Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

Prof. Mahendra K. Verma

Department of Physics

Indian Institute of Technology, Kanpur

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Lecture – 24

We will start our discussion on Propagators. So, Propagators is like Green's function. I did tell you, you are aware of Feynman propagator know that is what in single particle quantum mechanics, but we will do propagators for field theory. So, these are very useful for constructing Green's function, then renormalizing mass, so all that. So, Green's function is the critical thing which I told before and essentially we compute the Green's function for interacting fields. So, I am going to tell you now little bit, we are going to start some discussion, but we will do real fields in after sometime, I have real interacting fields.

So far, we discussed free fields know and they were not interacting except we put some potential small potential, but we will really do fields little bit later. So, let us start looking at Green's function for interacting or more complicated Hamiltonian. It is not more complicated, but I am just slowly beginning to make things complicated. So, our Hamiltonian H has free part and the interacting part.

So, H is H naught, H naught is the free part, so that is a typical thing now and so this is similar to perturbation theory you did in quantum mechanics, but of course, this more advanced mathematics plus potential that is what I am going to write and this potential can be function of phi itself or it could be external potential know, so that is a possibility. So, we write this is the definition of Green's function. Now, delta function I can write is x bra x ket y know bracket notation. Now, this you know from your quantum mechanics of a complete basis energy basis n, I can write insert n this identity operator inside this shifted in ppt, but that is what it is. So, what is this n x? This is wave function eigen function phi n x phi n.

So, this is eigen function x and this is phi n star complex conjugate of y. So, the right hand side is nothing, but sum over n this right hand side. Now, I want to construct Green's function in terms of these eigen basis functions. In many body physics this is one of the standard ways to construct Green's function. So, function of x and y and you can see right hand side is like this.

So, let us make a guess that I can construct my Green's function in this form with n being unknowns. So, remember this function of x. So, we substitute h the g in there. When you substitute g what will happen h acting on g what will it do? It is function of x. So, you will act on phi n x right that is what it is going to do.

So, first thing it is going to do. So, let us just write down h acting on g. So, it is going to be a n it would not do anything to y and h acting on phi n will give us e n right phi n is a eigen function for h with eigen value e n right. So, this is by definition h phi equal to e n phi n. I can write in terms of this notation as well e n n clear.

So, this is the first term. Second term will give us of course, yeah. So, this is phi n x will also come. Second term will be e e is a number. So, it does nothing e n I will keep it outside phi n star y phi n x and right hand side is some phi n x phi n star y.

Now, I equate the well these are basically functions. So, I can take it to the left hand side and just get the coefficient of this function to be 0. So, the coefficient here is 1 and coefficient in the left is a n e n minus e correct. So, I am equate the two. So, that is going to give me 1.

So, that means a n is 1 by e n minus e. So, this is the coefficient which will go in there. So, great now we have the Green's function is this. So, Green's function for this Hamiltonian is this. This called Green's function based on energy.

There is no time here, but it is function of energy. And in fact, it is very useful from the Green's function you can compute the poles of the Green's function will be give you the energy Eigen values. So, this function becomes 0 at the bottom. So, these. So, it has lot of applications.

And this is for with for full Hamiltonian of course, the problem is I need to find this Eigen functions for the full Hamiltonian that can be a difficult task. Any case, so let us do some more work. So, my Hamiltonian is written as this. Now, I am going to write the Green's function. So, this I am playing some algebra with this.

So, well I am writing in terms of time dependent. So, H is an operator. So, I am writing d by dt psi is H psi I H bar. So, H bar is 1. So, the corresponding Green's function will be d by dt I dt minus H Green's function equal to 1.

I am sorry is a delta t minus t prime. If I do the Fourier transform of that well I get minus I omega minus H Green's function equal to 1. So, we I go to from t basis to omega basis then I will get 1. So, this is nothing but that. Plus I have there Green's function of two types

I did mention it one forward and one back forward in time one backward in time which I am going to discuss it today.

So, the two types of Green's function one with a where so, there are two types we will see it today in more more concrete way. So, H is H naught plus V. So, let us write that down. So, I get minus omega plus. So, I am replacing this H by H naught plus V.

Now, this part is a linear part and this is an interacting part. So, what do I do? I can rewrite that. So, you may recall the Green's function, Green's function is I wrote minus I omega plus H naught. We in fact H naught may be in matrix, but we rewrite like that. So, look here the definition.

So, I am just going to write this I d by dt or less minus I omega plus H g equal g naught H naught equal to 1. That is a definition for H naught and g naught and thus I have definition for g naught. Is it clear to everyone? This is a definition. Now, I can multiply. So, what I will do is I will just do the.

So, this is what will that be? This g naught inverse know g naught equal to 1. So, this is g naught inverse or so, this is a basically short form you write do not write 1 by a matrix, but this is inverse is a we should understand that is a inverse. So, we write this part as a g naught inverse, but I just showed you. So, we rewrite this part as that and I take this to the right hand side.

So, this is my equation. Now, I we do one more step. So, I multiply by g naught in the left, I just multiply by g naught to the whole equation then I will get that. So, this equation is I should write it k omega which I did not write it. So, these all are function of k omega except v is typically function of k only. Well, it could be time dependent as well, but let us just write this function of k.

Now, Green's function is function of Green's function itself you see that. So, unknown this one unknown is function of unknown in the right hand side. So, it is a in fact, you might seen this called integral equation if you write into the integral. So, you have to solve it perturbatively and you might have seen this Born approximation perturbation theory in quantum mechanics it starts like this. So, now let us try to do this perturbatively.

So, in the first approximation if v is 0 then what do I get for g plus first approximation or 0th. So, let us assume v to be 0 free particle or free field. So, it is going to be g plus k omega A g naught. Next step I substitute the g naught here. You understand now the next step I just so, first order whatever I get I just substituted there.

So, then I do is the next second approximation g plus k omega this is my equation this is a full equation this in fact, I cannot well sometimes you can try to solve it analytically, but we I am doing perturbatively. So, I am going to replace this first order function here for g plus. So, I get g naught minus g naught v g naught. So, this is my new function new guess for g next what you can do is third approximation is getting better omega I can plug this in here keep plugging in. So, what will I get g naught minus g naught v now I plug the whole thing here.

So, that is g naught minus g naught v g naught. So, what is this g naught minus g naught v g naught plus g naught v v g naught v g naught these are matrices or operators. So, you have to put them in proper order and we have to just write like that. So, we can keep going on, but you can see that you can write this perturbative series and this is the first approximation first level first order second order you can go to third order and so on. So, this we write in terms of Feynman diagrams or diagrams.

So, this is the diagram. So, this is the full Green's function this full g plus this is g naught. So, the bold line this bold line thick line is the full Green's function, but then we have g naught which is thin line. So, this is this I already know how to compute then this is minus g naught v g naught. So, this is minus this is my g naught v g naught. So, it is a diagrammatic way to represent this operator.

Next is plus g naught v g naught v g naught. So, g naught v g naught v g naught. So, these are three g naught's and two v's and we can go on. So, this is a infinite series. We can also write in the following way we write this g naught minus g naught v n solid line full g.

So, this will correspond to this one this is represented here. So, this one representation for how to compute Green's function. So, in fact, if you are used to it then you can write this diagram very easily also it is a very visually it is nice. These operators are easy to write, but if your integrals then it becomes too tedious. Integrals are very tedious I am going to show you in the next slide, but these diagrams are very useful and this is by Feynman.

So, he constructed these diagrams for QED first time, but now it is we used in all field theory and for this reason I mean once we have this Hamiltonian I want to write down the full Green's function then I will follow this part. So, let us this is how we plug in this perturbation v. Now, let us do it one more thing suppose I want to do it in terms of time g k that part hand omega, but we will find fields of this sort g k t t prime and this guy is again g naught v g. So, I am going to leave it for you they do exercise this one exercise actually there should be a minus sign here really. So, this g naught v g naught will be of come of this form this you can do it.

So, d t prime g naught k t minus t prime. So, this Green's function t minus t prime and the source will be t prime source is at t prime. I might have done this in when I did the Green's function. So, this is in fact standard for given the Green's function how do you find the solution? So, given the Green's function how do I find the solution? So, if you have something like operator H psi is source let me write it not in the operators, but H of psi of x is rho of x H is a linear operator and I know the Green's function. So, Green's function will be H H g H of x H of x g x minus x prime is delta x minus x prime. So, I have a point source at x prime then the response it is the Green's function that is a Green's function.

Now, once you know the Green's function I can solve for psi. How do I solve for psi? Psi of x this is a beauty of Green's function g x minus x prime rho x prime d x prime. So, you know it already that you compute the potential for a point charge then you can write down potential for any set of charges. So, this is a general charge source and this is a potential for a point charge.

So, this is what I am done doing here. So, this is the Green's function is a response when my source is at E prime and then I can write down for this is a perturbation remember this is the this is perturbation in the right hand side. But, perturbation is function of g itself and that is why this is coming in the source this corresponds to source. So, I recommend that you can do this as an exercise and prove that indeed this is what we will get. So, I will end here for this part previous slide yes this line.

So, what is the problem? Let us see. So, this you agree. So, this is the definition of the Green's function right the delta t minus t prime. So, I just write we go to Fourier space. So, d by dt becomes i omega this correct now this one you have to have question now. So, how do you now here I put h equal to h naught here I put h equal to h naught and g equal to g naught.

So, g naught is the Green's function for free field free Hamiltonian. So, this is h naught and this is g naught no no no I made a mistake here this is my i h bar dt. So, no no no i h bar. So, i h bar d by dt this is correct no.

So, there should be. So, there is a this is correct this should be minus h. So, there is a the whole slide has a problem of sign. So, this should be minus h is true and. So, let us let us fix it.

So, I think we will let me just do the sign change. So, in the recording only we will quickly do the sign change. So, let me just see. So, this will be minus i h bar. So, minus h. So, this will be minus h naught and this will be minus v this is going to be this is i h bar.

So, this correct and this correct and this should be minus. So, now, here this will be minus. So, this will be minus and this will be minus. So, it goes to the right hand side this becomes plus and this becomes plus. So, yeah all these are plus plus plus no.

So, this is minus g naught inverse is this minus. So, this is plus plus plus. So, these are. So, here this will be minus these are plus. Thank you.