Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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So, I did flux for shell model, you can do similar thing for Navier Stokes. So, what is the idea? So, the formula is so is that wrote so this is a sphere of radius k naught. So, pi k naught is two sums. So, give her p is within the sphere p less than k naught less than equal to k greater than k naught k dot u q u p dot u star of k imaginary part of this. So, this is a formula derived by various authors. We have some interpretation of this which is interesting, but I will not it is not required here I need to compute this interval ok.

This formula we derived it ok. So, let me say this is let me take a claim that this is this form is done by first time by Dar my PhD student Eswaran and me in 2001. So, anyway so we use this formula to compute flux. Now I will not do the full algebra which is very involved, but you can see that there are three u's right.

You have u q u p and u of k they are vectors. So, I need to do this tensor stuff, but essentially we have this three u's and this is a pre factor there will be a vortex term. So, if you understand the shell model calculation properly then you can do this calculation here as well. So, to zeroth order assuming u to be Gaussian if I do the average so I want average of this. If I do average what will I get to zeroth order is Gaussian.

So, odd order should give you 0. So, zeroth order this integral is this product is 0 ok. This is equilibrium solution, but not the non equilibrium solution. So, zeroth order it gives you 0. So, I want non zero value for the flux.

So, what do I do? I expand it using Green's function. So, idea is so there are three Green's function sorry there are three Feynman diagrams. So, one of them look like that. So, this G of q t minus t prime and the two legs will come right. So, like before so Green's function and two u's.

Navier Stokes equation has this two u's in the right hand side and you get non zero values only when this is up and the u of minus p. This u of q the u of minus q and the two integrals the two sums right. So, that becomes integral. So, the two integrals here. Of course, there are two more Feynman diagrams which are like this plus like this.

So, what is this integral? So, let us write the integral ok. So, dp dk so this p sum and k sum of course, one of them is within the sphere other one is outside the sphere ok that will involve complications, but we ignore that part. So, that two correlation functions Cp Cq t minus t prime Cq t minus t prime Cp sorry Cp Green's function qt minus t prime. Integral dt prime 0 to t plug in again the same form Anzat's assumptions. So, dp dk dt prime exponentials right exponential minus nu p p squared t minus t prime.

I am just going to write them together nu p p squared t minus t prime what about this plus nu q q squared and the two correlation functions. So, Cp Cq, but they are equal time correlation for which I am going to substitute Kolmogorov. Understand this is an equal time and unequal time. So, unequal time has this exponential factor multiplied by equal time equal time is t and t equal to t equal to t prime. So, that is a spectrum know that is a spectrum and the Green's function will give you k squared Green's function this k.

So, k squared nu k ok. If I integrate with time what will I get like before dp dk. So, integral always gives you in the denominator nu p p squared plus nu q q squared plus nu k k squared and Cp Cq and there will be some pre factors ok. If I do the other sums I will get again two correlation function, but denominator is the same. So, these guys will basically here I will get p q p k and the other guy will give me Cp Ck and the pre factors are different know these pre factors are different ok.

Now, what do I do next? I substitute Kolmogorov formula for the correlation function and it turns out I will not do algebra here it turns out the right hand side is proportional to epsilon and a constant and low and behold key Kolmogorov three half by nu star ok. It is exactly same form because dimensionally we should get the same answer ok this constant will be different by stuff. So, this epsilon so that means the flux is constant and equal to epsilon multiplied by a constant. So, this will cancel, but this constant this guy must be equal to 1 know. So, I know nu star this constant will come from the integral complex integral.

So, that will give me k Kolmogorov and that is approximately 1.6 1.7 ok. So, if I do all these algebra correctly then I get this Kolmogorov constant ok. So, that is a outcome of flux calculation ok.

So, let us summarize I think it is the RG analysis gives me nu k it turns out nu k is not just for fun, this nu k is also very useful for real simulation which you call large-eddy simulation. So, we can define viscosity at different scales also for modeling diffusion in in atmosphere right. So, I need nu k for atmospheric diffusion. So, all that can be done. So, enhance viscosity a flux analysis shows the flux is constant in the initial range ok.

I showed you for field theory and you can compute Kolmogorov constant. So, this scheme I believe gives me around 1.6 something, but if I use some different field theory then I get different numbers, but they are all lying in this band ok which we find in simulations as well if you do carefully ok. These are non-equilibrium feature. Equilibrium feature exactly like what I argued for shell model we can show that this correlations will be 0 for the flux.

So, the energy flux will be 0 and other well for the viscosity is pretty somewhat complicated, but we can also show that nu k is not renormalized for Euler equation ok. The proof is bit complicated, but it can be shown. So, we basically field theory is telling us same story what we expect. So, this is a non-equilibrium field theory for turbulence ok. So, let us stop.