Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Alright, so we were doing statistical physics of phase transition. In the background I discussed systems, in fact spin systems where phase transition occurs, but what happens during the phase transition, that is what we are after. During the phase transition it turns out some parameters change with scale. We will see that like mass in field theory changes the scale. So parameter of the field theory will, if you look at one scale it is a number, but when you go to larger scale then the number changes, that is called renormalization. So renormalize means how the parameter is changing with scale.

Understand, you know scale is, so idea, well I discussed about fractal, but if you look at human body you know, so human body I see, you see this scale, but you go inside then you see your heart or intestine you know. So in fact your parts are changing, but just think of homogeneous system like gas in this room. At a larger scale we have temperature, but you go down you get the same temperature, right, as long as you do not go to microscopic scale, atomic scale. So these are, these how you should see, how I am going to see the system changing with scale, okay.

So I did some of it, but let me recap this description of renormalization group. So renormalization means how does the parameter change with scale. Renormalize how it is renormalizing, adjusting, that is what is parameter changing. So this field renormalization is used in lots of fields, particle physics, condensed matter physics, statistical physics. So these are the names in statistical physics, okay.

So this is a picture of Wilson who won the Nobel Prize for this work, but renormalization is became famous by the work in quantum elute dynamics. And the mass of electron renormalizes, right. I mean I did discuss some of it is a very crude way in earlier lecture. So we will do that, I am going to show you how mass changes, mass of electron changes with scale, okay. But let us, so this is a, I mean what I am going to describe is most, clearest explanation of why parameter change, alright.

So we start with spin system. So we have spins. I did this in previous lecture, but I am repeating it because it is long time before we did that. So we have spins at every site, is a square lattice here. It could be cubic lattice too or it could be 1D lattice, okay.

Depending on dimension we will have neighbors. Now so we apply mean magnetic field H, external field. So Hamiltonian or energy is minus beta H, minus sigma dot B, right. So this H times spin, total spin that is energy, right, external field. So this is due to external field and this due to interaction among spins.

And this coupling is, in most systems it is nearest neighbor coupling. So this spin here is interacting with neighboring spin, the four of them at the faces. We do not think of diagonals or the corners. So this spin X is interacting with four spins, these spins. And if you put in this system, this is called Ising spin, this shows very interesting behavior called phase transition, okay.

So at high temperatures it is in a disordered phase, T greater than T critical, but at T less than Tc it is in ordered phase. It could be either up spin or down spin, okay. These are phase transition. So this system has been studied by lots of people, very very smart people. So it turns out this system which I just described has been solved in 1D and 2D.

But it turns out that at the large scales, you know, so that was atomic scale, basically spin scale. But if you look at coarse-grained scale, so we have spins here, spins here, spins here. But if I look at the big scales, big spins, big spins, big spins here, then we can make a continuum. So here if I average over many many spins, I would not be get discrete numbers, right. 50 spins positive, 30 spins negative.

It will become continuum and so this is the field which I described phi of X. This is sum of all the spins in neighborhood of X, okay, this continuum. The way we talk about fluid velocity because this velocity is sum of velocities of all the molecules in a region and it is continuous variable. So phi of X is treated as continuous variable and it turns out all the properties of phase transitions, well, lots of properties, I can say all, lots of critical properties of phase transition can be described by this free energy, okay. So this is Wilson's idea, well, many people but Wilson kind of I think is given credit for this, leading credit for it, okay.

So he says the property of phase transitions we can understand by solving this, basically using this free energy formula. Interestingly the phase transition properties of Ising spin is very similar to phase transition properties of liquid to vapor transition, okay. Because both of them can be described by the same equation at large scales. At small scale of course they are very different system but the phase transition property depends on this equation, okay. So this is, that is why it is called universal theory.

Whatever details at small scale do not quite matter, at large scale this equation is works for theory of phase transition, okay. So analogies of course this R0 is like M squared. So we have done this equation before, phi 4, this we call it phi 4 theory. This H phi wasn't there but this is the term coming from the magnetic field, okay, external magnetic field. So this is called phi 4 theory.

And so this is grad phi squared and c0 is a constant, u0 is a constant, m squared is a constant. But I am going to use r0, okay and I will follow this book Kopietz notation now, okay. So he showed that small scale do not matter and we can get a universal theory of phase transition. I am going to explain that in like today and I think we need one more lecture then that will become clear what is a theory of phase transition of Wilson, okay.

So RG procedure, so this one well it is a basically it involves two steps, one thing is called coarse graining, other one is called rescaling, okay. So we have lot of spins, so at every point there will be up or down spin. Now we make bigger boxes, right. So each of the box has 16 spins, right. So this box has 16 spins.

So earlier there were small boxes, right, this is tiny boxes, each box had one spin. Now we have box with 16 spins in each box, okay. So you sum up the spins for the 16 spins, I mean you make an average and give one spin number for that. You can add them up, so let us say 8 plus and 8 negative then we will get 0 or 6 up, 9 down, well 6 down, 6, 9, okay, 15, so 15 minus 9, 9 down we will get minus 3. So you get a number, so you put numbers here, these numbers.

Remember earlier it, so this is what is put in here, okay, this is put the numbers. Of course the spins are interacting with each other, so the interaction between the spins, so the four interaction, this if you focus on this spin, the interaction four spins here and they are all Js, microscopic interaction. But when I coarse-grained it, what is the interaction between this spin with these neighbors? Will it be J or will it be different? It will be different number. So you can say that this J prime, but this is at level 1, so you can call it J1, okay. So this is after doing the coarse graining.

Now once you have done coarse graining, this the here the grids, well right now, well I have not, I should not go there actually, I should not write it here, there are two steps. Just focus on this box, interaction between these, I should have made interaction between this one, this and the four of them, but the distance is large, right. So just focus on the left one, do not go to the right. Now the distance is more, right, for the neighbors the infinite is 4, 4 units. For the Xs here, the neighbors are 4 units apart, but of course the spin is bigger number, okay.

Now, we do the next step is called rescaling. So I want the same 1 unit distance between two nearest neighbors. So scale by factor 4, so this B will be 4. So this 4 unit distance will become 1 unit distance here, this is 1 unit, 1 unit, 1 unit, okay. So now we look at what happens to coupling constant J.

Is it J or something different? We can call it at this level J1. So it has two steps, one is coarse graining, other one is rescaling. So J goes to J1. So this is called renormalization. Next step what will I do? I make the box of 4 again, this box, another box like this, okay.

And again you make a average and get some numbers, then again do the rescaling. So from J1 it goes to J2, okay. So we keep doing this operation and this operation is called renormalization. So operation is renormalization but another operation, the parameter J is changing to J1, J1 to J2 and J2 to J3 at every level, okay. So whole thing is, this is the process of renormalization.

So this is under scaling. So symmetry we are exploring, studying, study of symmetry under scaling. Scaling means going to small scales right now here. Now you have understood, you have thought of symmetry about rotation, space translation but these are different symmetry. Your coarse graining, well you are going to different scales, okay. So okay this operation, so symmetry is normally studied using group theory.

This I am defining the symmetry operation. So what is the rotation group? So if I take this system and I rotate by any angle, no? So if object is symmetric under rotation like this, this object looks symmetric under azimuthal rotation like that. So any angle rotation, rotation by any angle is an operation, okay. So we have rotation theta, this operation. And if the object remains unchanged during this operation is called, this object is symmetric under rotation, understood? And R theta is a group member.

You also know that R theta 1, R theta 2 equal to R theta 1 plus theta 2, right. So this call, if I multiply I get rotation by a bigger angle, theta 1 to theta 2, okay. So these are group properties. So for rotation or for group these are the, well I think before I go to that let me just give another example, before I go to groups, this is another example, okay.

So we have individuals in our country. So that is one level. We are looking at individual level, individual level, individual interaction. But I can make coarse graining, okay. So I can make bigger boxes. And I do not have individuals but I have societies or group mohalla, you know, so group of people.

And then at that level there are interactions. There are certain properties. You can say income or some properties. So I get this, these are not individuals, these are groups of people.

Now we scale it. The distance between, well the individual, these groups are, it is a bigger group. So scale it then you get this country, I will get the distance to be same as before between the interacting units, okay. So we can go from individual level to group level, then groups of groups and groups of groups, again we keep going up. So this operation, you have two operations, one is coarse graining and rescaling and but finally if the system was symmetric under this operation, India at this scale will look exactly like this scale.

It only look changed, okay. So if it is not symmetric then you see something different. So in the previous example, if J remains J, then this operation will, this system will be symmetric under this scaling operation, okay. So that is what we look for. Is it symmetric or not symmetric? So let us just state the group properties. The group, so there are four properties a group has to satisfy.

So there are elements of group, A and B are group, then A dot B is also in a group. For rotation, R theta 1 is a group member, R theta 2, these are normally matrices, you think of matrices. Then R theta 1 product will be R theta 1 plus theta 2. So if this is group member, this is group member, this is also a group member. Associative property, so we had two, three angles R theta 1, R theta 2, then you do R theta 3.

This is equal to R theta 1, well first you do R theta 3, then do the combination which is same as do R theta 2, R theta 3 together then do R theta 1. It remains unchanged, you know. So these are called associative properties. For rotation we have identity which is rotation with zero angle identity and inverse also, okay. So group should have a, for every rotation there should be any rotation so that I get back the original object.

This is symmetric but imagine a cube, okay. If I rotate by any angle I would not get the same configuration, no. I rotate by 90 degrees that is the thing. So for cube if I rotate by minus 90 then I get back the same cube. So every member should have inverse. For here, well this is symmetric object but well this is not quite symmetric.

In fact this part will destroy symmetry. So at what angle, so if I rotate right by 10 degrees then inverse of it will be rotation by minus 10 degrees. So a group must have an inverse, okay, which takes back the object to its original configuration, okay. So does this scaling operation which I described, does it, is it a group or not? So it turns out this scaling operation these three properties are satisfied, okay. I scale by factor 4, then I scale by another factor 4, I will get scaling by factor 16. So this will also satisfy this identity which I do nothing but its inverse does not exist.

Once I have coarse-grained it I cannot get back the original, okay. So inverse does not exist that is why scaling is not a group really. So we will call it a denomination group but that is incorrect name. It is really a semigroup because its identity does not, its inverse does not exist, okay. So you should really call it a denomination semigroup but this word has stuck with people so this is called renomination group, okay.

So what is the objective of renomination group? So we like to study how parameter changes the scale, okay. And it, this changing, well this property of change or how things change the scale tells us very important property of the system, okay. In fact phase transition will characterize phase transition using this property which will become clearer at the end of the lecture. So this is called renomination of parameters.

I gave the example of perfume. So this I think I did in the last class. I will not repeat it. So the diffusion of perfume at small scale is different than diffusion of perfume at larger scale, okay. So these parameters we will see that some parameters do not, well some parameters under, under the scaling operation become small and small, okay. So these are called irrelevant variables, something which is not mattering so it is called irrelevant variables.

And so, the unimportant ones go to 0. So these are irrelevant. It will become clear when I do the theory and these are called relevant. The one which becomes, which remains finite, in fact which may grow also that is called relevant. Thank you.