Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Lecture – 12

So, now let us look at relativistic field, I will not start with this in my field theory lectures but we need to do this I mean of course we will QD, quantum field dynamics will all be relativistic fields. So, we have four variables space time know. So, space time is this t and x, the counter variant and the covariant is d by dx, d by dt. So, instead of writing dt dx, we write d for x, space time together and this is Lagrangian density and we know this variable is covariant derivative know d phi dx. I will not delve into it because this is a huge topic. So, we will just well I think we will do bit more when we get to that but I am just going to state the results.

So, the equation of motion is this, partial L partial phi this same as before but those time derivative and space derivative are combined here. Well, d0 is dt, partial tt and d other ones like dk, space derivatives are dk. So, 0 is time and the 1, 2, 3 are space, they are combined here. So, if you write t and x separately we will get back the same equation.

Example, now this is for free particle, this is Lagrangian for a free particle. So, this is the kinetic energy term but remember it has both d by dt and d by dx square in this. So, if you expand this one, this is d by dt of phi squared minus grad phi squared. That is the first term. Why minus sign? Because this is a relativistic field.

So, you get a minus sign. No, I mean this part I do not want to belabor on this but invariant thing is t squared minus x squared not t squared plus x squared. This minus sign is product of contra variant and covariant vectors. So, the minus sign will come here and minus half m squared phi squared. This is a Lagrangian density for a relativistic free particle.

This is a free particle with mass m. This in fact is masss m. So, we get, so I can do the same algebra. Let us do it here. So, if I do dl by d phi what will I get? Minus m squared phi, this one.

Here I get d by dt of partial l partial d dt phi. So, this term will give us, so this second term will be, let me not, let me write this term. So, partial t. So, this is basically this term derivative. So, half and two will cancel.

So, dt d phi. So, this is this term, double derivative in time. The next term is this one, grad phi squared. So, I get d mu, well the space derivative. Let us look at x derivative, d by dx of partial l partial dx phi.

This one, same idea except there is a minus sign there. So, I get minus dx square phi, dx x, double derivative with x. Plug that in here, we will get this equation. So, this is a plus sign, well sorry, these are all of the, all are minus sign, these are also minus sign, these are also minus sign, these are also minus sign. So, how do I get minus plus sign? Let me just check.

So, 0 equal to that, so this is a minus sign here, minus phi squared. This is minus sign and this is plus sign because this is plus. I take a, this is a minus sign here. So, minus sign here and minus sign here as well.

So, minus minus becomes plus. So, look here, this equation is written like that, minus M squared. So, this has d by dt of partial dt dt phi and then with this is a minus sign and same thing with x. So, there is a minus sign here, so the minus sign is going to be plus, plug that in. This I have taken into the left hand side, this is minus M squared.

So, go to the left, this is plus M squared, this is minus d by dt dt. So, that becomes plus and this is plus, it becomes minus. So, this is the equation. So, we get dt squared minus Laplacian squared plus M squared phi is 0. For M equal to 0, we get the wave equation like classical electromagnetic waves, but M not equal to 0 is a is a massive field with mass.

So, in fact, this object is written as Laplacian, this one, shorthand in using relativistic fields, this square equal to dt squared minus Laplacian squared. So, we will encounter this, this is what this is one of the important equation for field theory. Of course, this is a free particle, but then we need to put interactions. So, interaction when you once you put in, it will become 5-4 theory, that is you might have heard 5-4 theory, 5-6 theory, 5-7 theory.