Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

Prof. Mahendra K. Verma

Department of Physics

Indian Institute of Technology, Kanpur

Week - 04

Lecture – 25

We are trying to now get to compute Green's function with perturbations. Now, perturbation could be, so what could be perturbation? You can easily think the free electron field, but now you put in electromagnetic field. So, electron will interact with electromagnetic field, so that is a part of it. Or we can think of electron in a magnetic field or electron in a, with some, we call some impurities in condensed matter physics or electron interacting with itself and their interaction will be become a potential. Of course, we will not do lot of it, this is not the course on detailed calculation as I said, but this how we bring in interactions. And then the effect will come out in the Green's function, Energy Eigen values corrections, so we can derive all that in that.

Alright, so let us look at some more ideas on the Green's function. Again, we will keep it simple for time being, bring in relativity. So, that is what is, I am not going to do this equation, but we will stick to Klein Gordon equation, but the idea will become explicitly clear in this discussion. So, we start with the Klein Gordon equation, which I have done in the past, but let me in fact, try to derive the Klein Gordon equation.

So, this is a Einstein equation, know, everybody knows about this one. I made C equal to 1, E square equal to p square C square plus m C square square, but so this is called E and p are 4 momentum energy and 3 component of momentum becomes 4 vector. Now, one thing you can see in fact, is was noticed by Schrodinger, Dirac and I am sure some other people that energy square root becomes plus minus square root, energy here takes positive and negative values both. Now, we are all used to positive energy, you know, I mean in classical physics energy is always positive, but in quantum mechanics energy can be negative, we have energy bound states where energy is negative. So, for free particle, this is the free particle, for free particle what is energy, negative energy state, what does it mean? And you can easily see there are problems.

I am going to show you in just couple of slides that negative energy states or negative energy particles will have negative probability, negative mod psi square and that is a problem. So, there are ways to overcome it by bringing in antiparticle. We some of it we discussed for many body problems, we say well we bring in holes, negative particles, negative energy particles are basically antiparticle. We discussed it, no, I mean that was done sometime back, but in relativistic field theory we will bring in antiparticles today. So, now we know that energy will bring in again make this operators.

In fact, this was first supposedly written by Schrodinger, this equation which I am going to Klein Gordon equation is called, but written by Schrodinger. So, energy operator is i h bar d by d t and momentum operator is i h bar grad, i h bar or minus i h bar, minus i h bar. So, substitute it here in this equation, the way we write down how do we derive Schrodinger equation? Energy is p square by 2 m plus v. So, we derive Schrodinger equation, but we do it for relativistic thing now. So, we put energy operator as d by d t partial partial d by d t and this minus i h bar grad.

Substitute it here and now I can do the derivation, but I will just leave it for you to just figure it out, this what you will get the equation. This just simple algebra and this is correct I checked. So, in fact let me just do it. So, i h bar d by d t so h bar is 1, so h bar we do not carry around. So, e square will be minus this and this one equal to square of this will become minus Laplacian plus m square, m square will remain m square, i will 2 i will give you minus.

So, when I take this to the right hand side and bring this to the left hand side I will get this. So, with m equal to 0 we get a free waves which is a photon, m equal to 0 mass of the photon is 0, but there are particles with non-zero mass pi on, there are lot of particles with non-zero mass and this is the equation for those particles. Now, this is historically I am not 100 well basically I am not historian, but this was written by Schrodinger himself and then is attributed to Klein and Gordon these two people. So, this equation for with h bar included this is the equation for a relativistic particle. This has a problem called locality problem and to overcome it Dirac wrote the Dirac equation.

So, this is second order in time right and second order in time leads to some problem which I will not discuss. So, to overcome it Dirac wrote the first order equation, Dirac equation is the first order in time, but then you have to bring in the spin matrices right spin or she had to bring in. So, that is known complication and the algebra is reasonably complicated. So, stick to this and we will try to do quantum field theory for Dirac equation for Klein Gordon equation. So, this is the equation.

So, it has all sorts of solution if you bring in negative energy E. So, I am going to try the solution e to the power i k dot r minus i E t. k is momentum h bar k is momentum. So, that is a very nice thing in relativity momentum is just h bar k and energy is h bar omega. So, omega and k omega k vector form a 4 4 vector right 4 relativistic vector as here energy and momentum also forms a 4 4 vector classically this what we do, but in quantum mechanics we bring in this and we can deal with particles.

Now, we write down waves like this in 1 d this corresponds to e to the power i k x minus omega t bracket. Omega is always positive and it comes with the negative sign right we do not want consider negative frequency energy is always positive in classical physics. We change the sign of k, k positive the wave will be going to the right right x is changing x is increasing and t is increasing both. So, that is the wave going to the right it is a positive momentum wave and if k is negative then wave will go to the left if t increases x decreases. Now, what if we have omega negative that is a problem in fact Feynman thought about this and this 2 people which I am going to give the name he thought before Feynman I forget the name.

So, they constructed ways to interpret this negative energy particles how they move in space time. And I will just show you hand waving way to show that this negative energy particles will go backward in time. We will do we will I will show you what how we can deal with negative energy state in field theory. So, this is this slide about so, omega can be both positive and negative this was nice observation. So, we keep like this omega positive and I just interpreted this, but omega negative that means so, omega negative this becomes e to the power i k dot x plus i omega t that is a argument and interpretation of it becomes a problem.

So, let us see how to interpret this. By the way you will have to deal with both particles and anti particles at the same time. So, we cannot say that I will deal with particle first or anti particle later you have to write down for all these particles and anti particles simultaneously and there must be a way to do it. Now so, let us look at the current assuming both positive and negative energy states are possible. So, this J psi no I derived this before.

So, the J 0 is a density and J vector three components are the probability currents. We did it for Noiser theorems when I was deriving it I derived it is J mu and del mu J mu is conserved we do all did all that. So, J 0 is this J 0 is supposed to be the mod psi square probability density rho. Now, I substitute this one E can be negative please remember E can be negative substitute it here what will I get I psi star will give me the I will become minus I, but d by dt will be so, psi star d by dt will give me I minus I E correct this will come here n psi that is the first term. Second term minus psi d by dt of psi star and psi star is what N e to power minus I e dot x minus I t.

So, this will be plus e right plus I psi star. So, when I take the derivative I will get plus I e. So, minus N I square will give you minus 1. So, this becomes e times mod psi square and here also I will get I square is minus 1. So, 2 mod 2 e mod psi square this is supposed to be the probability density.

So, there are particles and this is the probability density of the particle in the quantum probability density. Now, e is negative then what happens to rho? Mod psi square is positive. So, if e is negative then rho becomes negative for negative energy state and that is a problem. And that is the reason why supposedly Schrodinger abandoned this whole equation this is not these are physically possible equation. But there is a way to overcome this and there is a way to make a formalism and that is what I will come next.

So, this Stuckelberg interpretation. So, the idea is quite nice. So, electron and this is a nuclei electron is going forward and going to hit the nuclei. This is going to slow down if it is same charge in negative or depends on what is the sign of n. So, it is going towards direction.

So, imagine that this is negatively charged. So, nuclei normally positively charged no. So, it will accelerate right now it will accelerate. Positive charge means it will accelerate. Now, let us imagine that we turn this the movie I record a movie, but where the electron is moving forward, but now I turn the movie and play it backward.

I am just going to play the movie backward. So, what will happen? So, electron which is accelerating will go in the. So, it here so by the way electron is time t equal to 0 is here, t final is here. It has slowed down, but if I play the movie backward then what will happen? Electron will go backward and will accelerate. Correct? When nuclei is assume that nuclei sits there no problem it does not I have clamped it or it is too heavy it does not move.

So, how is it possible? So, well if I want to make a physical interpretation of this then I say that this particle is a positively charged and is going with opposite momentum. So, this is what is interpretation of this charged particle. So, this is a positron with positive charge and it is going with opposite momentum and its energy is also if I think of particle then it is a negative energy state, but you think of positron is a positive energy state. It is very similar to what we did for hole. If you think of hole then it is positive energy, but you think of electron there is a negative energy state.

So, to deal with positron we say that is electron which is moving backward in time. So, these how it is dealt and so the wave function for this particle is this. So, we can now deal with both positrons and electrons in interactions. So, I am going to write down in fact operators for antiparticles similar to what we did for holes and then I can do all the field theory. So, with minus with positive E this antiparticle.

Of course, you can break your head with this interpretation in fact is very interesting interpretation and very important interpretation. So, let me just give you a brief I am not sure how many of you know about this, but in classical physics parity is conserved. You know this what is parity conservation? So, if you have a some experiment where I see some well there are certain spins which are happening some particles are spinning or some helicity is there and I do the experiment, but this particles involve certain spins. So, this has a helicity spinning is always helicity if particle moves in that direction and spinning then you can compute spin dot velocity it will be a pseudo scalar and this guy must be present pseudo scalar. So, classically you may think that if I do the experiment in a mirror then velocity will flip, but spin will not flip.

And it turns out you say that classically you can perform an experiment where this helicity will be opposite but that is fully realizable. That is what one would think classically. But it turns out in quantum world parity is violated. That means this kind of quantity is non well basically you may think that some experiment will be positive some experiment will be negative if its parity is conserved, but it turns out your Hamiltonian has this kind of quantities. I will not get deeper into this, but you can just read about parity violation.

And this won the Nobel Prize for C.N. Yang and T.D. Lee and there is a person who did experiment she did not get the Nobel Prize, but unfortunately anyway this is the experiment. In terms of this parity is connected with a charge conjugation sorry charge switch and time time reversal. So, I am telling you that back going backward in time right I am and charge.

So, three things are related parity, time and charge. You can see here I was trying to go backward in time that is time reversal and charge right you can see that here. So, C.T. I talked about, but you can also think of something spinning and parity will also come.

So, it turns out these three are not conserved individually. Parity is violated time reversal is violated and charge conservation is violated, but CPT together is not violated together. So, this is a CPT conservation law. It is a discrete symmetry, but it is conserved. Anyway, so that is not our objective for this course, but it is an important thing for inter particles parity and time reversal.

Anyway, so let us look at how to do field theory for this. So, I construct this field operator phi. So, phi is a wave function or but it is an operator. Well, I should not say phi for wave function I construct an operator phi. So, this A p is a destruction operator for the particle.

This is a destruction is standard no A p is destruction and E dagger is a creation. This destruction, but B this is a creation operator for anti particle. So, A is particle and B is anti particle. Please see the sign there is a minus sign here and there is a plus sign here. And this pre factors I do not a time and I do not want to get into this.

So, this is a space part and the time part integral well this has come here it is going to be it has a connection with relativity, but we will not get to that. So, pre factor please ignore it and focus on the signs. So, P x in relativity notation this is 4 4 momentum and 4 4 time. So, is written as E t minus P dot x P x is written is like that E t energy times time minus P dot x. So, energy is coming with a minus sign right it is a minus I there.

So, this is the particle say minus I E t I like that no I wrote I p dot x minus I E t these are particles I want to say well positive frequency particles with linear momentum positive will go in x direction. So, that is why this is for particle. Here is a plus I p x it has totally opposite interpretation. This guy will come with I E t plus I E t minus P dot x it is a positive energy a negative energy. So, that is a there is a no minus sign here there is a minus sign here and there is a no minus sign here and this how we combine the two set of particles.

Now, let us dig deeper I mean it is sensible or not I am not going to do all the algebra, but I will just show you where to look for in that in the book. You can prove it that a dagger P. So, this one this is an operator. So, a time t equal to 0 we have a dagger P not a dagger a P a P operator a P and this is operator at time t. How do I get this in Heisenberg picture? I put this evolution operators and dagger operator.

Now, this is how Heisenberg picture we evolve operator by this. And you can show that this equal to minus exponential minus I E p t there is a minus sign sitting here a P. This is there in this book which 3 of t for gifted amateurs example 11.1 is bit of algebra, but essentially we multiply this by you operate on a field of N p N p N q state and then see what happens and you will find that this this pre-factor will come out. So, this is looks nice that this corresponds to a positive energy particles.

So, this looks like well I am not deriving it, but I am telling you this is a reasonable field operator for this destruction operator for particles, particle and the particle together. Now, this is for creation operator phi sorry destruction operator phi, creation operator phi dagger. So, what do I do? The pre-factor is a number do not worry about it, but now I need to do that transpose well basically dagger operator on a and I need to change the sign of this complex conjugate and transpose. So, a P become a dagger P and exponential minus I P x will become exponential I P x and b dagger P will become b P and this is become exponential with a opposite minus I P x. So, we constructed well I am just stating what is the phi operator and phi dagger operator for Klein Gordon field.