Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

Prof. Mahendra K. Verma

Department of Physics

Indian Institute of Technology, Kanpur

Week - 03

Lecture – 22

This is my operator V which is written as Vx, Vx is a real real function like coulomb gas okay, a coulomb interaction 1 by R. Now I have destruction operator at x and creation operator at x okay. So this is an operator no? Vx is a function but I have to an integrate over full space okay. So how do I compute this or how do I act this will act on some state and how does it work okay. So let us rewrite this in Fourier space. So yeah you have to just see this line which is looks complicated but it is not.

So psi dagger x I am just writing this part as this in Fourier space okay. So what is psi dagger which is in Fourier space? It a dagger p exponential minus I p dot x and integrate so this is p2 okay I made a mistake this p2 right. So this is the definition so this creation operator in real space is written in Fourier space. And as I said lot of calculations are done in Fourier space okay.

It simplifies and in fact all the Feynman diagrams you might see two photons coming going out these are basically the wave numbers photons with wave numbers k1, k2 coming scattering okay. So that is what we are doing it here. This this particle we are just going to specify this psi dagger as in terms of various wave numbers. Now this part psi x I am going to write as here okay. So this is psi x.

Vx is the function real function is not an operator so it can be shifted around okay. But I cannot change this cannot go to the left to the right psi dagger is to the left okay psi dagger cannot go to the right. So this is psi x which is coming with a plus sign I p1 dot x okay this we defined it okay. So this Fourier stuff please remember the definition I am doing it in 3D so divided by 2 by cube is is there. Now Vx I am going to again Fourier transform it but this is just simple function.

So this Vp is a real well complex number and is that. So this is a dx part right this is a dx d cube x is there. So what will I do with d cube x? So let us do it here d cube x. So d cube x will act only in the functions because A p, A dagger p they are all momentum space operators they d cube x cannot do anything on on these Vp, A dagger and so on. So it is going to act on exponential I minus p2 dot x so x we remove it outside plus put plus p1 minus p2 plus p dot x okay.

What is this object? These are vectors are is a delta function know so 2 pi cube delta function okay. So great so we got now I can compute this. So well I cannot compute this by I am rewriting it. So this is p1 and p2 you have A dagger p2 A A p1 they just coming from here and so this Vp right. So this is a delta function so I can write ps p2 minus p1 know from here.

So minus p2 plus p1 plus p is 0 so that means p equal to p2 minus p1 these are vectors of course. So this is what I have written it here. So I need to integrate this. So this is my operator written in Fourier space. Of course this will act on so imagine that I act V on number operator.

On state psi so psi will have some number operator like there will be 3 particles if it is boson 2 particles. So it will just see whether well look for wave number p2 and it will increase by 1 this one and look for wave number p1 and decrease by 1 okay. And Vp is a function so we compute in that wave. So basically these are going to act on the state which is just stated will given in terms of number of particles in various wave numbers. So this is how we write and if I write a Feynman diagram for this.

So how is Feynman diagram written for this? So we write V as a some kind of blob. You can think of this as a coulomb scattering. So this V is a coulomb scattering. Now this A is a particle coming the p1 is a particle coming p1 is a particle coming and is getting destroyed this is getting destroyed and what comes out of this thing is particle p2 comes out of this right this is a creation operator. So this part is written in like that okay.

So coming in is p1 p1 is so you just imagine the shoot a particle p with momentum p1 this guy will destroy it then V will act on it this V is acting well V is going to be appearing V will play a role and then it will eject a particle with momentum p2. So this process is amplitude coming from this potential okay. So a dagger and a dagger a p1 and a p a dagger p2 will give this amplitude square root p2 minus p1 square root p1 and that and multiply by V and just sum over all all all p1 p2. We will do one concrete example. So I hope this is clear.

Any given state we have to just do it for each piece and then just integrate it or sum it up okay and that is how it is done and this is not big magic but this is how it is done in field theory okay. We will encounter some of this in future as well. So a density matrix you might have heard of so the density matrix is defined as this. A dagger x I x is a one destruction and one creation so okay and this is a definition and we write this operator so this is operator and putting well this V operator is written like that okay and right now I am just giving the definition I am not really discussing more on this okay. Now let us do this more complicated example.

So Hamiltonian is this looks complicated know when this is a creation destruction operator here and this destruction creation operator here and their amplitude E naught here and this V by 2 but let us imagine that only 3 states are possible. So p is only 3 of them you can call it k1, k2, k3 okay. So 3 wave numbers you may call. If 3 wave numbers so what is this guy doing? He said destroying for that wave number and creating. So I write my state as number of particles in p1 state, p2 state and p3 state.

So what does this guy do first operator? First operator let us look at d dagger p1, d dagger p, dp I have to sum over all p's. So imagine that I have 1, 0, 0. So here this is 1 and these 2 are 0 so there is only 1 particle but that is in p1. So what will it give you? So this will I have to sum over all p's. So this implies d dagger p1, d dagger p2, dp1 plus these are operators this is d dagger p2, dp2 plus d dagger p3, dp3 acting on 1, 0, 0.

So what is this guy do? dp3, 0 no there is 0 particles so this is 0. Second one 0, third one 1 right because this is acting on here is creates is 1. In fact these are number density so there 1 particles will give you 1. If there was 2 then it will given would have given 2. So this gives you 1 okay.

So we can these are basically this matrix is going to be you can make a guess. So these are matrix is going to be 1, 1, 1 diagonal okay so that is what it will be. But this is more complicated. So let us act so you can act this operator. So I think I have okay so let me explain.

So I hope this part is clear this is diagonal this guy. It looks diagonal number of so you will get that. Now here I have to sum over p1 and p2 and this is destroying p2 and creating p1. So let us imagine I am working on this. This sum so there how many terms are there? The 9 terms okay.

There are 9 of this I had only 3 but there are 9 terms. So which ones will be non-zero? So destroying 1 this will destroy this one. It create anywhere it can create 1 at third. So is destroying this particle but it can create here. So so let us write d p1 which is the first this one d dagger p3.

What will that do? Remove 1 so will remove 1 this and create a third. So will create 0 0 1 okay. The one which is d dagger p1 d p1 which will be this 1 0 0 it it destroys and creates. And d dagger p2 d p1 will be 0 1 no no no is destroying here and creating at p2 so 0 1 0.

So this is what we got here okay.

So there are 9 terms when it acts on this state it creates this okay. So I can choose this in my basis vectors. My basis vectors are 1 0 0 0 1 0 0 0 1 and I create a matrix as operator right. That is what you do in quantum mechanics. And your operator is just to see destroy create and here I did not have to worry about square root n p plus 1 n p and that but you need to worry about that.

And that is it you can construct this operator and this is what I wrote this E naught is a pre factor. So this is diagonal matrix and this will be 1 1 1 all 1s right. 1 0 0 acting on this operator is 3 1s right in the first column. Then you act on 0 1 0 I will get a second row 0 0 1 I will get a third row.

So this is my operator clear okay. Now what is the eigenvalue of this? You can make a guess eigenvalue for this sum of two operators. So what is the eigenvalue of this? 1 1 1 three eigenvalues 1 1 1. What about this? Great some very good. So this is so this one the trace is 3 and determined is 0. So and you can see if you have vector 1 1 1 it gives you eigenvalue 3 okay.

So this one has eigenvalue 3 and the other two are 0 and 0. You can just compute them okay but this is a good guess some somebody guess it right. So we have this one is 1 and this 0 0. So you can just add them together. So eigenvalues E naught plus 0 E naught and this will be E naught minus 3 B by 2.

So these are three eigenvalues. In fact we know the eigenvectors as well. So this one corresponds to this one. This is eigenvector is that and 0 0 1 you can easily construct no. So we have to just construct for this. Diagonal means any vector will be eigenvector no.

So that is no worry. So 1 1 1 1 1 1 x y z is 0. So this 0 so we have x plus y plus z is 0. So just choose so I can choose minus half minus half and 1. So and that one more option is minus half that the two independent option minus half 1 minus half. Just choose two of them and we will get 0 0 eigenvectors.

So I have three eigenvalues and so this corresponds to this 1 1 1. This is eigenvector corresponding to this one okay. So we can write down operators with particle with this field theory. So it is not very mysterious I mean just had to work out and remember the rules. And this is taken from that book quantum field theory by Riester and Blundell.

So this book is top reference in my notes. The last one I think I have I know two more two more examples. So Tight binding Model you might have heard this no tight binding model.

In condensed matter physics this is one of the very important models. So we destroy a particle at site j and create one at i. So this hopping so this electron is hopping from j to i is called hopping okay.

So how do we write in terms of field theory? Very easy you know so you destroy a particle at j and created i that is it. So thus the Hamiltonian is written as destroyed j and created i and this is an amplitude for that. So once you know the rules and we can easily write and you can understand what they are talking about okay. So the last example the fourth one. Now this slightly complicated looking operator okay.

So we have two particles coming in it interacts with this potential. So this is like basically the two electrons coming interacting with the potential and getting off and they are getting off here. So we can objective is to write this in Fourier space okay. So I will do exactly same algebra what I did in the first slide okay.

So let us just do it bit quickly. So there are four operators V is not an operator V is a function. So four operators P1, P2, P3, P4 and these are the four wave number operators, operators in wave number space. So these two correspond to these and the creation operators correspond to these okay. So P3, P4 are the outgoing momentum and P1, P2 are incoming momentum okay. This is exactly like what Feynman language, incoming momentum and outgoing momentum and we have this exponential coming from this Fourier transform this one and this is getting cut V x minus y.

So this V x minus y okay. So clear know? So and this integral d cube x here. So this part is the real space integral okay. Now this is function of P1, P2, P3, P4 this whole object. X will be integrated out okay.

So potential will have all the momentum okay. So we write, so this is a bit of manipulation. So P3, P4 minus P1 is written as Q. Well you will see why okay. P3 minus P2 is written as minus Q.

So the operator looks better. So if I write like this okay, so then you will get x minus y okay. So this is just algebra. So this is x and y. So the two of them are with y and two of them with x.

So there is a typo here. So P4 minus P1 this Q. So P3 minus P1 is minus Q. So this is x and this is y okay. So I have to write this is psi y will be okay.

So let me tell you. P1 is coming with x. P1 is okay depends whose I am using. So what this is y? So this P2, P1 is coming with y. P2 is x, so P2 is x, so this is P2.

I am expanding this as P2 and this is expanded as P1 okay. So P1 is coming with y alright. And here also P3 this guy is coming with x P3 and this is coming with P4 okay. The momentum I these are dummy variables P1, P2, P3, P4 but I have to keep this track of x and y okay. So this is my real space integral okay. Now you see this is the only function of so I get rid of x minus y okay.

Actually there are two integrals d cube x d cube y. There are two integrals here. So this will be function of what? This whole thing real space integral only function of Q. Well x will be integrated out, y will be integrated out only momentum. So we write this as V V of Q is a real well is a complex number but V is V of Q okay.

So we write this as V of Q okay. So this is how I compute. Now given the number of given the state I just have to destroy two of them and create two of them okay. So how do I write this is the Feynman diagram okay. So let us bring this particles P1 and P2. So incoming is P1 and P2 know P1 and P2. Outgoing is P3, P4 okay and they exchange via this propagator well this is V V okay this is V of Q.

Now so this should be straight line they are not bending there is no. So these are straight lines. These are incident fermions two electrons. So P4 minus P2, P4 minus no I made a mistake this P1 this P1 and this P2. These are labels I have to respect this.

P4 minus P1 is P4 minus P1 will be so this is I add to this Q right. P1 no Q will be inside is it clear to Q1 so Q1 is coming in. So look at it here P1 plus Q equal to P4 right. Can you guys see this P1 plus Q is 4 P4. So P1 is coming in Q is also coming in and that gives you P4.

Momentum must be conserved okay. So P1 is coming P2 is coming in okay. So here P3 minus P2 is Q. So you get here. So Q plus P3, Q plus P3 is P2 right here P3 plus Q is P2.

So it is true. So this is true P2 equal to Q plus P3, Q plus P3 okay. So momentum is conserved momentum must be conserved at this called vertices okay and these are Feynman diagram okay. So we have two particles coming with wave number P1 and P2. They interact via this VQ and they get out with P3, P4 and now I have to sum out all possible momentums. So imagine that lots of particles are coming in.

You have to destroy and create and you have to just sum them up. These are we compute this V operator. So actually what will be there V operator acting on psi it gives you phi okay. So V or V operator acting on psi will give you another state phi okay. So this will be lot of number state with numbers in between. Now this will be another state with numbers in between and of course, there will be some pre factor.

The pre factor will be function of Q or rather V function of V. So potential will come in here potential effects will come in here okay. So this is how we compute the field theory. Basically this is how you calculate in field theory. You construct operators, you bring in states and then you will state will act on the on the operator and and give a new operator a new state sorry new state.

Okay any questions on this? So these are interactions. So we have free particles and they interact with this potential. We will have more complex interactions in future, but this is a kind of simple Hamiltonians. Okay so anyway so we yeah this is well I rewrote this in terms of we can get rid of one of them okay. So okay so if no other question then I will do the last topic for today and then we will quit.

Well right now think of external example one. Example two could be that two particles are coming and there is a some again coulomb potential that is or actually you can also think of potential well do not think of coulomb in fact you think of coulomb of these two particles itself. So repulsion of these two particles. So, one divide by mod x minus y with photons that was a v is photons you can think of photons propagator okay that is a that was what Feynman diagrams are.

So this v is a potential these are electromagnetic field. Right now we simplified it, but that is a coulomb interaction. And in between example that we had v of d dagger P 1 d dagger d P 2 that is more like solid state thing where there are three states in a in a in atom and we have photon coming and hopping. So, laser kind of thing that could be an example. Thank you.