Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Week - 11

Lecture – 64

Okay, so KPZ is a Kardar-Parisi-Zhang equation. In fact, this is the famous paper which is one of the highest cited papers, so 1986. And this paper proposed a model for surface growth. So, surface is growing and so surface has a height, you know, the surface has a height. So, the height of the surface is H, you drop particles and surface is growing, right. And this height is given by this equation.

Now it has a diffusion term, it has a non-linear term, it has a noise term and noise is delta correlated, okay. So, that noise which I have written here. So, delta correlated in space and time. Now we make some transformations.

So, x equal to bx prime. So, in fact, I write down the equation for h prime, p prime and so on. And so then h equal to h prime times b to the power alpha and so this is done there and t is here, t is t prime times b to the power z, okay. So, these are three parameters, alpha z and D is the dimensionality of the space, okay. So, it is 2D surface and it is 2.

If it is 1D surface, so if it is growing on a in this room then it is D equal to 2. If it is growing on a line then it is equal to 1, okay. So, that is a D. And other part I have done, so this part I will not repeat. So, how to derive this noise? So, in fact, we do not need a new relation for noise transformation, eta goes to eta prime times some stuff which is derived here, b to the power this stuff, okay.

Now of course, we substitute it here, this stuff. So, in fact, we write down equation for h prime and you get h. We rewrite this thing is in terms of h prime, D prime equal to and then replace h prime by h, okay. So, that is what we do. No, so I think, so we basically put this one, this is the x prime equal to bx, okay.

So, that is what is being done there. So, this has been done in the previous slide which you can refer to, okay. So, h is, so h will be b to the power alpha. So, h equal to so this is correct h prime. So, I am going to replace h prime by h to the power alpha, h prime equal to h times b to the power alpha, right.

So, h prime, so this h prime is replaced by h b to the power alpha and t will be, so this t prime is there, so t prime is t times b to the power z. So, this will be replaced by t prime is b to the power z. So, the b to the power z goes up. So, this is one term. So, now we look at the viscous term, this will look like that, okay.

This is simple algebra. In fact, you can easily see that Laplacian will give you b minus 2 and h will give you b to the power alpha, okay and grad square h. This one h will give you 2 alpha, there is h squared and grad will give you b to the power minus 1, so that comes. So, I need to put this grad h squared, okay. And this one we have done it for, called Edward Wilkinson equation and that gives you that, okay.

So, it derived from this, this one which I did it in the class, in earlier class, okay. Now, so here, so this equation looks similar to the old, this equation except the pre-factors have come. This pre-factor has come, I took it to the left and rearranged and this pre-factor has come and this pre-factor has come. So, you want the old equation, then this pre-factor must be equal to 1. So, from here, I can see that, so the exponent of b should be 0, right.

So, that gives you alpha plus z equal to 2 from here. This one gives you alpha plus z equal to 2, okay. Now, what does this give you? b to the power z minus 2 is 1, so z minus 2 is 0, so that should give you z equal to 2. That means alpha equal to 0, right. I mean, combine these two relation, then alpha is 0 which is disaster.

Because then surface roughness which is captured by alpha is, there is no surface roughness. In fact, H is not changing with time. H is as smooth as before, so okay. So, surface roughness is non-zero, we know it from experiments. So, this is giving us a nonsense result.

Why is it giving nonsense result? Because I have assumed that nu is constant, okay. But nu is not constant, nu is function of b. Nu, nu prime equal to nu b to the power some, some, like we did for viscosity renormalization. In fact, this guy should be renormalized. Nu should be renormalized and we will get some additional b dependence and that will give us a correct result, okay.

So, we need to renormalize nu. I can also tell you that lambda also should be renormalized, this coefficient is a parameter. In fact, it is coupling constant, but it turns out because of Galenian invariance, lambda is not renormalized, lambda remains same. And eta also should get renormalized, this forcing, okay, forcing, different scale may look different, okay.

And we will see. So, what we find by RG calculation, which I am going to show you, that eta and nu are renormalized, but not lambda, lambda is not renormalized, okay. So, this next RG procedure follows. Thank you.