Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Lecture – 61

So, I am going to start another equation which is, the idea is very similar to what we did for hydrodynamics ok. So, now we go to another phenomena ok, of course surface growth I am going to explain what is surface growth and there are very famous equation. In fact, this equation called KPZ equation, Kardar-Parisi-Zhang and that equation has huge following in statistical physics. This is more famous in statistical physics compared to Navier Stokes, the Navier Stokes they do not work with that, but they work with KPZ. So, for completeness I thought we should look at KPZ, but there are similarities. So, I am going to show you the similarity, but the approach which is used here is with the use noise and it is similar to Phi-4 theory for Wilson.

So, you will see that. So, I think you should do at least connect what is done where ok. So, Phi-4 theory will see lot of similarities and with noise ok, it is a dynamical thing. So, there are certain changes which is done here and we should see the difference ok.

So, I took it from this book Barabasi and Stanley, it is a famous book ok.

So, we will do another phenomena. So, I work basically with turbulence so far, but there is one very famous equation called KPZ equation ok. And there lot of work in this equation and the field theory for KPZ is well is a same lines, but it is not self-consistent, it is with noise and is extension of Wilson Phi-4 theory. So, if you please recall Wilson Phi-4 theory which was non equilibrium theory, no frequency, no time, but now it is extended to non equilibrium theory. And so, we will see that KPZ, I want to do today.

So, let us get there in 15-20 minutes. So, but I need to set up the framework ok. So, surface growth phenomena. So, this became very popular in 1990s ok. Though the phenomena is I think very common.

So, if you put a blotting paper in ink. So, the ink so, this is a blotting paper and ink is flowing up right. So, you can see that there is a profile may look something like that. So, and if you wait for longer time then this surface will keep moving up right. So, this is called surface growth surface growing, but this is for fun right I mean this is not really of practical use.

But we can do real stuff by making chips right. So, what how do you so, there is a deposition of silicon or deposition of material on top of surface ok. And this used I mean one surface you drop gold particles. So, there are lot of material science connected with surface growth. So, it is a very very practical problem and so, people try to make theory out of it ok.

So, there is some successful theory of course, theory does not really match exactly with the experiment, but there are certain overall picture we get from simple equations ok. So, this picture is again from this book. So, my discussion right now is basically from this book Barabasi and Stanley and I taken quite a bit of material from this book. So, so surface is growing know I mean that this blotting paper the ink is going up. So, it turns out we look at this roughness this called roughness W is called roughness and h is a height variable.

So, this is x axis and for every x I have height of the ink column right. So, this is certain profile. So, h is function of x and time ok. So, now or here it is I is in x variable. So, normally people also write in discrete variable.

So, I is a height. So, h minus average height at given time. So, take this h of x t minus h at time t height is changing with time. So, I subtract. So, this will be if I take the average I will get 0 at a given time right by definition the h of t is a average of x t, but if I take the square of this then I will get non-zero value if I average.

So, take the square and then take a square root and divide by length of the system. So, this is average rms value of height at a given time ok. So, this function of L well different system size as well and depends on t ok. So, this is called roughness ok. Now, this is plotted for a generic reserve these are theoretical plot, but we find that in experiments too.

So, this roughness is increasing with time is a power law is a time is x axis. So, this in fact, goes as t to the power beta ok, beta is exponent and that depends on how you model it and then it finally, becomes flat. So, rough whether the height is increasing, but finally, the reference does not change with the with time ok. So, it will fluctuating going up, but this roughness this object flattens ok. And this turnover I mean the change of behavior occurs at certain time t is equal to t x.

So, from this picture we can see that this in the early phase it is 2 to the power beta, beta is an exponent and this level the saturation level depends on L. So, W of L saturation is L to the power alpha, alpha is also an exponent. Now, we have two exponent alpha and beta, but turns out I can combine these two relations using same idea as Vidham scaling. I did in the class this Widom scaling I did earlier ok. So, this Widom scaling I can combine in the same manner.

So, the idea is that I write single function for roughness is L to the power alpha, but then there is a non dimensional variable t by L to the power Z and is some function of that ok. And now we got a new variable for Z and what is this Z? So, we can get this Z by the following ok. So, I write this is 2 to the power beta know the at early time or in the early phase 2 to the power beta. So, now from this relation now t and L are related ok. So, if these two variables are related.

So, I write t as L to the power Z ok. So, if I take two systems I increase the system size by factor 2, then the time will also be increasing for reaching that particular roughness, but this will not be factor 2, but it will be some function of Z. You understand what it means? So, for given L I get this ok, but if I increase L then my what happens if I double it then what how will the curve look like ok. So, then the curve may look like that ok. So, what is the factor? So, this is the relation for given time t these are the two different values for different L's ok.

So, now, I can rewrite this well I mean from this you can see I write this 2 to the power beta, but t is L to the power Z right. So, it is L to the power Z beta ok. Z beta must be equal to alpha because my alpha is exponent for L. So, we get the relation Z equal to Z beta equal to alpha well I am just using this relation to replace t by L. So, Z equal to alpha by beta ok.

So, this is another exponent is called in fact dynamical exponent ok. So, let me just write that this is very important relation how does the frequency scale with wave number? So, you see one by time will be frequency and one by L will be wave number ok. So, that goes to Z. Now, we know for hydrodynamic turbulence what is Z? So, frequency has dimension of nu k square nu k k square these are renormalized viscosity. So, what is nu k is k minus four third right I derived that k square.

So, this k two third which is k to the power Z. So, Z is two third for hydrodynamic turbulence ok. So, this Z is a very important exponent for all dynamical phenomena which connects frequency with wave number or time with L ok. Of course, this is I am deriving it for surface growth, but this relation we already derived it for hydrodynamic turbulence.