Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Lecture – 47

We need to get this field interact with the electromagnetism that is my goal for today's climax the Higgs will be connecting with the electromagnetic field. So, we need to first write down the electrodynamics in terms of proper variables you write with E and B right I mean that normally that is what we write electric field and magnetic field, but that is not what is common in quantum field theory or in field theory. Field theory what is more important is 4 potential. So, we are doing right now relativistic so, this is so, 4 potential is you may know phi and vector potential the 4 components first is the scalar potential and next is A ok. So, I have no time and no energy right now to tell lot about electrodynamics, but this is called covariant formalism ok. I am not sure whether you done in your course, but we will just look at some of the forms which appears in field theory.

So, first A mu is a 4 potential the first component is phi and the third other three are like d t and d x know. So, they are 4 vectors energy and momentum the 4 vectors then we define a the tensor called F mu nu you might have heard of this F mu nu which is del mu A nu minus del nu A mu these are partial derivatives ok. And how many components this F mu nu has 16 components right nu goes from 0 to 3 nu goes from 0 to 3 now if you work them out it is not difficult. So, we get this matrix of this form.

So, we have 3 electric field well electric field here is a B sitting here. So, this is the 16 component. So, F mu nu has electric and magnetic field is a component is a tensor ok is a anti symmetric tensor and it turns out you will see very soon the coupling is I am going to show you the coupling what is coupling interaction between. So, we have free electromagnetic field we have free matter field like which is phi now how to couple them right you need interaction and that is what we are after right electron and photons how do they interact ok. So, I will write down the formula for that.

So, this is a Lagrangian for free electromagnetic field B square minus E square and the Maxwell equation is derived like that. So, take the derivative of F 4 tensor and then I will get mu naught J. So, this has it covers 2 Maxwell equation del dot E is 4 pi rho and curl of B is 4 pi J by C plus or minus C square ok. I hope my sign is right ok this equation these 2 equation what we write is covered here. Other 2 equations which are constraints in fact del dot B is 0 and curl of E is minus dB by dt.

So, this should be plus I think right this is plus or minus well let us keep it. So, this equation this should be plus actually this should be plus other way equation will be problem. So, curl of so these 2 are constraint and they are certain equations for Bianchi identities ok. So, these are all can be derived in terms of F mu and it is bit of algebra, but it is a very nice algebra setting point for covariant electrodynamics. Lagrangian that is what we need now.

So, Lagrangian coming from the electromagnetic side. So, these are Lagrangian I told you about this is a free Lagrangian for the for the photons ok which is E square minus B square right. E square plus B square is total energy Hamiltonian, but E square minus B square is a Lagrangian and this is the interaction. Now, are you happy with this J mu m u? A mu is a electric field and J is a current ok and it looks fine because for charge particle Q phi is a potential energy you know charge multiplied by potential. So, this is J 0.

So, J mu is my current and what is the component J mu? Charge density and current right that is a thing and A mu is I wrote before phi and A. So, take a dot product of these then that is what we get interaction which is we already know ok. So, J dot A and rho times phi with the minus sign will come ok. Now, please note that this Lagrangian has no term of the form M square A square right. This one is del mu A mu minus del mu A mu same thing with a superscript.

So, where is the term like M square A square? None right. So, that means the free particle which is photon has 0 mass ok. So, this is by construction of course. So, photon has a 0 mass and one more constraint which is comes in electro dynamics is for given wave vector k in Fourier space I could have 3 components for A right. If I do Fourier transform of this vector A then I can have 3 components, but it turns out be only this component the transfer the longitudinal component is 0 ok.

So, longitudinal component is 0. So, we have 2 transfers component and these are the polarizations right circularly left circularly polarized or it is polarization in this way or that way. We do not have polarization of photon along the k vector ok. It follows from mathematics ok. So, photon has 2 independent particles one this way this way another one this way, but not along that this is not along and action is this which is I just integrate Lagrangian with d 4 x ok.

So, this is connected with electromagnetism ok. Now, so, Gauges now Gauge we all seen in electro dynamics. So, this vector potential A or phi they are not unique what is unique is the electric field and magnetic field right ok which is B is curl of A and E is gradient of phi minus grad phi. So, in phi I can always add a constant. So, in phi in coherent formulism is d by dt 1 by c ok.

So, we can always add something. So, that my B field remains unchanged in A I can always add grad gradient of a scalar. So, because curl of gradient of a scalar is 0 ok. So, this so, since my interaction so, please note that what participate in the interaction was J mu A mu there was no J can be with electric field or magnetic field. So, in quantum mechanics you may know that what we work with is A not E or B right.

So, the Aharonov-Bohm effect right. So, that A is important quantity. So, there are some subtle things. So, A is a the central object ok. So, now let us look at the Lagrangian and how to write the Lagrangian which contains both matter field and the electromagnetic fields and interaction.

So, it is written in a called minimal form. So, this is a minimal form. So, this is part is we already saw for the free photon field. Now, I am doing only phi-4 theory ok. Right now I am not doing QED direct.

So, I am doing only phi-4. So, this part you seen it already for the scalar field. Now, in this some of it is you may recognize right del mu psi dagger del mu psi this is what I wrote before know. So, what changes did I bring I have added minus i q A mu in the left and plus i q A mu in the right. So, my derivatives are replaced by derivative plus or minus i q A mu ok.

q is a charge of the of the matter field ok and m is a mu is a mass of the matter field. Now, is this object what is symmetry of this object this Lagrangian it is an amazing symmetry ok. So, remember my psi I have to make e to the power alpha alpha was constant, but now I can make alpha as a variable ok. But if I just make variable then there was a problem know there was some subtraction had to be cancelled out. So, that I can absorb in A.

So, the transformation this is very nice that psi going to psi to alpha and A going to A mu minus gradient of alpha. So, if I do this then my Lagrangian is unchanged ok. Let us we can do it easily. So, let us look at this part. So, del mu will be what will be del mu del mu psi let us do del mu psi.

So, del mu psi will be del mu psi well I have basically psi prime called it this psi prime psi e to the power i alpha then take derivative of alpha psi del mu alpha i e to the power i alpha ok. What about i q A mu? So, i q, but remember A mu has to be also transformed A mu has to be changed you cannot keep the same old A mu. So, what do I do? Old A mu minus 1 by q del mu alpha alpha x ok. So, and e to the power ok. So, I am multiplying this e to the power i alpha will cancel with the e to the power minus i alpha in the left.

So, this part I can get it off right. I mean because when I do the product e to the power minus i alpha will go away. So, not plus add I am making mistake not multiply plus. So, what happens to this term psi grad alpha and this q will cancel with this is a minus psi this exactly cancel that got it. So, we get back the original gradient operator the derivative operator with multiply by e to the i alpha, but if I do to the left guy then I will get a same derivative operator, but e to the power minus i alpha out there.

So, e to the power i alpha and e to the power minus alpha will cancel and we get the same Lagrangian for this part. So, these are very nice interaction term which preserves the symmetry. So, this is my Gauge symmetry you know this is in classical physics we know that this is important Gauge symmetry right. A can be replaced by adding a gradient of a of a scalar and psi this is also very important symmetry. So, my Schrodinger equation or this wave function psi I can I would like to it is independent of the phase, but of course if I make it arbitrary then I have to do something to A it is not only one or the other.

So, this is called gauge interaction because the interaction is determined by the gauge field. Now, this is for electrodynamics people have done it for electroweak this electroweak theory and QCD. So, this is the cornerstone. So, this is called gauge interaction that is like very big thing in particle physics even in condensed matter physics superconductivity we want this gauge invariance. So, does not change or also it does not change E and B right because this thing will not change E and B.

So, we have same electric field magnetic field same Lagrangian, but this is a transmission. So, now we are all set the interaction term is J mu A mu. So, how do I see that J mu A mu coming here? So, let me look at one term interaction term what is this interaction term. So, del del mu psi dagger and multiply with this i q A mu psi. So, this is the term know I have take this one this derivative multiply that this is one term is A mu A mu mod psi square.

So, here so, what is this object psi del mu psi dagger multiplied by A mu what is this term this is the current know well I have to add the complex conjugate psi grad. So, what is the current in quantum mechanics psi grad psi star minus psi star grad psi well this is i h bar by i or something. So, this is part of the current function. So, current is multiplied by A mu and the coupling constant is Q is that clear.

So, this is the current J mu A mu. So, this is the Gauge interaction. Now, we are going to look at the Higgs after this. Thank you.