Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

Prof. Mahendra K. Verma

Department of Physics

Indian Institute of Technology, Kanpur

Week - 02

Lecture – 13

Hamiltonian formalism, okay. So, we need some more energy for this one. So, the way we are write Hamiltonian for particles, we can write Hamiltonian for fields. Okay. So, momentum density, we have momentum for particles. So, what is this dL by dx dot, partial L partial x dot.

So, instead of that we have partial L, not partial, okay, it is a functional derivative. L is a function, so we get partial L partial delta L delta pi dot. Now, i is a component, it could be x component, y component, g component, I get partial L partial pi dot. Now, L is integral, L d4x.

So, when I go inside, then it is going to be partial. So, this is total L, d cube not d, sorry, d cube. So, total L I have to do the functional derivative because it is function of any phi. But when I go inside, this Lagrangian density is function of phi, then I do partial derivative. Okay.

Now, that is momentum. I defined, this is a definition, Hamiltonian density is pi i, this is pi i, pi i dot minus L is very similar to what we do for particles, pi x i dot minus L. Okay. And we define total Hamiltonian is integral over space. Okay.

So, now, equation of motion. So, like we have x dot is derivative of H, you know, with pi dot. So, we have phi i dot equal to partial H, no, there is no derivative. This is old slide actually. Delta H delta pi i and we have pi i dot is derivative delta H delta phi i.

Okay. So, I think I have an example. So, we have this example. For this I am going to write down the Hamiltonian. Now, this Lagrangian, Lagrangian is function of phi, phi dot, but Hamiltonian is function of phi and pi.

Okay. So, this dot is not there. So, delta L delta phi. Now, this dot is there, delta L phi i dot, like delta L delta x i dot is there, no, for, and that is, will act on derivative, which is phi i dot, this one, square. You take the derivative, I will get phi dot. So, pi is nothing but phi dot, which is like momentum is x dot, very similar.

The formula is in fact motivated from particles. So, Hamiltonian will be pi phi i dot minus L. Now, pi is nothing but phi dot. So, this guy will give us, okay. So, what will I get? Phi dot into a phi dot, which is phi dot square minus this one, minus half phi dot square, that is this one.

Okay. Then this minus minus becomes plus, so plus grad phi square plus V phi. Okay. Now, phi dot is nothing but pi. So, this is my Hamiltonian density. It should be function of pi and phi and grad phi, but not phi dot.

Okay. Don't write phi dot for Hamiltonian. The way for particles, we don't write x dot for Hamiltonian, we write only momentum. Okay. So, equation of motion. So, phi dot is delta, this is not there, delta H delta pi I and I equal to 1 for this is 1D.

So, this is going to give us phi dot equal to delta H by delta pi will be pi. Okay. Now, this is not dot here. So, pi dot is delta H by delta pi with a minus sign, this is a minus sign here.

Okay. Now so, we get delta H by delta pi. So, this is a functional derivative. Okay. So, functional derivative, I again apply the logic of functional derivative is function of the derivative phi as well as grad phi will come.

Okay. Both the terms will come. So, I get partial H partial phi minus this divergence of partial H partial grad phi. Okay. This is, this will come and this one is the Hamiltonian. So, function of derivative phi will give us, will come from here V phi.

So, that is V phi and partial H partial grad phi will give us grad phi here and divergence. So, divergence of a grad is Laplacian. Okay, that. So, this is pi dot, pi dot and pi dot is, well pi is phi dot. So, this is nothing but pi double dot or this, it is square.

Okay. Right. So, that I am just using this, this relation. So, this equation. Okay. So, field with a potential, equation, Euler Lagrange equation is this. And there are potential, there are in fact, we will encounter fields with potential.

You might have seen the free energy, you know, for, for phase transition is phi squared minus phi 4. So, these are the potential. Okay. And so, this is the example. I think I will do the other example later, because the thing is already is close to saturation.

So, this one is a quite important one. So, you can write down Lagrange for Schrodinger equation, non-relativistic, non-relativistic Schrodinger equation and I will do it in the next class. Thank you.