Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

Prof. Mahendra K. Verma

Department of Physics

Indian Institute of Technology, Kanpur

Week - 11

Lecture – 62

So, now let us look at a simple equation before KPZ. So, it is a linear equation. So, in fact a diffusion equation which is forced with noise, and noise again we will assume it to be delta correlated, ok. So, the noise is delta correlated like this. Now, we can derive without we can solve this equation exactly, ok. Exactly means is function of noise I mean this parameter d.

So, in earlier slide I had used gamma, but d is a strength of the noise, is a measure of strength of noise, it has dimension of eta square, ok. Well, there is a delta function as well, but so it is basically proportional to eta square, I cannot say equal to, but it is proportional to eta square, ok. So, we can derive certain interesting property. In fact, you can get alpha and beta and Z by dimensional analysis, ok.

So, this is a nice way to do it, I am going to show you, ok. This is done by this whole study is done in 90s, 30 years back or 40 years back. So, let us do the scaling, ok. So, this is what I want to do scaling. So, I am going to scale these variables x goes to x prime.

So, scale means I change the system size, right, the way we did for fractals. So, x prime is B of x. I write t as t prime, t goes to t prime which is t times B to the power Z, ok. So, Z is a scaling with length, right, of time, same Z which I discussed in the previous slide and third is height, height goes to H prime. So, I have two systems where I have one is scaled, H prime equal to H to the H times B to the power alpha, ok.

So, I am just going to make this change a variable, scale variable. So, we rewrite this equation H equation in terms of H prime. So, let me just rewrite this for H prime. So, I should get the same equation. If I, so my equation should not depend on what is the box size, right, how I scaled it.

So, it should be the same equation. So, dH prime by dt prime equal to nu Laplacian H prime. Let us assume nu to be unchanged, nu can change, ok, but let us assume it to be unchanged plus eta prime, eta prime in a scale variable, ok. So, let us look at what how

how will be H prime I am going to replace by H times b to the power alpha. So, this is dH by dt.

So, I will get b to the power alpha and t prime is t times b to the Z, but it is below in the denominator. So, it becomes b minus Z, correct, it is just straight forward. What happens to the next one? Nu Laplacian is 1 by length. So, it is going to be b to the power minus 2, H is alpha. So, this becomes Laplacian H plus eta, eta is with involved.

So, I am going to use scale variable for eta. So, eta prime x prime t. So, x prime is remember scaled. So, I am going to write this as d x, x prime know is a scale variable b and this for b to the alpha, b to the Z t, ok. And next is eta prime b x prime x bar is I am replacing x prime b to the power Z t prime that.

So, 2 d, ok, d also we will assume it to be, ok. So, let us see how it scales. So, delta will be, so it has dimension of 1 by length. So, by the way this is going to be d del d bx prime minus b bx minus bx prime, right. I replace x by bx and delta bz t minus bz t prime.

So, what does it look like 2 d, now this delta function also will give you some dimension, right. So, what is the delta bx in terms of delta x? Exactly, very good. So, delta x by B, thank you, right. So, this for 1 d, but in d dimension is going to be B to the power minus d and for this I will get minus z, right. Minus z will come below and delta x minus x prime delta t minus t prime, x minus x bar, t minus t bar.

So, prime, so this x bar is a new position in the same length, but prime is a my scale variable. So, my D, so these my new D's, ok. So, eta prime, so this is, so my eta prime variable eta prime squared, ok. Magnitude of eta prime, how does it change with scale? So, eta prime this is the coefficient, no. So, Db to the power minus z minus minus d minus z will go like that, correct.

I mean just comparing that this term. Earlier system in terms of x it was 2 2 d, but now I got a new factor. So, eta prime will be square root of this, correct, yes or no? So, I get a pre-factor B to the power minus d by 2 minus z by 2 times eta, ok. So, this is my new equation. So, now let us get rid of this term in the left.

So, dh by dt equal to nu, I, so this alpha alpha will cancel both sides or the second first term in the right and I get z minus 2 Laplacian h plus b. So, this minus z and minus z by 2 in the thing, so I will become minus d by 2 plus z by 2 minus alpha eta, ok. Now, this equation looks very similar to prime, well this equation except this B is there, ok. So, I do not want my equation to should not depend on b, correct. So, how can it not depend on b? It cannot depend on b if z equal to 2, right from here.

So, I want bz equal to 2 and here this also becomes 0, should the exponent should be 0. So, minus d by 2 plus z by 2 minus alpha equal to 0, but z is 2 already, right. I know that z is 2 from here. So, minus d by 2 plus 1 minus alpha is 0, alpha equal to 1 minus d by 2, correct. I can derive it always without doing any algebra or without doing any analytical calculation of this equation.

So, for d equal to 1, well this is our formula, ok. So, z equal to 2 and alpha is 1 minus d by 2, ok. So, I am going to do then go to the next slide, rewrite that, alpha equal to 1 minus d by 2 and z is 2. But look for d equal to 1 what do I get for alpha? Alpha is 1 minus half is half, z is unchanged. Alpha equal to 2, what do I get? Alpha is 0, d equal to 3, alpha becomes minus half and minus half looks odd, no, the roughness cannot decrease with time.

So, alpha is roughness, right. So, width was L to the power alpha. So, if width if I increase the system size, my reference should increase. So, if it is negative, then it will decrease with L. If I take large enough L, then my roughness will become constant, it will not change with L, it will be flat, which is wrong.

So, this equation is not working well for d equal to 2 and 3, ok. Well, 2 is ok possibly. So, 0 means it could be log, it could be log of L is possible for width. Log of L has 0 dependence no, when this is standard thing, but this is does not look good, but d equal to 1 half is ok. So, it turns out this system is good for some 1D systems this equation, it is called Edward Wilkinson equation.

So, alpha is half is ok and I can write z is equal to alpha by beta, which is half by 2 is one quarter, ok. So, Edward Wilkinson equation is good for some systems. In fact, it is very nice easy to do this analysis. But please keep in mind the nu was not renormalized, there is no non-linear term, so we do not get nu to change and d is also does not change, d is a number remains number, ok. So, these are not normalized equation, these are basically free equation and get ok, we get some reasonable number and it has exact solution, which you can look at the book, we can solve it by in for Fourier transform and you can see that here T is L to the power squared no, x prime is like length.

So, z equal to 2 is seen from here and if I put d as 1, then h squared as goes as L right, x 1 is x prime is like length. So, if I take square root I get half right, h is L to the power half. So, exact equation is basically telling you the answer. So, you can get this answer alpha and beta by two methods either by solving the equation exactly or by doing the scaling algorithm, ok. So, this is a lesson we learn from this exercise, ok.

So, Green's function and correlation functions are easily doable, I will just state the result. Green's function is a linear equation, so it is basically we did it before this one and correlation function we will do exactly the same way like what I described in the previous half an hour back. So, h of k omega is G naught, k omega times noise k omega right, yes or no? Agreed? So, G inverse, k omega, h, q omega is noise, q omega right. So, G inverse is minus i omega plus nu k squared, so this is the equation. So, I can do exactly same thing what I did before and that gives me correlation function C is the noise part which is 2d divided by omega squared plus nu squared k squared, ok.

So, this is exactly same as what I did before for for Martin CGR Rho's formula. So, differential equation has certain merit, we can easily derive the formulas, ok. So, I think this, so Edward Wilkinson equation is solved by this method. So, there is more activity at different different sites, so you are dropping more and more particles at different sites, so there will be more particle coming at, so there will be more variations with length. Imagine that yeah, so more activity, so basically in statistical physics we normally find that fluctuations should increase with box size, right.

So, if you have more people and more more variance in income, more variance in height, so that is a general paradigm which is assumed in statistical physics and I am talking in that same manner. Like entropy, entropy is per particle is constant, where entropy increases with the system size, so where roughness is like that thing is you cannot get entropy go to 0 with system size increasing. . Yes, so very good question. So, we need to bring in some real physics, so some non-linear term has to be introduced and that is what people did and that equation is KPZ equation, so I am going to describe that now.

So, in D equal to 2 we will get a reasonable answer, D greater than 2, we will not get negative, we will get a good answer. So, this will take bit of time, so we can exceed 20 minute I mean you can go up to 20 minutes from now, I have to reserve my energy.