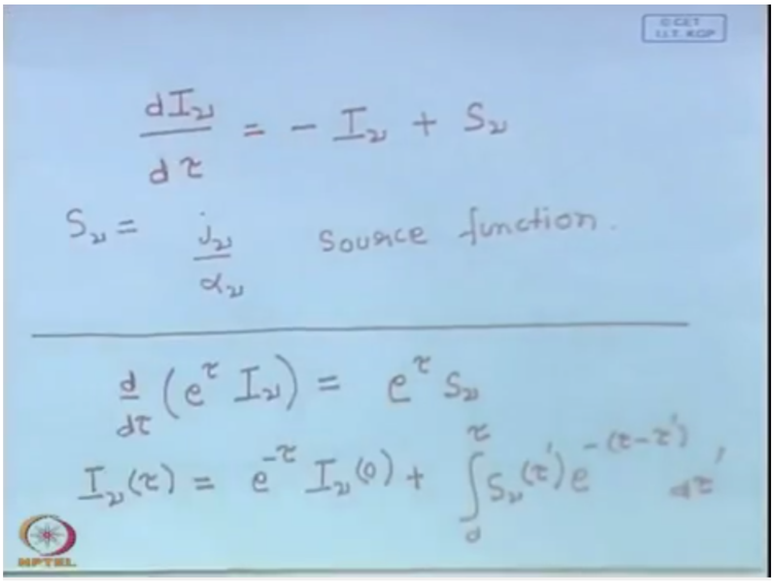

So if you divide this by alpha nu the equation now becomes let me write down the equation, the same equation now becomes so we have divided this equation by alpha nu and written this term first, which gives us -I nu and we have this S nu where S nu is the ratio of the emission coefficient to the absorption coefficient and this is called the source function of the medium okay.

And quite often these are very useful quantities both of these, the optical depth and the source function and it is convenient to think of everything now as a function of the optical depth okay and the optical depth is the very useful quantity. It is convenient to think of this equation in terms of the optical depth. It tells us the appropriate intervals as far as the interaction with the medium is concerned okay.

So that it quantifies the appropriate intervals along the way as far as interaction with the medium is concerned. The length may increase a lot, but if the optical depth does not increase much it is of no relevance for the propagation of the radiation because the specific intensity is going to remain same, whereas if the optical depth changes drastically over a small length then it is that interval, which is important right.

So if you work in terms of the optical depth you are now going to see the intervals that are important for interaction with the medium okay so that is one of the very great importance of using the optical depth okay. So we are going to work in terms of the optical depth everything here is now a function of the optical depth and not of the length along the ray okay everything is a function of the optic tau.

Second thing is the source function is a ratio of the emission coefficient to the absorption coefficient. Quite often this is a physically simpler quantity to calculate than the emission coefficient itself. So this also has got great utility of the source function okay. So we are going to solve this equation in the radiative transfer equation in this form. The way to solve this equation so we are going to work out the formal solution to this equation.

The way to proceed is as follows. We multiply this entire equation with e to the power tau. So let us multiply this equation with e to the power tau and we define so let us do that. So if I multiply this equation with e to the power tau then I can write it in this way and we can now integrate this equation straight away because this is what you want. This is the known function on the right hand side.

We are assuming that the source function is known so this is a known function on the right hand side, this is unknown you want to find this, this just integrate this. So it is quite straight forward now to write down the solution, just integrate this with respect to tau and then divide throughout by e to the power tau and we will integrate from tau = 0 to some value of tau okay.

So I will write down the solution straight away. So we are going to integrate this equation from 0, 0 is where the light enters the medium and tau is the value of the optical depth where we are interested in the specific intensity. So this will be equal to the e to the power -tau I nu 0 at tau = 0 this term does not contribute. So when I divide by e to the power tau I get e to the power - tau I nu 0.

And we have one more term, which is this into an integral which is I have to do the integral 0 to tau integral of the source function. So S nu tau prime e to the power now you see I am doing this integral and the variable is now becoming tau prime so I have e to the power tau prime d tau, but I am going to divide with e to the power tau at the end of the day so we can write this as e to the power -tau -tau prime d tau prime where that is the formal solution okay.

So this formal solution to the radiative transfer equation so we have solved it in general, but life is not so simple okay. So the formal solution does give us something, but it does not always not very easy to apply in all situations okay. Let us first physically interpret this equation the formal solution. What are the 2 effects that have gone in? So the first thing that you see is this term over here.

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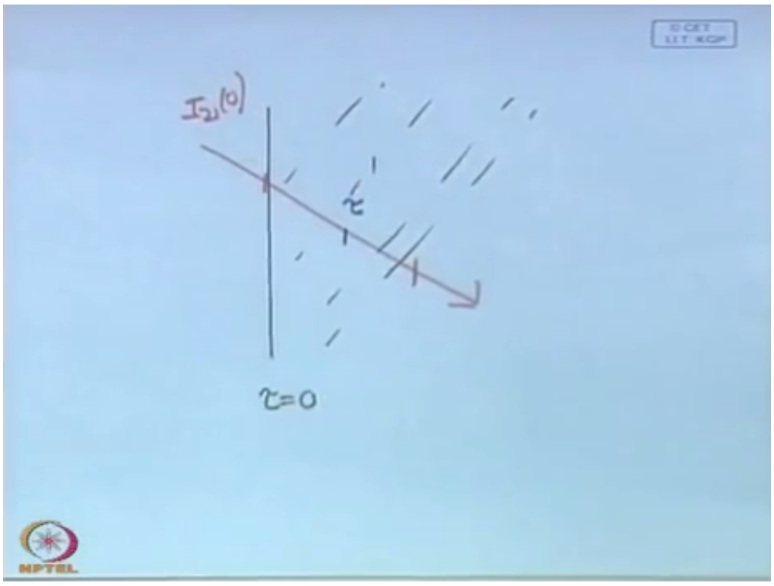

So let me draw a picture. This is my medium on this side, this is tau = 0 and this is the ray that we are following so at tau=0 we have some incident specific intensity I nu 0. As it goes inside, we are interested let us say at this point so as it goes inside it gets attenuated by an amount e to the power - tau, which we have just seen that is the solution when you have only absorption so which is the first term that you have over here.

Whatever is incident gets attenuated by an amount e to the power - tau where tau is the integral of this absorption coefficient along this line. The second effect that you have is that each point over here is itself going to contribute to the specific intensity and that is going to contribute by this source function okay. So the point over here at some tau prime is going to contribute and it is going to again suffer an attenuation that is not the entire tau, but the difference tau - tau prime by this much right.

So we have to take the source function at this point and attenuated by e to the power -tau -tau prime and then add up the contribution from all of these points, which is this term over here okay. So this is the contribution from the radiation along the path and each point the contribution gets attenuated again as it propagates through the medium okay so this is the formal solution to the radiative transfer equation.

Now as an example so we can first of all straight away see that we recovered the very simple situations that we had considered right at the start where there is only emission and where there is only absorption. Those you can recover straight away.

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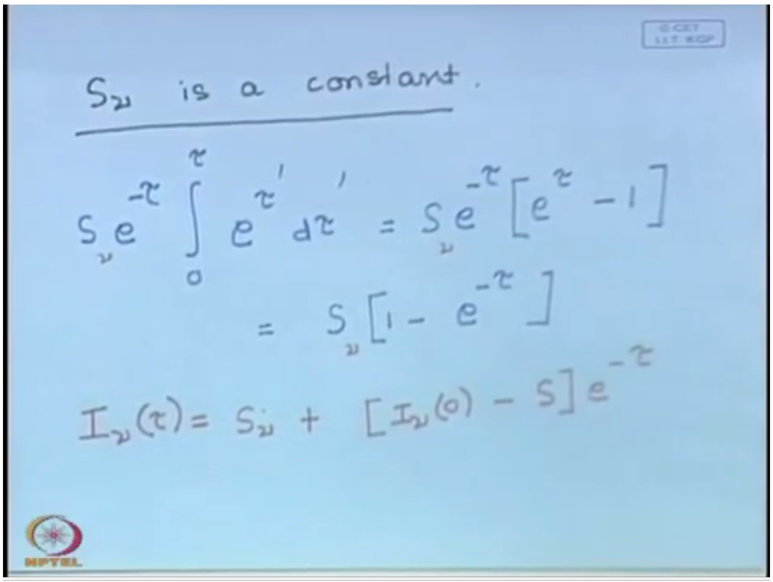

Let us now consider a situation where the S nu is the constant, it does not change with position, it is a homogenous medium. The source function is a constant. So we are going to assume that S nu the source function is the same throughout this entire region okay. If the source function is the same through this entire thing, we could then do this integral, this term will come outside and e to the power - tau will also come outside so we have an integral of e to the power tau prime d tau prime right.

We have just an integral of e to the power tau prime because this - and - sign here e to the power tau prime d tau prime and the limits of the integral are tau to 0. So if you do this integral e to the power tau prime let me write down the integral. So we have the integral the second term now becomes e to the power this S comes out, e to the power - tau comes out and we have e to the power tau prime d tau prime that is the second term.

The first term is unchanged anyway, second term becomes this and this can be written as S e to the power - tau and we have e to the power the integral of this is again e to the power tau prime so and the 2 limits we have e to the power tau - 1 okay. So this is S 1 - e to the power - tau. We have just evaluated the second term and we can now plug it back in this.

So if I plug it back in this what do I get? I get the fact the solution is that I nu the specific intensity so we will have the constant term S nu, the subscript nu is the throughout at particular frequency. So this is going to be S nu+ we have this term already here and we have this term which has to be added to it so what we get is +I nu 0-S e to the power - tau okay. So that is what happens when light propagates through a medium, which has a fixed source function. It does not change with place okay.

Let us try to understand what happens. So the light when it enters this medium its specific intensity is I nu 0, the source function is S nu inside. Now let us see what happens here as it goes deeper and deeper into this medium, the value of tau increases and as the value of tau in the limit where tau goes to infinity tau is very large, the specific intensity then becomes just the source function okay.

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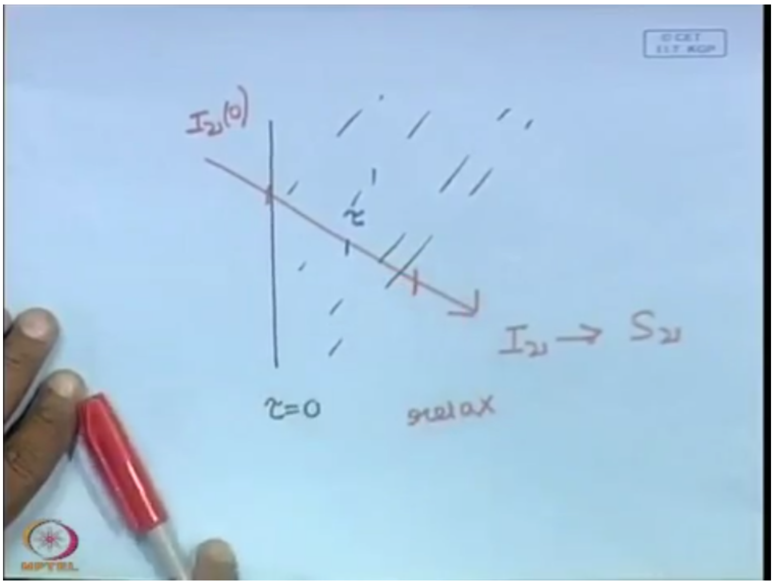

So given sufficient optical depth, the specific intensity will tend to the source function, it will relax, this is the relaxation. The value of the specific intensity will relax to the value of the source function so provided there is sufficient optical depth okay. So if there is sufficient optical depth, the specific intensity inside the medium will tend to the source function, it will be just equal to the source function whatever it comes in with.

If it comes in 0 also does not matter it will finally just become the source function inside provided it is optically thick okay.

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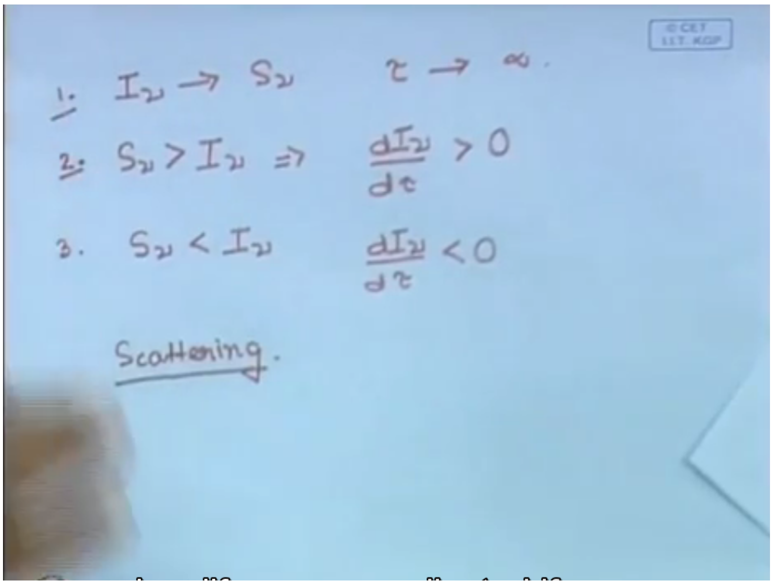

So that is the first thing so the first property is provided the optical depth is very large. The second point suppose the light enters a medium where the source function is more than the specific intensity, so the light here is entering a medium where the source function is more than the specific intensity of the ray what happens then the medium essentially pushes the specific intensity up that is what we see here.

It pushes the specific intensity up so this implies that the specific intensity will go up inside that medium. Similarly, if the source function is less than the specific intensity, it will bring it down and given enough optical depth it will bring it to exactly the same value as the source function okay. So the source function rises the specific intensity up if it is more than the specific intensity, it brings it down if it is less and given sufficient adequate optical depth it will bring it to the same value, it will relax to the same value as the specific intensity okay.

So this is a rough picture of what happens when radiation propagates through some medium where you have both absorption and emission okay. So picture is quite simple, but its life is not so simple okay. There is a process called scattering, which we have not discussed. We have not discussed any specific process as yet, but scattering makes life very complicated if you are dealing with radiative transfer.

If you have scattering, the light that is incident gets spread into different directions so the emission coefficient or the source function is essentially related to the incident specific intensity. I cannot specify it a prior right because the source of the light is the incident light itself in scattering. In scattering light comes in and it gets scattered into different directions okay. Example, light is propagating through some let say glass, which has rough surface okay glaze glass.

So the light that is incident itself gets scattered in other direction so that is again a source for the light in some other direction so the source itself is dependent on the specific intensity that is incident and this makes the problem extremely complicated. The formal solution is just a solution, but you do not know then what this S nu is. It is dependent on the I nu okay so this is of not much use there okay.

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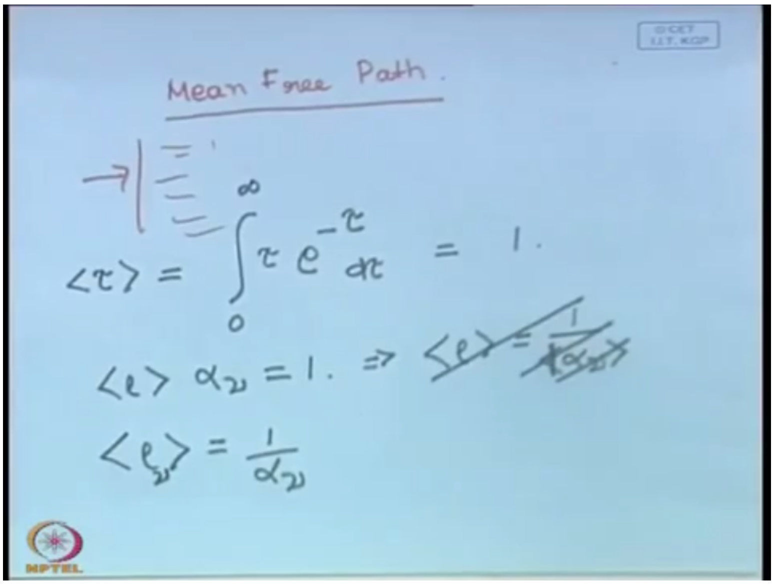

So having discussed this let us now ask the question what is the mean free path of radiation inside a medium. So this is a medium, the radiation is propagating, this is radiation incident on a medium again the same question okay I have a picture over here already. So there is radiation incident on a medium. We know the idea of a mean free path on the average how much distance does a photon propagate before it gets absorbed that is the mean free path. How much distance does a photon propagate before it gets absorbed?

Here we are talking in terms of distances not in terms of optical depth okay. So the question is how much distance does a photon propagate inside the medium before it gets absorbed a typical photon on the average okay. So we want to calculate this. Now we have seen that the probability of a photon getting absorbed after an optical depth tau is e to the power - tau. The probability that a photon gets absorbed at an optical depth tau is e to the power - tau.

We know this because the specific intensity falls as e to the power - tau so that many photons the fraction of the photons that have been absorbed is e to the power - tau okay. So that is the probability that a photon is absorbed at e to the power - tau at this optical depth tau. So let us ask the question what is the typical optical depth of a photon inside a medium before it gets absorbed.

So that is easy to calculate we have to do this integral tau e to the power - tau you want to calculate the average optical depth of a photon where it gets absorbed and we have to integrate this from 0 to infinity okay. Considering an infinite medium where every photon will get absorbed we want to calculate the average optical depth at which a photon gets absorbed.

So this is the mean optical depth at which a photon gets absorbed and this we see = 1. If we just do this integral it comes out to be 1 right d tau. We have to integrate over all possible optical depths and this integral comes out to be 1. Now the main optical depth we can write as a mean free path, the mean value of L the distance it propagates into the absorption coefficient so this is 1 that is the definition of optical length.

The optical depth is the distance into the absorption coefficient and the mean optical depth is the mean distance*the absorption coefficient. We are assuming that this is a constant throughout. So here we see that the mean free path is essentially the inverse of the absorption coefficient okay. A point I forgot to mention, the optical depth is dimensionless. The absorption coefficient has dimension of 1 by the length okay.

That you can check from the definition and the mean free path is 1/the absorption coefficient that is the mean free path of a photon in a medium and if the absorption coefficient there is no mean over here it is just the mean free path is at a particular frequency nu is 1/the absorption coefficient okay. Finally, let me discuss one more thing very briefly.

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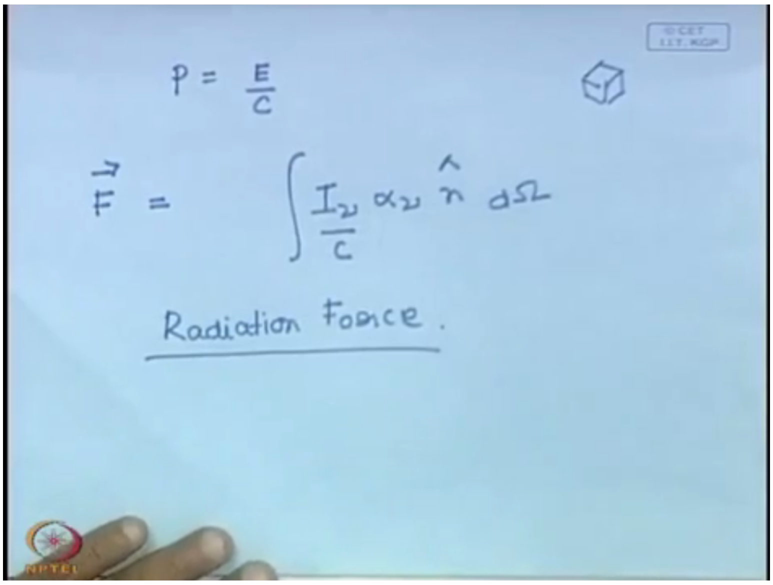

The question is when a radiation propagates through a medium so it transfers energy to the medium. Now if it transfers energy we know that a photon also has momentum and the energy and momentum are related as P = e/C. So we ask the question what is the force exerted by the radiation to the medium okay force per unit volume, so that force in a medium, when a radiation propagates through a medium not all of it gets absorbed in 1-unit volume.

The part that gets absorbed is essentially I nu * absorption coefficient alpha nu per unit volume that is what gets absorbed and this has a direction of propagation n that is the direction of the force. So if you want to calculate the total force on a volume element dV, we have to integrate this divided by C over all solid angles. This is the radiation force on some medium.

So when a medium absorbs radiation it also absorbs momentum and you have a force therefore you can have a net force, the radiation is coming from one direction you will have force in that direction being transfer acted on the medium because of the incident radiation getting absorbed okay and this is the force. It is the incident radiation specific intensity into the absorption coefficient divided by C.

Because that is the relation between the momentum and the energy integrated over all solid angle and n is the direction we have to consider rays in all directions okay. So let me end today's class over here, tomorrow we shall continue our discussion of radiation little further.