Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Week - 01

Lecture – 01

Welcome to the class. So, this is kind of unique kind of field theory class because I do not work on quantum field theory, but I work on classical field theory and the tools are roughly the same of course there are lot of differences. So, I thought I will try to mix all together and see the similarities and dissimilarities and also look at what are the unsolved problems. If you work on particle physics field theory then you do not really know what are the issues is with the turbulence field theory and vice versa. So, this course to best of my abilities I am going to contrast various fields and there are lots of unsolved problems and that is what I will try to highlight. So, you can see that the scope of this course is very very wide classical quantum equilibrium and non-equilibrium.

This aspect equilibrium and non-equilibrium is not emphasized significantly in most field theory courses. I can just tell that many body physics as well as most of quantum field theory of particle physics they focus on equilibrium field theory. They do not tell it with explicitly, but it is equilibrium field theory, but turbulence is a non-equilibrium field theory and also there are many dynamical systems in statistical physics for which we use non equilibrium field theory. So, there are lots of issues with this aspect and that is what I will cover significantly on this non equilibrium field theory.

So, you need to look at that screen if you want to see the notes otherwise yeah. So, in this introductory lecture I am going to tell you the overview then we will get into specific details. So, this field theory course is about I am going to cover all of it, but not in depth. So, there is no time to cover in depth this is required four courses. So, we will do in one course.

So, you may know that the field theory is the most fundamental physical theory of nature. It includes particle it includes quantum theory and relativity statistical physics all put together. In fact, it has been built very nicely by lots of expert experts. So, you see some names in the course like Feynman, Dirac then Wilson and then in turbulence in classical field theory we have star works like Kraichnan, Kolmogorov has contributed.

So, it is a basically culmination you know the peak of understanding and combine it is all combined, but do not assume that it is all solved.

For example, this one field theory essentially is perturbative most of the calculations are perturbative like we do in Taylor series to term by term. So, there are very few exact solution. So, perturbation is it valid not valid if it is not valid then what we do. So, there are issues which are still a problem I mean it is still unsolved. So, fields as I said there are we can classify in four main categories like particle and nuclear physics, condensed matter physics which is like many body physics is called particle physics also many body physics, but there are names given know when many body physics is first basically condensed matter physics like semiconductors, metals, liquids.

So, that is condensed matter physics. Then statistical field theory that is for phase transitions fluctuations. So, that is statistical field theory with mostly connection with basically statmech. So, the non-equilibrium statmech is in statistical field theory with phase transition being one major theme and turbulence and but these are one general topic it includes other systems which I am going to cover later part of the course like coarsening like pattern formation we will do some of the field theory for that. So, now let us look at specifics of these four topics.

What is field theory? How field theory comes into these four topics? So, before that let me so in fields so like most people have done quantum mechanics course in fact all of you have done quantum mechanics course statistical physics course. So, in quantum mechanics and classical physics we deal with particles. So, Schrodinger equation for particle in a box or electron in a hydrogen atom. So, this is particle but we have electromagnetic field as well know. So, electromagnetic field should be quantised because everything is supposed to be quantum quantised.

So, when you quantize the electromagnetic field we get photons. So, electrons interacts with photons waste particles. So, this in electron itself can is in fact there is no single particle that is what this my next point is there is no single particle in quantum field theory. Just imagine electron will generate electric field. So, the electric field will generate photons where basically photons will is quantum version of electric field and this photons will generate positron electron pairs and in fact other particles too.

So, this photon P photon will generate electrons and positron E plus and E minus. So, in turns out a huge number of particles if you start with a single electron. So, that is why it is called many body physics. In nature there is no single particle each single particle will generate fields and that in general will create particles. So, field theory is combination of particles and fields.

So, it is a comprehensive description that is why it is really in fact is fascinating description very successful one. You can imagine that because of this what I just made a little sketch here this particle number is not conserved because they keep generating and destroying particles and mass is not conserved. So, like photon if you think of as energy then it will create mass and vice versa. So, classical physics assumptions are not is destroyed yet. A single particle theory is incorrect.

It is just said we cannot have one electron and write down the electric field per electron that is not possible in field theory. In fact in nature it is not possible. What we do it is a good approximation if a charge is very very big very large charge. So, single particle like we do a particle in a box electron and hydrogen atom these are to be modified. In fact you may know that in quantum electrodynamics we so whatever we do in classical physics like magnetic electron in a magnetic field know there is a Zeeman effect and there is a correction because of quantum field theory and that is what Feynman, Tomanaga, Schwinger they calculated it and they found the experiment and theory matches very well and that is supposed to measure in 10 digits of accuracy and that is what won them a Nobel Prize.

So, that is basically field theory effect many particle effect. So I hope you understand I mean this is reasonably difficult is not easy. But we will get a flavour of it and you can dig deeper if you are interested. So, in particle physics QFT combines relativity in quantum mechanics. In fact as I said field theory is used in many many fields.

So, I am just telling you what what are the main well in one slide I cannot cover all of it. But I will just tell you just of what we do in particle physics and we will cover more in more details in the in the course without this we cannot cover a field theory particle physics has to be covered. So, QFT quantum field theory is a short form of quantum field theory combines relativity and quantum mechanics. So, equal MC squared and that is definitely there is one of the ingredient of field theory and provides a consistent theory for particles and fields. Particles can be created and destroyed as I said that is what field theory is about.

So, QFT has predicted and explained many phenomena unification of forces. So, we may know that nature has four fundamental forces gravity, electromagnetic, weak nuclear and strong nuclear. It turns out for three of them have been combined in a single theory except gravity. So, that is called unification and it is a big achievement and people are trying to combine gravity and not succeeded so far. But this is a one major thrust arising out of field theory.

Quarks is a prediction from field theory, Higgs boson which has become very very hot topic or after the Nobel Prize came some several years before renormalization. Some of it we will do in the course not unification not quarks but Higgs I will cover in a in a broad way a renormalization. This is renormalization is a general phenomena in field theory in particle physics the charge of a particle is not constant. The charge of an electron what we say is 1.6 10 power minus 19 coulomb that is a effective charge at a low energy.

If you increase if you probe with a higher energy I mean if you take a charged particle and you probe deeper then you will experience larger electric charge. So, electric charge what we record is not the basic charge of electron. If I supposed to be charge is very very large infinite charge but it differs because of this many particle effect we observe this effective charge. Similarly mass of electron so we will see some of it this called renormalization. So, charge of electron depends on the energy of the probe.

In fact it is quite easy to see if you have electron here which has infinite charge but electron will create electromagnetic field and then we will have lot of positron electron pairs. So, electron will attract the positron closer and repel the electron further. So, E plus will come closer in this particle and E minus will be far away. So, we will have lot of E plus in the neighborhood that will shield this real charge of the electron. So, real charge of electron is shielded by this virtual particles surrounding the electrons you understand what I am saying.

So, that is why if you are far away I mean if you are somewhere here then you will you are not seeing one electron please remember you are not seeing one electron you are seeing lots of particles collectively from far. And so it is shielded by this lot of positrons and that is why you see less charge not infinite charge and depends. So, the charge will depend on how far you are from the source you have too far away then you will see less if you are too close and you see more and this is called renormalization. Renormalization means you are normalizing charge with energy. Energy is proportional to the distance well higher energy less distance inversely proportional.

So, that is what is renormalization about which again Feynman and these are the pioneers for this but we will see that same phenomena occurs in condensed matter physics, phase transitions, turbulence. So, this is what this will be one major theme of our course. So, that is what if you are far away then the effective charge is that 1.6 10 to the power minus 19 coulomb if you are too far away. So, field theory has is in fact very beautiful theory and it is the most fundamental theory and it is it is the basically fundamental theory for all parts of physics particle condensed matter physics, statmech.

So, key issues is particles and fields are quantised. Fields are quantised you know particle being quantised everybody is aware of it but field being quantised if you heard about it but we will we show you how it is done. Like electromagnetic field we quantise it then you get photons. Same thing happens for other fields like Higgs or nuclear but I am not going to deal too much with that we just stick to many body and electromagnetism. This course has a broad agenda and I am not also expert of particle physics.

So, I should not claim any of those stuff but you will get a broad picture which is I think important. A particle number and mass is not constant single particle theory is not correct. So, what I just said here that the electron creates a field electromagnetic field and that in in turn in turn will create particles like photons is a electromagnetic field and that will create virtual positron electron pairs. In fact other pairs too photon will create quarks and antiquarks also if you give enough energy and the electron will attract then positive charges and repel the negative charges. So, I think that is the question I hope I so in fact you must have done this know the effective charge effective electric field potential easy to power it is not 1 over r e to the power minus alpha r by r.

In plasma physics you have seen this potential. So, we have big charge but there are lot of lot of positive and negative charges plasma has positive negative charges. So, the charge is plus Q what is it going to do you are going to attract the negative charges and repel the positive charges. So, you are not going to see plus Q you see less reduce charge and that is why potential is not 1 over r for this Q by R but Q times exponential minus alpha r is called shielding Debye shielding. Now look this quarks you have to be you have to be taking things from everywhere if I just try to derive Debye shielding here quarks is gone.

So, some of it I am not asking you to go back to plasma physics and read in detail it is go to Wikipedia and look at Debye shielding. Exactly what happens in Debye shielding happens for QED quantum electrodynamic. Is that clear? So, Debye shielding you should just look at it same phenomena in fact this is very generic this called shielding effect and particle physics this was where I was. So, I just said that in fact this is a what mostly highlighted the lot of Nobel Prizes were given for this which are important I mean I would say that these are extremely difficult problems which were solved by lots of very very great faces including Feynman. So, Tomonaga, Stringer, Salam, Weinberg all Dirac, Dirac was I think he is the first person who can whom we can give credit the field theory starts from Dirac.

Creation annihilation operator I think you can possibly tell Dirac starts it. This was where I was so any questions on this. So, the next topic is many body physics condensed matter condensed matter is broad field know when you this Ashcroft and Mermin that book superconductivity metals liquids. So, basically start condensed matter physics by band theory you know chronic model and you describe band.

So, this is a single particle theory. So, chronic model is the electron in a set of delta potentials periodic delta potential and you get bands no problem but that is not enough. So, the like superconductivity cannot be explained by naturally by this theory. So, we need many body physics in fact metals forget about superconductivity metal itself requires many body physics this is again creation and annihilation operators mode electrons interactions. So, some of it will cover in the course broadly again. So, metals in as a electron fluid this is Landau theory Landau theory of electron liquid super fluidity superconductivity.

So, this requires many body Bose Einstein condensate which is become very very important nowadays and thus many body of course some of it we approximate like superconductivity is a we deal with a effective wave function. So, that is many particle wave function. So, you will get gist of it what is meant by many particle wave function and why many particle because you see as I said we are dealing with many particles many body many body is many particles. So, you see that is in fact very interesting very nice theory band structure, band structure fundamentally we get from conic-ponic but there are modifications. So, typically we assume equilibrium nature I will describe what I meant by equilibrium when we deal with it.

So, my interest in field theory is non equilibrium. So, I will looking at what are the new assumptions are to be made and that is where I had to see which assumptions are not going to work which assumptions are going to work. Statistical field theory this is a really important aspect this again has many Nobel Prizes. So, statistical mechanics all of you are aware of know when is thermodynamics is explained formally using statistical physics like you can derive PV is nRT ideal gas but ideal gas is simple but if you go to phase transitions from paramagnet to ferromagnet then those in fact assumptions of thermodynamics does not work the standard thermodynamics does not work. There are fluctuations which are pretty large and that requires field theory.

So, one important point how do you connect the quantum field theory with the statmech in fact this is very nice connection path integral of quantum field theory is mapped to partition function. Now right now it looks like word for you or sentence for you I will show the connection there is a beautiful connection. So, all the formalism of quantum field theory can be ported to statmech because of this connection you know how to solve the path integral or you have the formula then we do not need to do much work you just have to translate it to statmech and using that we can derive lot of interesting stuff many body tools are employed here. Phase transition is one important thing like the Wilson name is Wilson very important contribution we will cover that in the class. Thermodynamics does not work I mean for phase transition it does not work spin chain many electron systems there are lot of interest right now okay in this quantum phase transition that is another many body problem but we are now going to deal with that in this course. Typically equilibrium system but there are non equilibrium systems also have been studied in major details I mean lot of people have worked on it.

So, dynamical field theory so dynamical means time dependent dynamics know statistics versus dynamics. So, dynamical means time dependent so turbulence is one such phenomena okay here we had to it is not in equilibrium okay now so I can just say this following so this is called detailed balance. So, what is detailed balance? So, there are lots of people in the in a room but they are not going to nobody is going to be richer than other person everybody has equal money of course they need to transact. So, somebody has some toffee some other guy has some other sweets so they will transact but nobody will get basically somebody x gives something same value will be given by either the same x to y, y to x, or z to x. So, in this interaction like in this room thermodynamics well not in this room okay so in a in thermodynamics this particle no particle has more particle in general every particle has equal energy right that is what we assume k half m e square is k B T in this collision process each of them on the whole sometimes they lose sometimes they gain but on the whole energy remains constant but that does not happen in turbulence.

Turbulence some of them are richer than the others we have large scale structures so large scale structures are more energy than small scale structures if a large scale structures give energy to small scale structures okay so that means somebody is giving to somebody else net and some in fact large scale structures are fed energy by some other source. So there is a net energy transfer in this system in intervals unlike thermodynamics where there is no net energy transfer everybody is equally rich or every easily communism system okay this is like everybody is equal in that in a lot of natural system it is not the case there are hierarchy there is a multi scale description there is a energy transfer. So, energy transfer means detailed balance is broken there is no balance okay it is non equilibrium this transfer in fact the huge transfer so that is what is called time dependence okay and their frequency and time plays a big role the energy transfers are to be computed by looking at that aspect of physics okay. So in fact equilibrium we I am going to show you that Green's function which is very important object for field theory and correlation function they are equal but in non-equilibrium field theory they are not equal and more complex computation compared to equilibrium ones many assumptions still unsolved issues these are some examples okay like turbulence KPZ, Kadar Parisi Zang, so KPZ is short form for these three people time dependent, Ginzburg-Landau, Cahn-Hilliard. Coarsening is pattern formation okay essentially so if you see that in nature lot of pattern formation can be explained using this equation coarsening equation okay.

So theme of the course is that broad agenda covering many fields, comparative mode I mean basically comparative study of this thing highlight equilibrium non equilibrium aspect which I gave one example highlight unsolved issues we will avoid detailed calculations I will I mean if I get into detailed calculation which some of it I can do it just going to be three lectures okay and then we will not be able to see the broad picture. So this course is basically broad picture okay that is my objective for this course so we are not going to detail Feynman diagrams for a phenomena I am just going to say that okay these how it is done and these are result. So renormalization of charge I will broadly tell you how it is done and then I will just state the formula okay that is what we will do. So the charge renormalization is something like three lectures okay I mean is four hour class okay. And these syllabus which I shared with you in the in the first hand out okay so quantum field theory SFT is SFT is statistical field theory then quantum field theory 2 which includes Higgs mechanism mass renormalization you know I just said and then we will do dynamical field theory okay so turbulence.

So we will cover these aspects in a broad framework as I said we have one canvas I am going to put pictures of various aspects of field theory and you will be basically seeing how the connections and is your interest now after that I mean if you are interested in this field then you find what you like and then you explore more okay. References I think it is important so as I said this is a very broad course so we cannot find one book to cover all these aspects but this book I really find to be interesting Quantum Field Theory For Gifted Amateur it is like the title is itself is pretty interesting. So these authors they say this that okay look field theory is very nice interesting beautiful and they want to give exposure to a person who is not trained in field theory or who is not a practicing field theory not a particle physicist or a condensate physicist. So it has almost all the topics included my sequence in this course will be different than this book but I am going to cover half of this book. The standard book is Peskin and Schroeder this particle physics is these two are particle physics.

So in particle physics I think this is very standard textbook. Some of the thing which I am going to use well I mean this is not really but some of the things I have done in the chapter 12 of this book I have field theory done there for turbulence I will take material from there. In turbulence this is a standard well I think this is one of the best books by McComb on who talks about turbulence I mean field theory of turbulence. The person who worked on this is Kraichnan, Kraichnan's papers are very hard to read. So Kraichnan is the person who pioneered field theory in turbulence there are various articles which I will use on the way. Thank you.