Introduction to Astrophysical Fluids

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Lecture - 57
Anti-dynamo theorem and turbulent dynamos

Hello and welcome to technically the last lecture of Introduction to Astrophysical Fluids. So, in this lecture, we will continue our discussion on astrophysical dynamos. And in previous lecture, we just introduced the concept of the necessity of dynamos and why basically we need to think about a dynamo type of mechanism.

And we actually saw that there are two parts of the game, one is that to produce the magnetic field, some large scale magnetic field starting from some seed magnetic field which is caused by the statistical fluctuations of magnetic field in a plasma, and then once this is produced, the question is to sustain that. So that we can see the magnetic fields as they are in the common astrophysical systems. And we talked about three typical systems, one is for the star, one is for the Earth and of course the third one is for the spiral galaxies. We mentioned about the pulsar and magnetic field but this is which is quite strong and this is the story is even quite complicated.

So, in this lecture, we will give you a very basic qualitative idea of how a proper dynamo action can be understood. But before that we will start by a very interesting theorem which rules out some possibility of having a dynamo action under certain simplistic condition.

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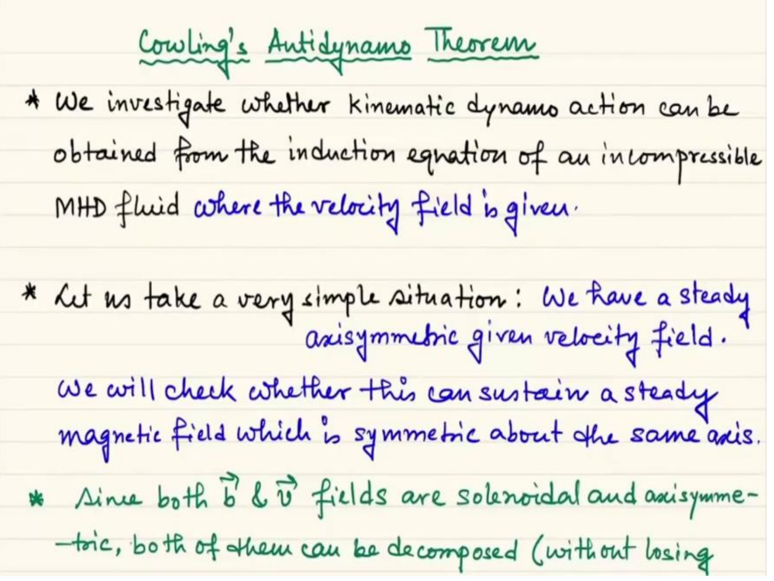

So, this theorem is known as Cowling's anti-dynamo theorem because Cowling was the first one who gave this theorem. And here we mainly investigate whether kinematic dynamo action can be obtained from the induction equation of an incompressible MHD fluid where the velocity is given.

So, once again for recapitulation, kinematic dynamo means that we will try to solve the induction equation, and the velocity is given and magnetic field evolution equation is just a linear equation in $\boldsymbol{B}$. And the magnetic field does not give any feedback to the velocity field through the Lorentz force.

Now, in this situation if we again take for simplicity, a very particular situation where we imagine that we have a steady axisymmetric given velocity field. Now, the question is that whether this type of velocity field can sustain a steady magnetic field which is also axisymmetric about the same axis, whether this is a possibility or not.

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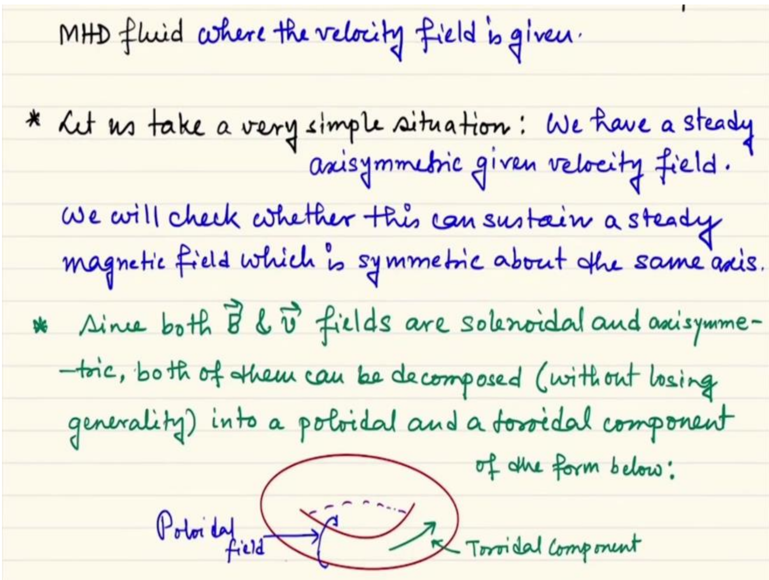

So by Cowling's anti-dynamo theorem one can actually show that this is not possible. Now, how to do that? Cowling's original proof was not really very very rigorous, but there is a short but much more rigorous proof of it and we are now trying to a follow in this lecture, this proof.

So, since both $\boldsymbol{B}$ and $\boldsymbol{v}$, that means the magnetic field both are solenoidal that means, they are of divergence 0 which is the case for incompressible turbulence, and if they are axisymmetric then both of them can be decomposed without losing any generality, that is very important, into a poloidal and a toroidal component of the form below.

Now, any solenoidal field vector field can be decomposed in a poloidal and a toroidal component. But since they are axisymmetric then this component should have a specific form and that is coming from mathematical physics. One thing I have to tell you that what really is a poloidal field and a toroidal field? just imagine at torus, and there are two type of degrees of freedom on a torus.

So, if you cut the torus then you will have a cylinder, then along this axis of the cylinder we will have one degrees of freedom and that is the so called toroidal direction. And if you just follow the lateral surface of the cylinder, this is another degrees of freedom.

And then actually the axis will be exactly the axis of the cylinder. So, here in the first case the direction was the axis of the cylinder, but the axis was in the vertical direction. And in the second case, for the poloidal case, the axis is the axis of the toroid or the axis of the toroidal cylinder.

So, this is roughly what poloidal field is which you can see in blue and what toroidal component is which you can see in green (see figure above).

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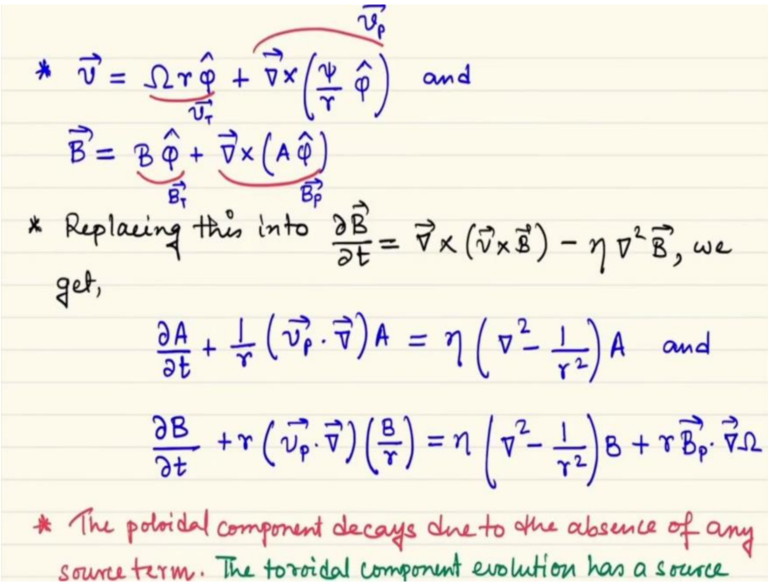

Now, for our case, velocity and magnetic field can be written like $\boldsymbol{v} = \Omega r \hat{\boldsymbol{\phi}} + \nabla \times \left(\frac{\psi}{r}\hat{\boldsymbol{\phi}}\right)$.

So, Ω which should be a constant and it is something like an angular speed, along $\hat{\phi}$ is exactly the toroidal component (v_r) and the curl part is the poloidal component (v_P).

And for $\boldsymbol{B}$ we can also write $\boldsymbol{B} = B\hat{\boldsymbol{\phi}} + \nabla \times (A\hat{\boldsymbol{\phi}})$. Where $\boldsymbol{A}$ is the so called vector potential.

So, this type of writing is very general for an axisymmetric solenoidal vector.

And now if we simply replace this expression for $\boldsymbol{B}$ and also $\boldsymbol{v}$ in the induction equation, now you see in this induction equation we have this the non-linear type of term which is no longer non-linear, here it is the linear term according to kinematic dynamo approximation. And you have another resistive term which comes with a plus sign of course.

And then if you do that correctly you will see that for the vector potential part, you will have the evolution for vector potential as

$$\frac{\partial A}{\partial t} + \frac{1}{r}(\boldsymbol{v}_P \cdot \nabla)A = \eta\left(\nabla^2 - \frac{1}{r^2}\right)A.$$

This is a scalar equation. So, the evolution of $\boldsymbol{A}$ is nothing but the evolution of the poloidal component of $\boldsymbol{B}$.

And then the toroidal component for $\boldsymbol{B}$ has an evolution equation which is

$$\frac{\partial B}{\partial t} + r(\boldsymbol{v}_P \cdot \nabla)\frac{B}{r} = \eta\left(\nabla^2 - \frac{1}{r^2}\right)B + r\boldsymbol{B}_P \cdot \nabla\Omega.$$

Now, you see this Ω is no longer actually a constant, this is a scalar, . Now, you see in this point for A, the solution is simple, it has an advecting component, but mainly its fate will be decaying because it has a diffusive component.

Now, what happens for the B? Now, B has once again an advective part, one diffusive part which lets it to decay, but there is a source term. Now, that can be something of interest. But the problem is that the source term is proportional to B_P, and B_P is nothing but proportional to A, now with time A decays. So, B_P will also decay, so B will also decay. So, the source term of B will stop in time and finally, B will decay just due to the action of this diffusive term.

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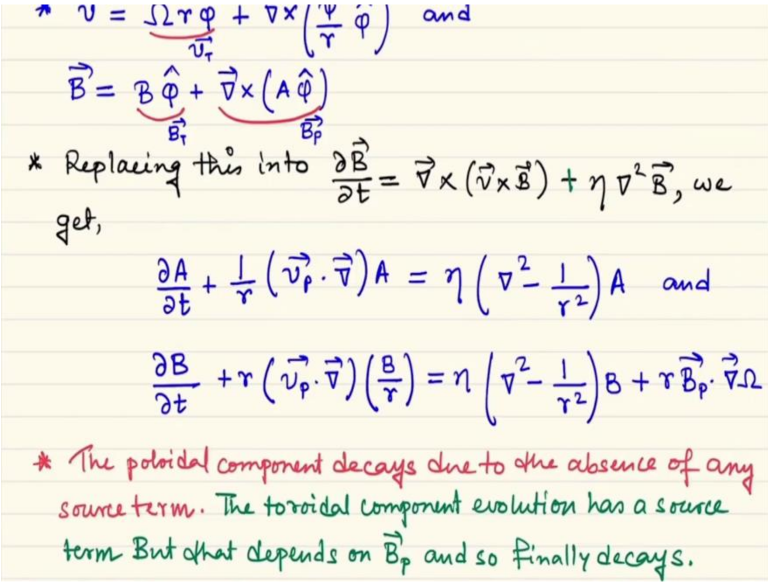

And that is why in this case, under this specific writing- that is very important, under the specific writing the poloidal component decays due to the absence of any source term. And the toroidal component evolution has a source term, but this depends only on the poloidal part. And so, it is stopping at one time because the poloidal part decays in time, so the source stops.

And so, finally, the toroidal component also stops in time. So, that is exactly saying that we cannot have a self-sustaining kinematic dynamo where both the magnetic and the velocity field are steady and symmetric about the same axis.

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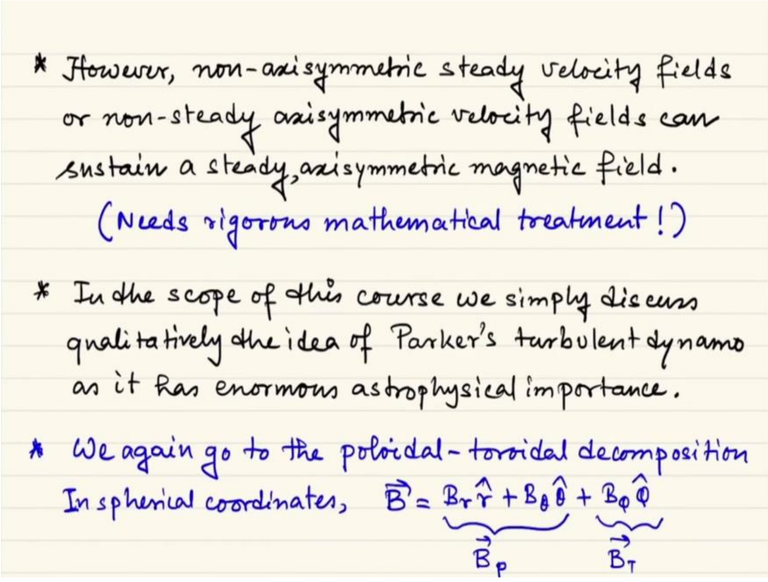

Then what can be possible? Then the thing is that in order to have a sustaining dynamo action, we need non-axisymmetric steady velocity fields or non-steady axisymmetric velocity fields, that means any of the two conditions of steady and axisymmetric, if violated can give rise to a sustaining dynamo equation.

So, they can sustain a steady axisymmetric magnetic field. But this needs rigorous mathematical treatment and there are researches on it, there are papers on it, you can search over internet and this is beyond the scope of this course.

Here we will do another thing which is much simpler. So, in the scope of this course we will simply discuss a qualitative description or qualitatively the idea behind the Parker's turbulent dynamo.

Parker's turbulent dynamo is very very important for this course because it has an enormous astrophysical importance. And actually, most of the astrophysical dynamos, except solar dynamo, in a very simplistic way can be understood using Parker's turbulent dynamo.

So, it is a highly useful theorem or highly useful model, but for that we again have to go to the traditional poloidal-toroidal decomposition of solenoidal vectors. So, in spherical coordinates, if we write any vector let us say the magnetic field, $\boldsymbol{B} = B_r \hat{\boldsymbol{r}} + B_\theta \hat{\boldsymbol{\theta}} + B_\phi \hat{\boldsymbol{\phi}}$, then this azimuthal component can be identified exactly to be equal to the toroidal component. The rest is nothing but the poloidal component.

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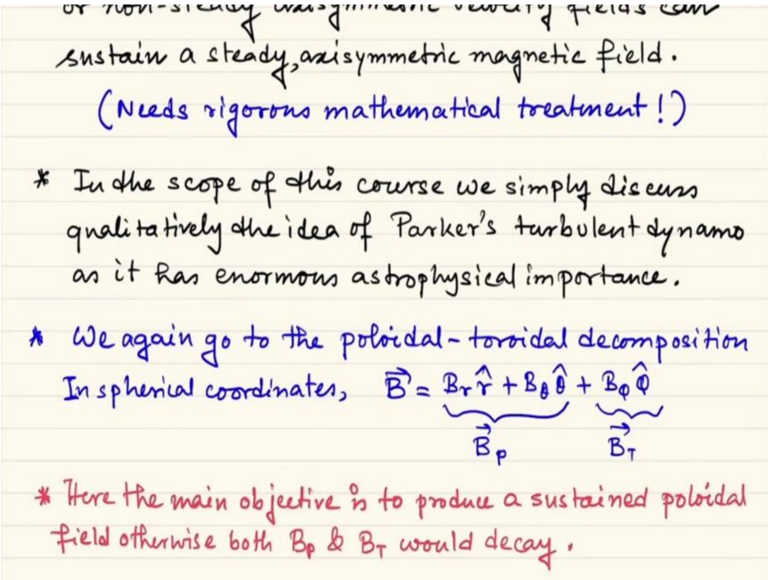

Now, here our main objective of the study is to produce a sustained poloidal field otherwise both poloidal and toroidal would decay. So, the poloidal part, if it is sustaining then as this has a source which is related to the poloidal part, it can also be sustained.

So, of course this is roughly true, we cannot talk about this model here, but the roughly the idea is that if in any ways we can produce a sustained poloidal field, then actually poloidal and toroidal, both can sustain. Otherwise, if the poloidal field is somehow decaying then poloidal field would stop and the toroidal field would also decay in time. Of course, it could continue a bit longer, but finally, it would stop.

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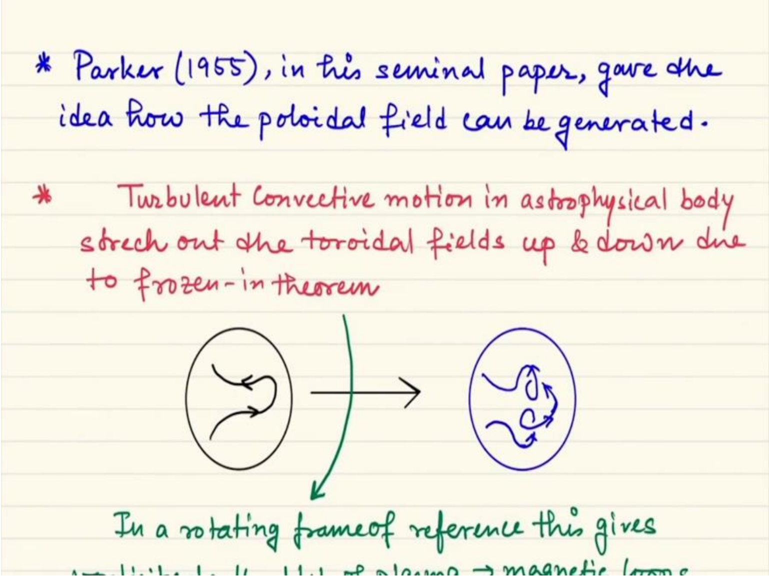

Now, Parker in the year 1955 in his seminal paper gave an excellent idea, a revolutionary idea on how the poloidal field can be generated.

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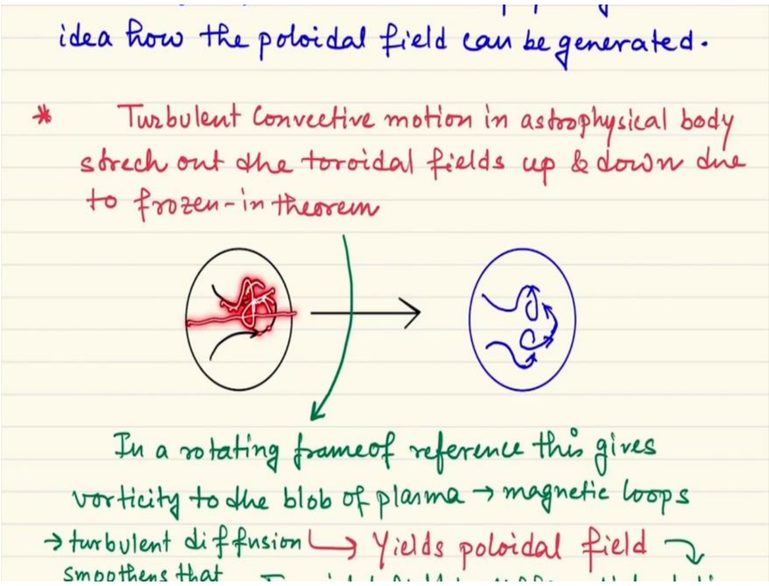

And at this point you will see that how useful for you was to learn turbulence. Because Parker said that it is actually the turbulent motion of the plasma, inside the astrophysical body which causes all this sustained dynamo action. According to his theory, the turbulent convective motion of the plasma in an astrophysical body stretches out the toroidal fields.

If the body has effective conductivity which is much larger than the viscous effects, so the frozen-in field theorem is approximately valid. And when frozen-in field theorem is valid then the plasma would drag the magnetic lines of force. And actually it is seen that finally, this can, in one hemisphere it can make the lines of force up a bit, and in another hemisphere it is like down (see figure above).

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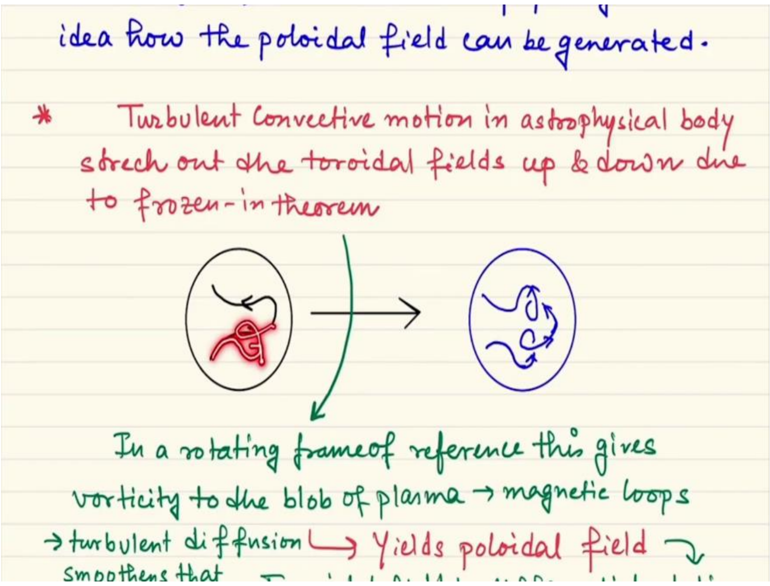

So, finally, starting from some toroidal direction we have something which is now nonzero in the poloidal direction as well.

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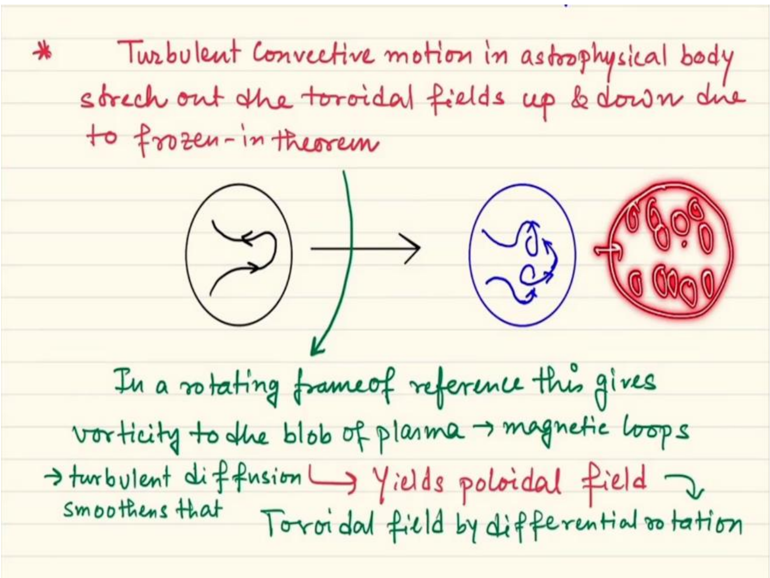

Now, there are other part of the story as well, in most of the cases what happens that we are considering rotating frames of reference, and rotating frames of reference, it adds to the vorticity of the blob of a plasma even if the initial vorticity is 0, that we know from the effect of rotation in a rotating frame of reference, that for an incompressible fluid we do not need an initial vorticity.

But the solid body rotation of the plasma itself gives the plasma a vorticity, and that vorticity will evolve in time. Now, once the vorticity is given then magnetic loops are formed. So, the story is very complicated. It is just schematically said here. And those magnetic loops get smoothed by the turbulent diffusion which you already have learned.

So, when turbulent diffusion smoothens that, finally you will see you can have small magnetic loops and actually very interestingly it can be seen that all the magnetic loops in both the hemispheres are of the same sense. That is very very interesting.

Now, this finally, leads to the poloidal field, a large scale sustaining poloidal field. Now, here we can see that the generation by turbulence actually sometimes we say helical turbulence because this is a turbulence with helical motion, so by helical turbulence, toroidal fields can give rise to poloidal field, and again if the system has a differential rotation as we can see in Sun, then again this poloidal field lines are stretched to give again toroidal fields by differential rotation. So, this is the whole story of the turbulent dynamo.

And sometimes some people call this as the $\alpha\omega$-dynamo, this just for a vocabulary. Although, the proper analysis of $\alpha\omega$-dynamo is not discussed here, and it is much more analytically vigorous. So, that is something which I suggest you to check, in any of the books of reference or over internet, how can using mean field magnetohydrodynamics we can account for the possibility of a large scale magnetic field by turbulent dynamo proposed by Parker.

Although, once again as I said, solar dynamo actually cannot really be described by this simple framework. So, this is something you have to understand just as a basic tool of understanding astrophysical dynamos. And now the question is of course, what about Earth’s magnetic field and what about Earth’s dynamo!

So, once again Earth does not have any differential rotation as such. But my question to you is that, can turbulent dynamo of Parker, put some light on the Earth’s dynamo problem? Why this is sustaining? If yes, why? If not, why not? This is also a very good homework for you.

So, turbulent dynamo model which I described here qualitatively, is just the starting point of the dynamo theory in its regard, so the rigorous treatment is not done here, but just to tell you that already the very simplistic dynamo model includes so many pictures of rotation, frozen-in field theorem, turbulence, rotational effect, everything, including poloidal-toroidal decomposition.

So, and of course, the situation becomes much more complicated if the fluid is compressible. So, a normal astrophysical fluid in a dilute interstellar medium is highly compressible. Then how to do that? Then you even cannot do very simply this poloidal-toroidal decomposition.

So, this course was just to give you a very brief overview of what are the building bricks of astrophysical fluid dynamics. Of course, in the scope of this course I could have just told you may be less than 0.1 percent of the whole story. And now it is up to you, if you feel motivated. you go to the research papers, you go to several books, you go to the videos.

Nowadays, you have billions of billions internet resources, so just search over that. And there is always a very big scope of doing theory and simulation. And observation is also very much popular already in astronomy, but theory and simulation there are so many scopes possible. I strongly suggest you, friendly but strongly suggest you to just go through several problems which are done in the last 25 or 30 years.

So, thank you very very much for your kind cooperation throughout the course. I personally may not have been able all the time to express the best possible way what I wanted to say, but you see that once again this was just an introduction. If you simply feel motivated by this course to the problems of astronomy or astrophysics my work is done. I am more than happy.

So, thank you once again. Thank you very much and best wishes. Best of luck for the examination. Thank you. Bye-bye.