**Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives**

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**Lecture – 34**

Alright, so we will do an application, where we will employ the perturbation technique, what are the perturbation methods I will use to compute some quantities of phase transition. I need to explain exactly what we are after. And phase transition by itself is a very interesting topic. So, I am going to cover very important aspect of phase transition, which won several Nobel Prizes. So, I will tell you what is the question we are after. So, phase transition everybody knows know, when liquid to vapor, solid to liquid, paramagnet to ferromagnet.

So, we see solid liquid vapor transitions. And so this called phase transition, phases three phases and their transition, we will stick only to that. So, there are many examples, I said para to ferro, water to vapor, solid to liquid. Now, you might have heard this word first order and second order.

So, this is a free energy F. So, free energy if I take the derivative with relative to temperature, then I get entropy. I may have missed a sign, but this is entropy. Entropy is the first derivative of free energy. So, if entropy is discontinuous, then we say it is first order.

So, you find that for solid to liquid, there are density is different know, solid to liquid density is different. So, that is a first order transition. But that situations where the entropy is not different for this two phases. For first order transition, we need a latent heat to compensate this change in entropy. It turns out sometimes we do not have discontinuity in the entropy or density.

So, this is a special situation. So, I do not want to get too much into this. So, the phase diagram well, so basically temperature and pressure not temperature. So, it goes like this liquid to vapor. So, I am not thinking of solid is this.

So, this line. So, liquid to vapor I need certain latent heat for typical temperature like for water we need at 100 degrees, we need some latent heat. But you keep pushing the pressure in such a way that the transition temperature is beyond 100. And at some critical temperature, you keep pushing keep pushing the critical temperature higher and higher at some pressure. So, I am increasing pressure at some pressure and temperature, there is no change in the density.

I am not sure whether you heard about this or not before. So, here if I go from liquid to vapor. So, this vapor phase and liquid phase I go like that there is no change in density. But here there is a change in density. And this point is called critical point and here since density is not changing or entropy is not changing, this is second order transition.

This important difference it turns out that. So, this for liquid and we are going to focus on this fixed point that is where several Nobel Prizes went to. So, the ferromagnet to paramagnet or para to ferro. So, if you decrease temperature. So, beyond critical temperature there is only para, mean magnetization is 0.

This is critical temperature magnetization, but below critical temperature we get the magnetization curves look like this. Here magnetization is start from 0, there is no change in magnetization well at the critical point. It increase start should increase slowly if I decrease temperature. So, this is also second order transition. So, para to ferro is second order transition.

Now I will not do this in detail how the entropy is and so on, but this is called second order transition. Because here the change in magnetization is increasing slowly from 0 to non 0 values. The way it is happens here density 0 here, but if you go lower temperature then the density difference will increase slowly here. I hope this point is clear at this point you can see my cursor. If I decrease my temperature my density difference varies slowly.

In fact, there is a formula given by Landau that magnetization is t minus T c to the power half. If I decrease more than my magnetization will be more with that is we will derive this formula this half. It turns out in experiment then I exponent was not found to be half. It was different than half and that is where problem was people did not know exactly why it is not half. So, I will so that correction to half is the what we will discuss today, but in including today plus some more lectures.

So, it is a difficult theory. So, now this is for phases, but we can apply this to other systems as well for example, mob behavior. So, mob is not individual. So, individual may act very differently and mob acts very differently. So, it turns out this is a mob is not collection of individuals mob is well mob is a collection individuals mob has its own behavior and we will find some of it happening in phase transition during the phase transition.

So, presently well we will we do not want discuss this. This is more complicated, but we will discuss second order with a generic free energy. We will assume equilibrium behavior. So, time dependence will be 0 there is no time dependence. So, it is a heat bath because I have temperature heat bath and I look at the behavior of the system.

And so we are looking at physics during phase transition. Well physics around phase transition we will we will focus on this region this region and I am going to take magnetic system as a example not liquid vapor system. It is slightly more complicated, but this one to one mapping it turns out these two systems behave exactly the same exactly not approximately exactly the same near the phase transition, but that requires mapping we focus on magnetic phase transition magnetic transition to parra to ferro that is what will be my discussion on. So, one of the leading theory of phase transition P T is short for phase transition by Landau this is a picture of Landau. So, what is the theory of Landau theory which you might have done in your statistical physics course.

So, we start with free energy. Free energy is like Hamiltonian, but it is an a heat bath. So, free energy is slightly more technical, but for this course we will not get into the technical aspects. So, free energy so, this phi is you can think of magnetization. So, this is a local magnetic magnetization.

So, we have lot of spins some imagine is ising spin up down up down like this. So, these three together gives you non zero magnetization right when down these two gives you positive magnetization. In ferromagnetic state it is totally zero, but there are fluctuations know some place it is small positive negative some place more positive. So, more negative paramagnetic it is zero, but ferromagnetic has non zero value. So, this phi is the magnetization and phi is function of both a position and time, but we are going to suppress time.

Of course, function of time, but I am going to look at equilibrium behavior. So, time is basically silent I do averages. So, what is the free energy look like? So, this postulate by Landau. Landau proposes that this is a constant part and this is a part which is phi square quadratic. Remember we had phi square m square phi square plus grad phi square plus phi 4.

So, you can see all the terms are appearing here and these are connection of statistical physics with particle physics and other fields. This is one of the famous theory called phi 4 theory please remember this this phi 4 theory. Because other than the linear part we have phi 4 term and this is the reason why phi 4 is a leading order term I will tell you in a minute. So, first term is r naught phi square r naught is Landau postulates is proportional to T minus T c, T is a temperature and T c is a critical temperature. And the coefficient in front, but I will not keep the too many coefficients already r 0 c 0 u 0 h, h is a magnetic field.

Now the second term is coming from the fluctuations. So, there is a gradient of fluctuation which also matters c naught grad phi square is a very important term is a potential energy current time kind of term. Now this is the interaction now I did not tell you the derivation, but the coupling between spins is captured by this u 4 u u naught phi 4 term. So, this is a important term which is responsible of phase transition without that we will not have phase transition is going to come in a minute. So, phi 4 is plays a critical role for phase transition and h phi is interaction sigma dot B you know.

So, h phi is a interaction of magnetization with the external field and h will also like act like a source term. So, if we need something but, h is a source term if you like h is a external magnetic field. Now why do not we have phi cube term? The reason why Landau says no phi cube term because the magnetic system magnetic energy is same for magnetic up and magnetic down. This is symmetry up down symmetry in the for ising spins. Why should a up spin system with up spin have more energy than the system in down spin? So, that is why all order terms are not included because phi cube will change sign.

So, it will decrease for minus and increase for plus. So, these are ruled out because the system has up down symmetry. So, rotation symmetry say the system must be absurd in form and that is what is happening here. So, we keep only the leading order term is phi 4. There could be term like phi 6 and we do not keep phi 6 for second order transition.

First order transition requires phi 6 term but, I think we will not get into it because that discussion itself is a reasonably complicated term. I just want to if you are interested you can look at it. You put phi 6 term then you get hysteresis and first order transition. But we are doing second order for which phi 4 is sufficient. So, Landau theory how does it proceed? So, t is this thing.

So, please remember the r naught is like m square term what we discussed before is the mass term mass squared. Effective action is the integral no ignore right now this lambda do not worry about it. It is a integral of the whole system.

So, this is the volume integral. And so we will do mean field theory. So, Landau does mean field theory. I will do more than mean field theory later but, Landau assumes mean field that means grad phi is 0. So, it is a constant field everywhere grad phi there is no variation of phi.

So, this is my potential ok. First transition says H is 0. So, when have ferromagnet it is in the absence of magnetic field. When you put in magnetic field of course, this is a net magnetization. But we are looking at magnetization in the absence of magnetic field. Is it non-zero or 0? So, we are looking for average m is it 0 or non-zero when H is 0.

So, if it is 0 then it is paramagnet if it is not 0 then it is ferromagnet. So, let us plot the free energy. So, when I plot it. So, my parameter is I change the temperature u naught you are not changing u naught is a coupling constant which will freeze it will say this some constant coupling. But I am changing r naught if r naught is positive.

So, this function of phi is a number phi is constant. So, you can take a phi is a number. So, if r naught is positive and u naught is also positive. This is always positive. If r naught is positive then an f naught is you can set it to 0, but f naught is a constant.

So, ignore f naught. So, this curve everything is positive. So, phi square plus phi 4 looks like this. For a small phi which term will dominate? Phi square will dominate. So, this should be quadratic a large phi phi 4 will dominate phi 4. Now, what if I change sin of r naught? So, you will get a at near phi equal to 0 I will get a inverted parabola because this term will be changing sign.

So, for negative r naught I will get inverted and for large phi of course, I get positive because phi 4 is positive u naught phi 4. So, it goes like that. So, it goes like that well I mean. So, this is where that and it is like this. For large phi they should be going together know because phi 4 is not changing and this symmetric curves are not very good.

It is symmetric both left and right correct I mean. Now, we can find this positions this position. How do you find this positions? Take the derivative know these are minima. So, I take the derivative. So, del f is 0 del f del phi is 0. So, that gives us r I think I have it type r phi plus you will 1 6.

So, that is this 24 actual 4 and 4 multiply you get 1 6. Some people just keep 4, but I am following a book that book I should Kopietz and that I am just following his book. So, 1 6 and so phi is a constant now please remember. At a given temperature I am having a some phi and of course, it will go to the minima. We know the free energy minima is what is chosen at a given temperature. So, the solution will be either this or these one of the two.

There will be unique solution at a given temperature. Now, what are the solutions for this cubic equation? One is phi equal to 0 in fact, this is pretty simple theory and other is this. Now, this solution is valid only when r is negative or temperature is less than T c T minus T c that is why you are choosing r as T minus T c T is less than T c. So, when temperature is below the critical temperature then I will get two solutions and these are the two solutions. So, what is this? This is positive spin and this is a negative spin. So, overall magnetization is either up or down and both of them have same magnitude because of up down symmetry.

So, these are the solutions when Landau says well for T greater than T c we will get a 0 solution which is this is paramagnetic or we get one of the solutions negative or positive this is a ferromagnetic solution and that is it. Now, how does a well of course, this is how does this point vary with temperature. So, let us look at that one. So, you see phi goes as I think I have it here.

So, phi is a magnetization is a phi is a number. So, it goes as square root T minus T c I put a mod you know the minus sign. So, it will be T c minus T well take a mod of this. So, and it is square root.

So, this beta is half. So, how does it if I make a plot. So, this is a temperature and magnetization then it is 0 for T greater than T c and then this part is like that that is what I was trying to plot and this is a parabola which is inverted parabola square root T. So, this is a Landau's theory of phase transition in a simple way, but it turns out I mean I hope this is clear. Now, these are assumed fluctuations are 0 and we get a mean value which explains this stuff. So, second order transitions are explained nicely by well not explained this is a one model. Thank you.