Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Alright, so, we did classical field theory so far, no? So, classical field theory has a definite evolution for the field like particle in classical physics move takes a different trajectory following the equation of motion. Similarly, fields also follow equation of motion and it is definite trajectory like we can think of a string which is oscillating and there is no ambiguity right. I mean the given time we can compute what should be the profile for that string or same for electromagnetic field which is classical is definite given the initial condition and boundary condition we can find the evolution. But now we are getting into quantum mechanics where the field will not be definite, field will have uncertainties, field will become operators. So, the complications start now and of course, the richness also start.

So, we are getting into quantum field theory from now on. So, let us just do the particle quantum mechanics right now first three slides just to review and some of it is going to be directly ported into fields. So, reuse of quantum mechanics. So, general principles uncertain relation know that is so position and momentum are not accurately measurable in quantum mechanics.

So, it is not I mean particle in fact, leaves in a abstract space it is not in real space. So, particle position momentum become indeterminate in fact, they are operators and they follow commutation relation this is our commutation relations. A Schrodinger equation yields a wave function evolution. So, from the wave function you can compute position momentum energy and so on. This is a Schrodinger equation using which we find psi and from psi we find expectation value of the physical quantities like position energy and so on.

Properties derived from the wave function and so for bounded system we have energy Eigen value right for Schrodinger equation for hydrogen atom we have bound states and for scattering scattering states we have scattering of the wave function and so on. So, normally is easier to work with evolution of psi t right psi is evolving from here right from Schrodinger equation or sometimes we make the operators evolve in time. So, the

Heisenberg picture, so Schrodinger picture, Heisenberg picture and third is interaction picture. I am not going to get too much into it, but we will use well field theory uses Heisenberg picture most of the time. So, we will come to that in 10 minutes.

So, first this is called first quantization we have position and momentum for classical particles, but now we quantize it and we basically come go to wave function instead of x and p and from wave function we compute various physical quantities which are expectation values they are not determined. So, this is called first quantization for particles now this is loose word, but you can just think that is a first quantization. Now, let us look at an example linear oscillator which is going to come again in field theory. So, linear oscillator right half m well, so we know the oscillator m x double dot is minus m omega squared x. So, mass of the particle m frequency of oscillator is omega and it is linear and this classical equation.

Now, quantum mechanics of course, we have to solve the Schrodinger equation and x and momentum become operators. So, the Hamiltonian for quantum oscillator is this p squared half plus half x squared and both p and x are operators. So, operators I am going to write as a hat this did not come out well in PPT. Now, by change of variable all of I am sure you know this change of variable this creation operator a which is x plus iP by square root 2 a dagger which is a dagger I made. So, this I think is a dagger, so a dagger is x plus iP and a which is a is a destruction operator and its meaning becomes clear when field theory.

So, right now it is not clear, but we will come to that this creation I am not 100 percent sure about the sign, but let us go ahead with that. So, Hamiltonian is a dagger a plus half, so this you can just substitute it here use a commutation relation and you will get it. Half is coming from the commutation relation and it has a very important consequence the ground state energy is non zero for oscillator because of that half. So, a dagger a this is our number operator and there is a number state this is derivation I will not do it is done in standard quantum mechanics course, so no point repeating it. So, n is a Eigen state for a dagger a which is an operator and it gives you number n, n is the level that is what is supposed to be.

So, n is a positive definite integer, so n can be 0 1 2 3 4 like that n cannot be negative it is can be proven energy is n plus half. Now, I am normally going to put h bar is 1 and omega in fact, this is a total energy, but h bar we set it to 1, so energy is n plus half h bar omega and these operators a n is square root. So, is a a is a destruction operator, so it will basically bring down the state from n to n minus 1 and the pre factor square root n we need to remember this square root n is a pre factor when it comes down to state n to state n minus 1. This is oscillator it has all this energy Eigen states, so we start with 0 1 2 like this. So, if I apply this a operator then I come down from let us say on 2 then we will go to 1 comes down by 1 state.

Hamiltonian of course, if I operate H on 2 then it will remain on 2 and will give the Eigen value n plus half a dagger will take it up a dagger is a creation operator which takes it to n plus 1. So, a dagger acting on state 2 will give you 3 is the pre factor and what is the pre factor square root 3. So, higher of the 2, so you have left state and the right state you the pre factor is always square root higher number. So, here it is square root n plus 1, where n plus 1 is bigger than n well is a maximum of n plus 1 and n here it is n and n minus 1, so it is square root n that is a easy way to remember. The vacuum state or the ground state for here right now we should think of is a ground state for single particle a acting on 0 gives you 0 it cannot go below 0 below the ground state.

So, this is in fact vacuum is for future this is ground state. Ground state does not have 0 energy classical oscillator if I it is a equilibrium state or ground state then it has 0 energy it is particle is sitting at a position with spring which is unstretched. So, that is a lowest state and which has 0 energy, but oscillator because the unsteady relation has non zero energy and its energy is how much is energy half h bar omega and we will see that in field theory this this plays a big role and this purely quantum. So, this is not this cannot be derived classically. So, this is a brief review of quantum mechanics which everybody knows I am sure there is no question on this.