**Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives**

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**Lecture – 36**

One important point is I said this, I gave the example of mob behavior. People form opinion small batches and then suddenly thing looks like if the opinion is reasonable the whole student body can rise, right, which does not happen often but it could happen, right. So, this is the phase transition, whole institute or country thinks the same way. So, but it happens in a small packet. There will be some people who will not be following that trend or the feature. So, it turns out we have large blocks of opinion or large blocks of coherent behavior and but it is also lot of depend on scale.

So, that is where this object called fractal becomes important. So, what is the fractal? So, this is an example of a fractal. So, how do they construct this fractal? So, this slightly more complicated but let me make a simple fractal. It is not complicated but there are just too many cubes to describe.

Well, we can do it. Let us keep this one only. So, this is a big cube and not cube, big square. Center region is black and there are eight of them in the periphery, right. So, this, if you look at from the largest level, we have big square.

Well, in the big square we have one big black square in the center and eight in the periphery. But now let us look at the peripheral ones. This guy. So, we have eight of them. So, just think of eight of them.

Each of these eight, you look at the surrounding. What have you got? We got each guy. Let us focus on this. There are eight of them. This is around the center.

Now, you go further inside. I look at these square. You can make again eight of them. Can you see the green one? I mean it is tiny but that again has the same behavior as a big block except it is shrunk. So, these are fractal be fractal is defined.

We have multi scale structure where at each scale it looks like the parent. If we look at the parent, then it looks like its grandparent and so on. So, that is how fractal is defined. Now, this object is has very funny behavior. You look at what is the area of black regions.

I can ask what is the area of black regions. Normally area for a in 2D is like R square, L square, no? So, if this is the size L, then I would say well black region should be like L square. It turns out it is not L square. It is some in fact L to the power alpha when alpha is less than 2. That is the area of the black square.

You can compute it but I will not do it right now. This is less than 2, in fact greater than 1 but less than 2 and it is a fractional dimension. Alpha is fractional. So, that is why it is called fractal. So, dimension is fractional.

So, similar thing happens during phase transition in magnetization. So, this is a pretty ordered one in the phase transition. You do not have to expect ordered structure but you expect something like this. This could be big plus, this could be negative. Green is probably not very clearly visible.

This is big plus. All the white, bright ones are plus and dark ones are negative. So, we have big plus here, big plus here, big plus here and there are small tiny pluses here but they are equally large minuses. So, this is how during the magnetization we will have big plus chunk, a smaller plus plus chunk, a smaller plus chunk and similar numbers of minus chunks and we need to describe them. So, one important part to remember this is not a course on fractals but if we find in field theory occurring often and not only field theory but in all fields that these objects have power law behaviour.

For example, free energy if I write s function of a magnetization it will be like m to the alpha or free energies. So, we saw that it is 1 over r to the power beta, you know, not free energy, correlation function. So, you will get power laws. Of course, we will get like that but typically m will go to 0 or m will be small. So, you will get power because this is we are finding same behaviour at different scales.

I think you should know this part. If some function is same, well, has a very similar behaviour at different scales. Now, for example, I can say that mass of the inverse in different different scales, I want to look at the mass of the galaxy, then mass of the planet, then mass of super galaxy and so on. What should be the mass m is the function of r. So, I define r like that.

So, I am looking from r going from r to lambda r. So, mass in this band, is it clear what I am asking? Lambda is a big number, lambda could be 2. In astrophysics you do not consider mass flowing from earth size to earth size plus few kilometres or few thousand kilometres. Well, there is a full gap, there is nothing in between. So, you go from earth size to star size, you make a jump in length.

And a star size you do go to, well, I mean planetary system size. So, you always make a jump in all these spectral structures, you make a jump. So, it is not r plus delta r, it is r to lambda r. So, when you say mass in a band, so I mean mass from r to lambda r. So, what should be this object, this function, a function of r.

So, I say well, what is the mass at planetary scale, then what is mass at this is planetary system scale, then mass is galaxy scale. Now, this right now we do not have a scale in the problem. So, what do you expect this function to be like? If there was scale for the mass, then it would be something like exponential or sign of something of r times beta, something of that sort. But there is no scale, beta will give you the scale for length. There is no scale, so this mass is of the like r to the power alpha.

So, well in fact r higher, so r to the power alpha plus alpha. In 3D you have to have been r cube, no fully dense mass will be r cube, no mean let us imagine the full inverse is dense with same density. You expect mass to be r cube, I make any sphere of radius r is going to be r, volume is r cube. So, but it is not r cube, there are holes and the alpha number I do not know, it could be between 2 and 3. So, that is how we write this dimension for m.

So, you find similar behavior during phase transition. Magnetization will not be some sign or cos or exponential, it will be power law. So, let us see how the some forms look like when we do the detail derivation. So, the free energy, now I it is written behind, free energy looks like this. We will derive it, free energy is a t minus t c, this t is power law in t, y t is a number and so this is the power law in t, that is what I meant.

So, control parameter to the power something. So, I stop right now, when I can take questions.