Introduction to Astrophysical Fluids
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Lecture - 56
Introduction to Astrophysical Dynamos

Hello and welcome to another lecture session of Introduction to astrophysical fluids. In this session, we will discuss a new topic which is the topic of Astrophysical Dynamos. Evidently, this is going to be the last topic of this course. To be very honest, in the scope of this course, I can only present you less than 1 percent of the whole picture of the astrophysical dynamos.

What I would try to convey is the necessity of a dynamo theory and in which sense, we can apply dynamo theory to Sun, Earth or galactic magnetic fields. So, once again, I will not be emphasizing much upon the mathematical rigor, rather some qualitative discussion and some conclusion of some theorem or some theory, without doing the mathematical development or the proper analytical development, in this lecture .

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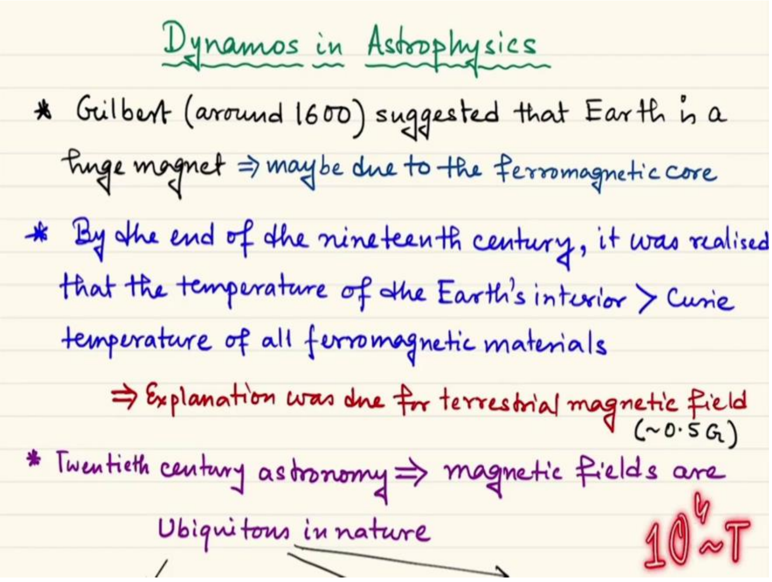

So, for the first time, it was Gilbert, who was actually by profession a physician of Queen Elizabeth I, announced or he suggested around 1600 that Earth is a huge magnet. And that was the first time that someone suggested that Earth has some magnetic property on its own and he actually suggested that while explaining the directional properties of the magnetic needles or compass on Earth.

Now, at the very outset people thought that the Earth's core is consisting of nickel, iron, this type of ferromagnetic metals. So, due to this ferromagnetic core maybe we can have magnetic property.

So, this was totally at the first hand looked somehow very reasonable. But by the end of the nineteenth century, people when they came to know about magnetic phase transition and the existence of a point or the temperature called the Curie temperature, they realized that the temperature in the Earth's interior was much greater and actually is much greater than the Curie temperature of all ferromagnetic materials possible and that means, that in the Earth's interior all these ferromagnetic materials can actually behave or actually would behave like a paramagnetic material, that you all know from your knowledge of Curie point.

So, that is why this explanation that the Earth's magnetism is due to a ferromagnetic origin, due to the ferromagnetic materials at the Earth's center was discarded. Now, actually then explanation was due for the terrestrial magnetic field.

Now, the terrestrial magnetic field is in a sense, weak, moderately weak. This is of the order of 0.5 Gauss to 1 Gauss as you know that 10^4 Gauss make 1 Tesla. So, this means, it is almost of the order of 10^{-4} Tesla.

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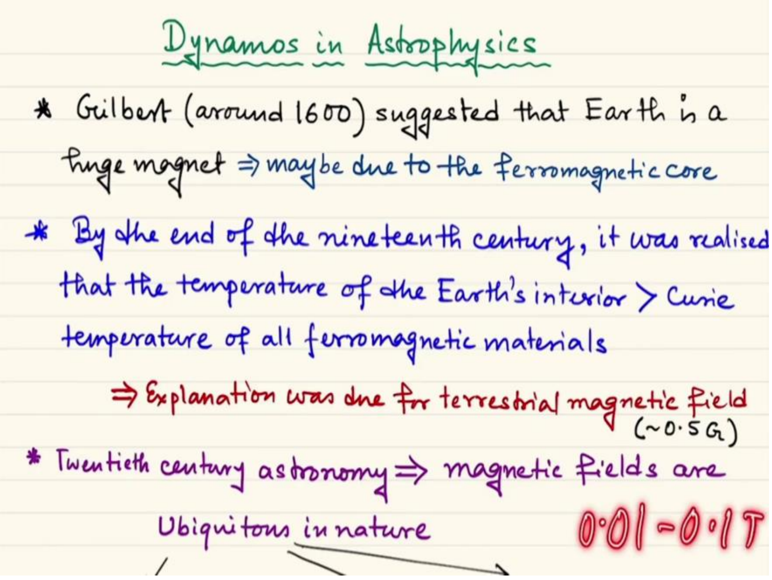

Now, as you know that in laboratory magnets, in general, you can have like magnetic strength from 0.01 to 0.1 Tesla. So, with respect to that Earth's magnetic field is quite weak.

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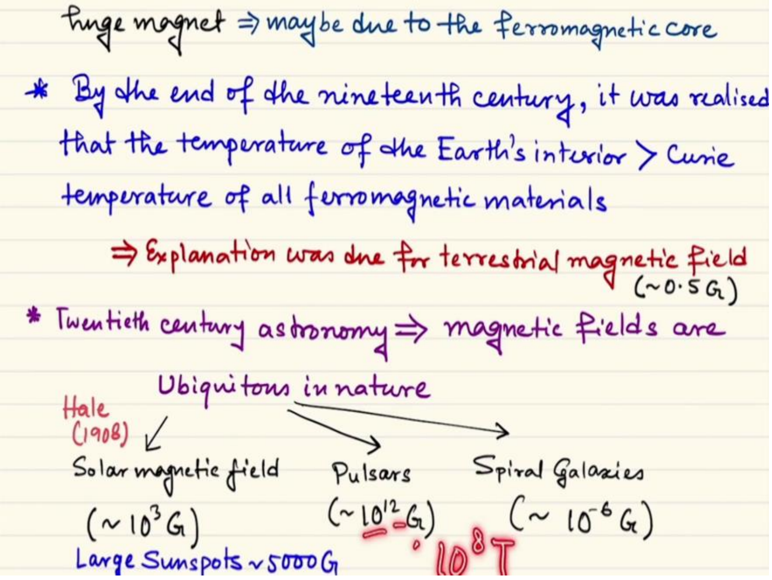

Now, in twentieth century, astronomy had a remarkable progress and that actually confirmed that magnetic fields are not only something which is a property of the Earth rather it is ubiquitous in nature, in galactic physics, in the galactic domain, so the Earth, Sun, stars, intergalactic medium etc.

Now, other than this terrestrial magnetic field, three instances are of very much interest, like the solar magnetic field, in 1908, Hale (I already have discussed this person while discussing about the sunspots), for the first time concluded about the Sunspots to be the region of extremely strong magnetic field. And the solar magnetic field in the near the sunspots, they are of the order of 10^3 Gauss.

So, this is almost like the magnetic field of normal laboratory magnets and even for the larger sunspots this can go up to 5000 Gauss. So, it is a moderately strong magnetic field, not very strong magnetic field, but moderate magnetic field. Now, another thing is the pulsars. We already have talked about pulsars while discussing the effect of rotation in astrophysics.

Pulsars are the rotating neutron stars and these pulsars have a very very strong magnetic field which is of the order of 10^{12} Gauss. Now, you can understand 10^{12} Gauss, this means 10^8 Tesla, it's enormous. So, this is one extrema, the other border line is the magnetic field of the spiral galaxies, that is 10^{-6} Gauss, very weak, but still it has a magnetic field.

So, we see that terrestrial magnetic field, solar magnetic field, pulsar magnetic field and spiral galactic magnetic field; 4 cases are of very very interest in the astrophysics.

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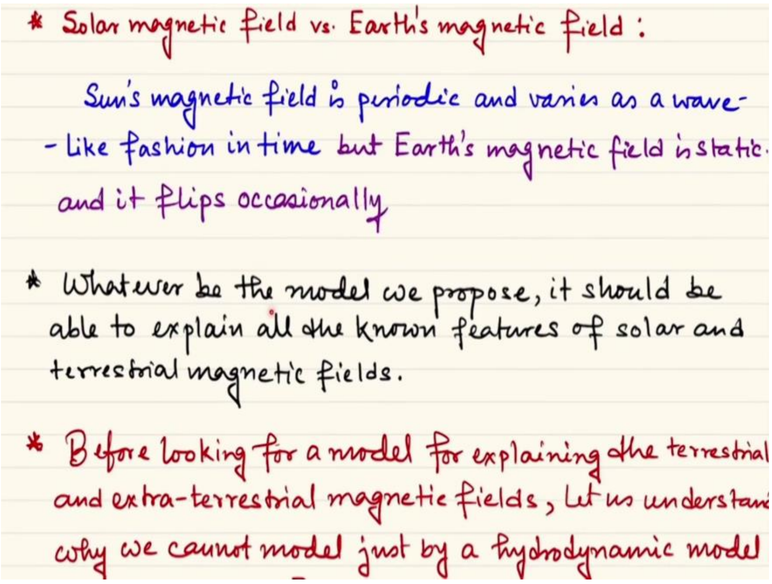

Now, first of all, since we are from Earth, we are the representatives of Earth and we are in the solar system, Sun is the our head of solar system family. Then, we will be interested first in the comparison of solar magnetic field and Earth's magnetic field just for information.

So, solar magnetic field is periodic and actually varies as a wave-like fashion in time and actually flips its direction in every 11 years. But on the other hand, Earth's magnetic field is very much static and it actually flips very occasionally. So, till now for example, after our astronomical and scientific progress started, Earth's magnetic field never flipped.

So, it was conjectured that in the past maybe it flipped, but very occasionally. Now, whatever be the model we proposed for the origin of the astrophysical magnetic field, the central question is that why there should be a magnetic field in this Earth or Sun or any other systems, where we know that the normal ferromagnetic origins are discarded to be the cause of those magnetic fields?

Then, we have to think of several models of course. Now, whatever be the model, the model should or rather must satisfy all the known features of solar and terrestrial magnetic fields.

So, if we want to propose a general theory of the astrophysical and interstellar magnetic field, then that should also include the solar and terrestrial magnetic field as well. If it is a general theory, it should be able to explain the origin of the magnetic field from our planet to the spiral galaxies.

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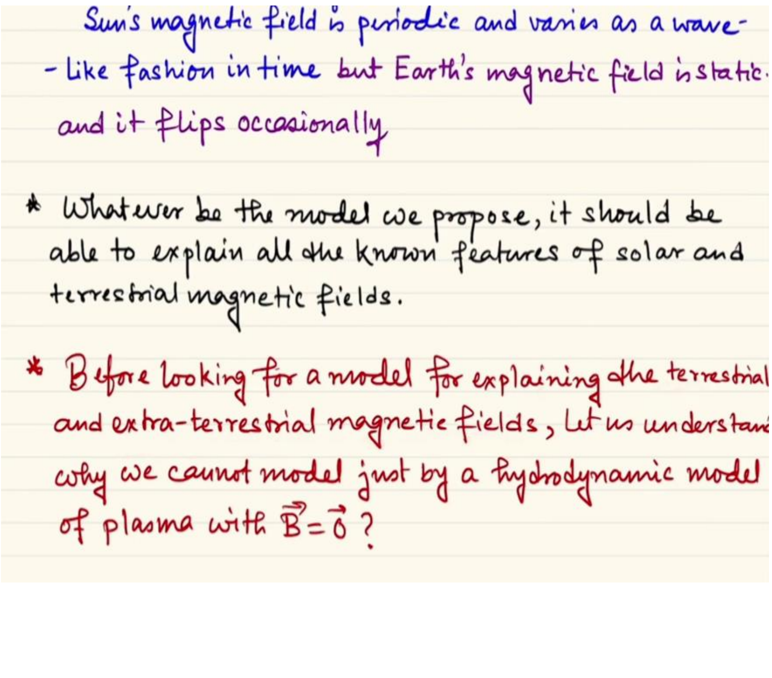

That is very straightforward claim. But before looking for a model in order to explain the terrestrial and extraterrestrial magnetic fields, let us first understand why we cannot model just by a hydrodynamic model of plasma with $\boldsymbol{B} = \boldsymbol{0}$. So, that is the question, why we are claiming that all these models should have some magnetic field and that is what we are searching for.

We have a hydrodynamic plasma, but the plasma has a magnetic field which is 0 and so, this plasma should have some electric field maybe, but its magnetic field is 0. So, electric field is nonzero, but electric force is 0. So, finally, the total system should behave in a mono fluid framework. The total plasma should behave like a hydrodynamic fluid. Why this is not possible? Well, of course, the first point is that by observation, we have already found that there are strict evidences of magnetic field in various cases as I just mentioned.

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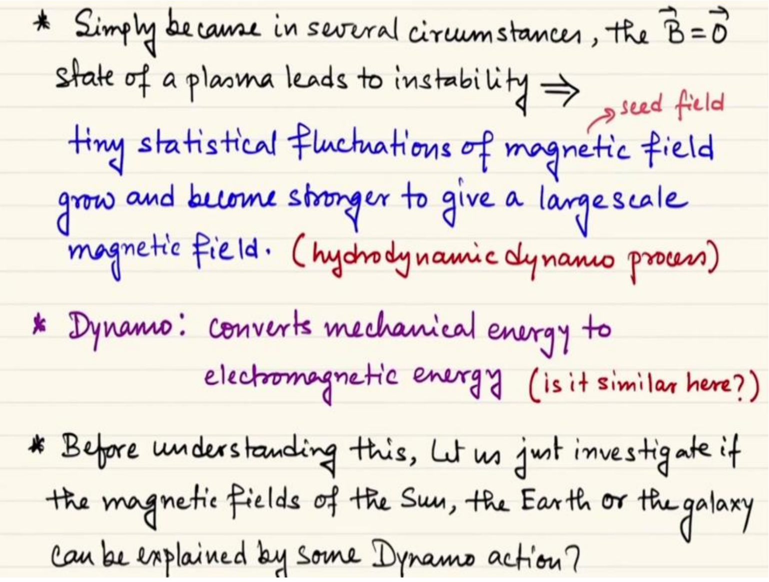

Another thing is that even theoretically, in several circumstances $\boldsymbol{B} = \boldsymbol{0}$ state of a plasma leads to an instability. And that instability leads to the growth of some statistical fluctuations of magnetic fields which is called the seed magnetic field, which comes from the statistical fluctuation of negligible magnitude in general.

But due to that instability, if you model the plasma with $\boldsymbol{B} = \boldsymbol{0}$, actually it will never remain or retain its stability. So, it will tend to such a condition where the very small statistical fluctuations of magnetic field which were neglected in general, now would lead to some stronger considerable magnetic field of large scale and this is known as the hydrodynamic dynamo process.

Now, why the what dynamo comes here? Well, from lexical meaning we know that dynamo is something which is exactly opposite to a motor. So, in a motor, you give electric energy and out of that electric energy you make some physical mechanical work.

On the other hand, and for a dynamo, you supply mechanical work and using the very basic principle of the electromagnetic induction simply, it creates a situation, where you have a variable flux situation and that is why finally, it creates an emf or electromotive force which thereby leads to a current, and another very straightforward way of saying is that you give mechanical energy as input and you have electromagnetic energy as an output. So, conversion from mechanical energy to electromagnetic energy is a very simplistic way of explaining dynamo.

Now, the question is, is it something similar which is happening here because we have till now, what we have said we cannot see anything like where we are giving some mechanical force or something mechanical energy and the electromagnetic energy is converted.

Well, of course, there is something similar. But before going to that question let us start by saying what happens if the magnetic fields of the Sun or the Earth or the galaxy can be explained by some dynamo action? Of course, if this can be done, then everything is solved.

That means, that the ferromagnetic theory is no longer there, now if the dynamo action can be used to explain this, it means that you are giving some sort of mechanical energy to the system or the system takes some mechanical energy from some source and the system in response creates some current and that is why we have some magnetic fields.

Because you know by Maxwell's equation, for non-relativistic limit, where you neglect the displacement currents, you have normal free current, then you can actually write $\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$, your simple current means a magnetic field, so in this way people started to think.

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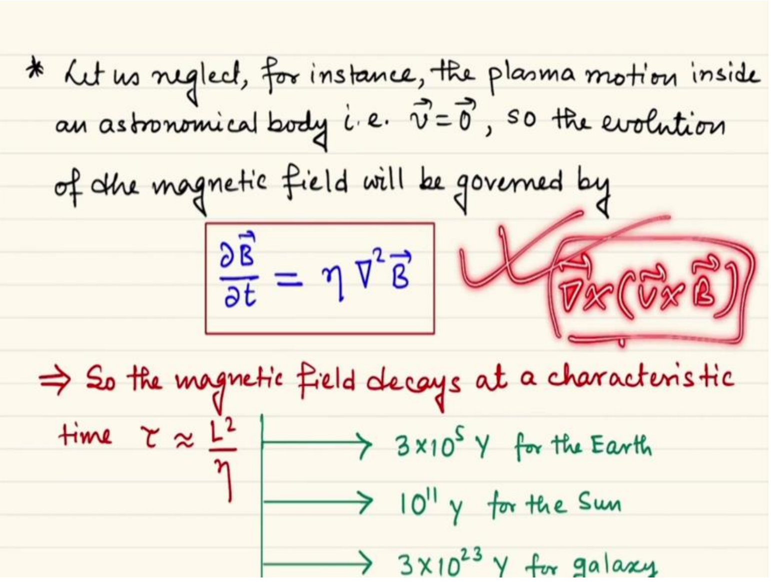

Now, in order to approach this problem, we have to start from some point which is very simple and then we will try to sophisticate it, step by step. So, let us first neglect for instance the plasma motion inside an astronomical body.

So, one thing is very clear that the plasma cannot have $\boldsymbol{B} = \boldsymbol{0}$, because once even if you make $\boldsymbol{B} = \boldsymbol{0}$ just by the fluctuations in $\boldsymbol{B}$, the statistical fluctuations which are inevitable in a realistic plasma will always tend to a condition, where you have a large scale sustaining magnetic field.

Now, let us say you have a magnetic field, but inside an astronomical body, what happens that the plasma motion is very very negligible. So, you can practically claim that the plasma velocity is almost equal to 0. So, if the plasma velocity is 0, then the momentum evolution equation does not have any meaning and the only evolution equation for the magnetic field is nothing but the induction equation.

But again, the $\boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B})$ term would vanish because $\boldsymbol{v} = \boldsymbol{0}$. Then, effectively, your magnetic field which is now created by some instability, from some seed magnetic field would simply diffuse with time, as

$$\frac{\partial \boldsymbol{B}}{\partial t} = \eta \boldsymbol{\nabla}^2 \boldsymbol{B}.$$

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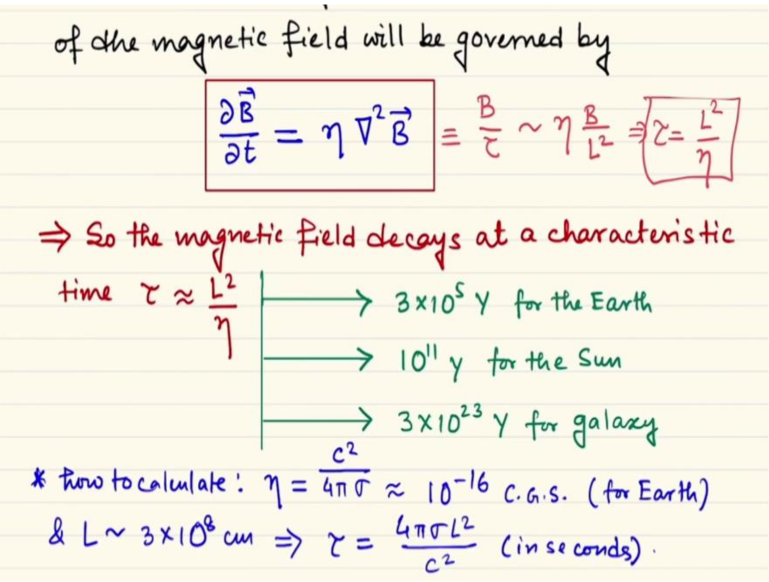

And the characteristic time of that diffusion can be simply calculated from the diffusion equation. So, you can see here, from this equation, you can write simply write that

$$\frac{B}{\tau} \sim \eta \frac{B}{L^2}.$$

So, then $\tau = \frac{L^2}{\eta}$, where η is the magnetic diffusivity.

And then, if you calculate this thing using certain process, which I am now going to describe, first the way of calculating η. So, η is related to the conductivity, since we are all talking about the standard practice in astronomy, we express the quantities in C.G.S units. So, if you know the conductivity (σ) in C.G.S unit, then $\eta = \frac{c^2}{4\pi\sigma}$.

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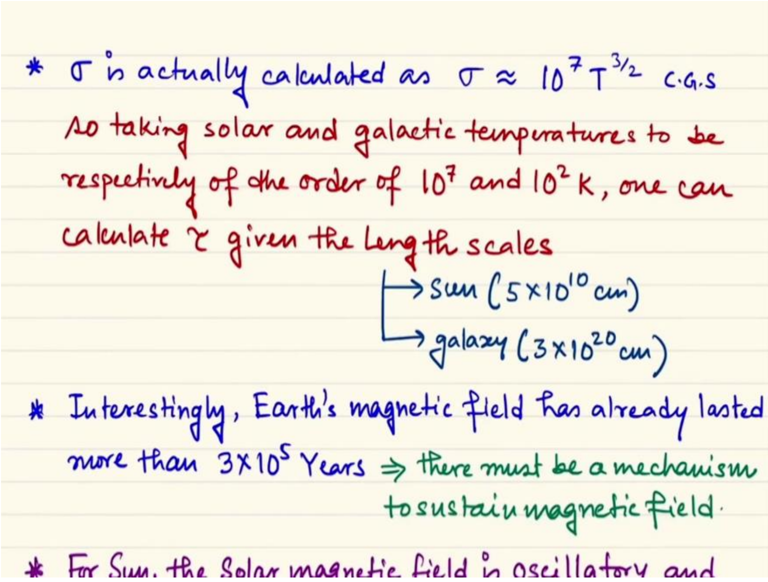

And this conductivity can actually be calculated as in C.G.S, $\sigma \approx 10^7 T^{\frac{3}{2}}$, T is the temperature of that medium. So, now if we know the core temperature of the Earth or the temperature of the Earth's core or the temperature of the Earth's core or the temperature of the solar plasma or the temperature of the galaxy, then we are done.

Now, taking for example, solar and galactic temperatures to be respectively of the order of 10^7 and 10^2, we can calculate the corresponding σ. Now, it is a very small task for you, what can be the temperature of Earth's core or the Earth's central part in order that σ which we know for Earth is giving something which is 10^{16} e.s.u.

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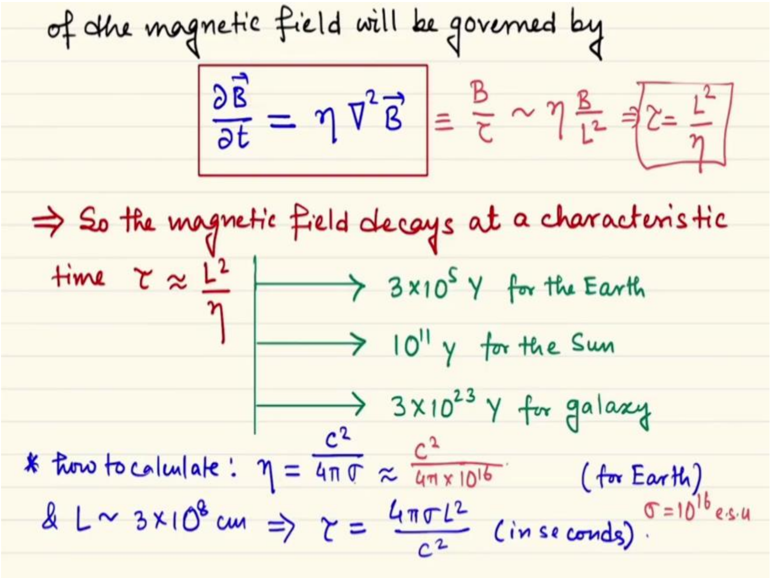

Because σ value for Earth is 10^{16} e.s.u. So, now, we use the formula η and thereby, getting η to give us such a value which leads to $\tau = 3 \times 10^5$ years, if you just take the L, the length scale to be of the order of 3×10^8 centimeters, that is nearly 3000 kilometers which is roughly of the half of the radius of the Earth actually.

And finally, you will have the τ or the relaxation time or the characteristic decay time for the magnetic field, 3×10^5 years. Now, this is of the order of 10^{11} years for the Sun and this is 3×10^{23} years for the galaxy, actually the spiral galaxies.

Now, these three characteristic times are very important because depending on that now we will discuss some specialties or some facts about all these three types of magnetic fields. Just for information that when you will try to calculate τ for Sun and galaxy, just take their corresponding length scales as 5×10^{10} centimeters and for galaxy 3×10^{20} centimeters, this is a small home task for you to calculate the τ for Sun and the galaxy and to check whether we agree or not. Of course, that should be in agreement.

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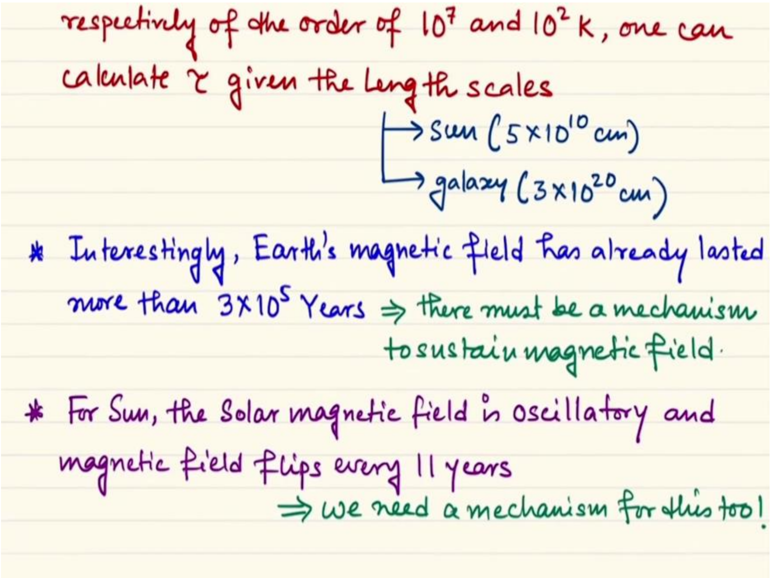

Now, interestingly, a very strange fact for Earth's magnetic field is that the Earth magnetic field has already lasted which can be done in other ways, that I am not entering now, has lasted for more than 3×10^5 years with a very considerable value. So, this is not really following the diffusion equation then.

So, there is something in addition to this, which actually counter balances this diffusion or the decay by diffusive effect. So, there must be some mechanism to sustain the terrestrial magnetic field. So, that is first necessity. Now, for Sun, the problem is that there is no problem with the decaying time, because the Sun's corresponding characteristic decay time is 10^{11} years and the Sun's magnetic field is well within.

But the problem is that the Sun's magnetic field is oscillatory and it actually flips every 11 years, whereas its diffusive decay time is 10^{11} years. So, we have to also find some mechanism in order to explain the oscillatory nature of the solar magnetic field.

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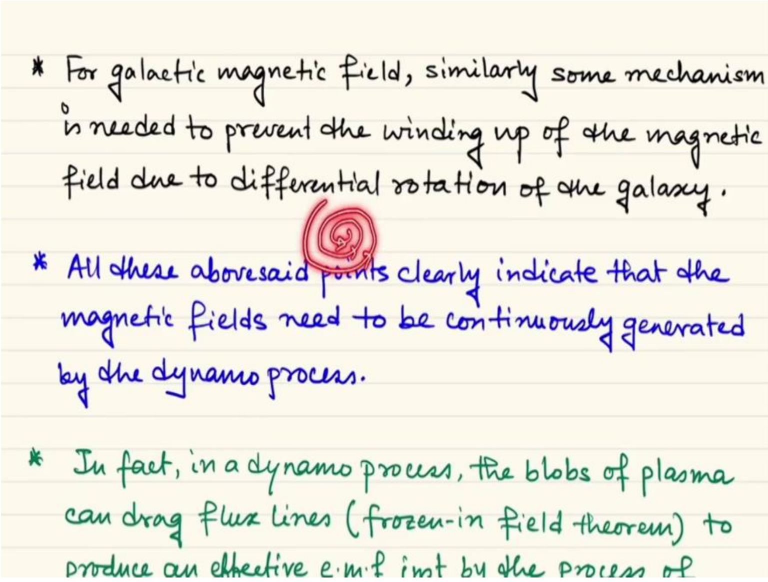

Finally, for galactic magnetic field, similarly some mechanism is needed to prevent the winding up of the magnetic fields due to the differential rotation of the galaxy. Now, for the galaxy diffusive decay is not a problem because 3×10^{23} years is much larger than the age of the universe.

So, there is no doubt that the galactic magnetic field cannot decay by present time just by diffusion, but in spiral galaxies there is considerable differential rotation which can wind the magnetic lines of force and thereby, destroying them and that is not possible if we have a mechanism which sustain the magnetic field.

So that the magnetic fields of the spiral galaxies are actually always acting along the spirals and that is a very famous thing, you can actually see several images that for a spiral galaxy, the magnetic fields are actually directed along the arms of the spiral.

So, all these above said points finally, indicate that magnetic fields need to be continuously generated by a process and which is likely to be the dynamo process.

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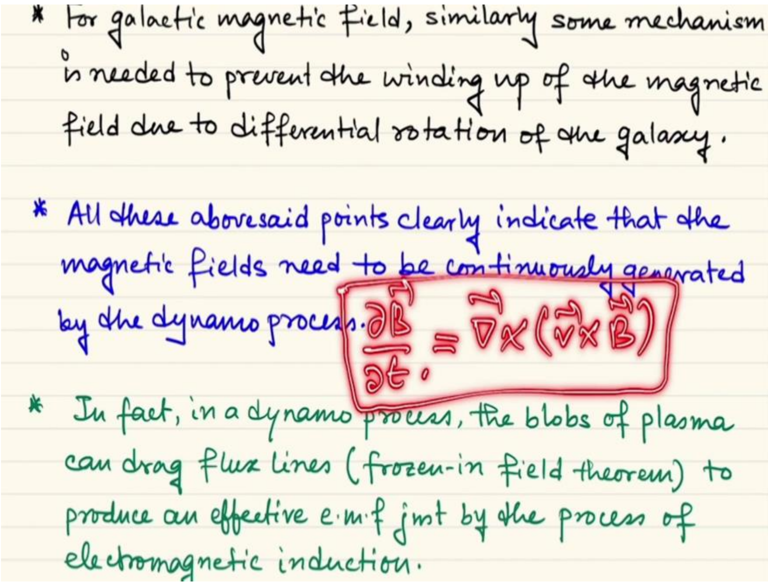

Now, it is true that now we come to this question that why this is called dynamo? Because when we are talking about a dynamo process, we are talking about plasma and the blobs of plasma, in case where you have the dissipative effect to be negligible with respect to the $\nabla \times (\boldsymbol{v} \times \boldsymbol{B})$ term.

Then you can actually write the effective induction equation like $\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B})$ and then, you have your magnetic frozen-in theorem for magnetic field to be approximately true.

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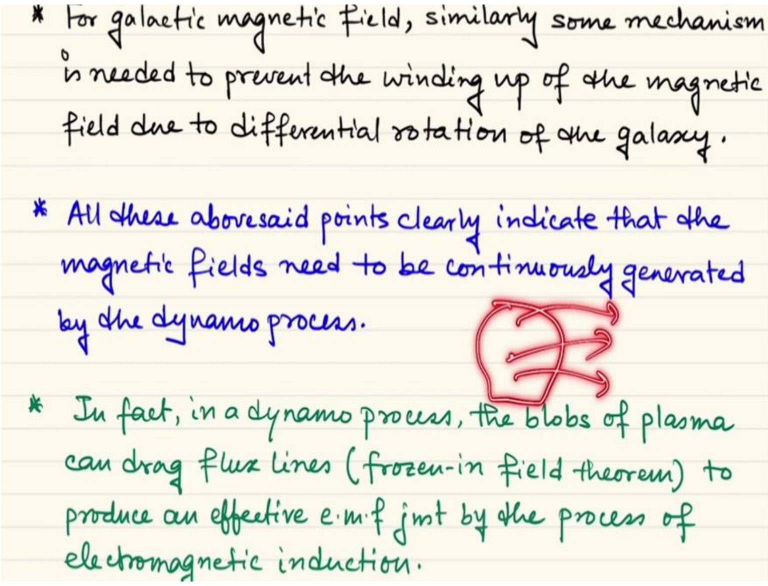

And if this is true, then this simply says that the blob of plasma has some velocity lines, because this is moving and the this simply drags the magnetic field lines. Due to this frozen-in field theorem because the magnetic field lines are frozen in the plasma to produce an effective electromotive force because of the convective motions or irregular motions.

So, you can actually see that in various situations, where you can have a situation of changing electromagnetic flux and that creates an induced e.m.f just by the process of electromagnetic induction and that is the basis of calling this whole process as a dynamo process.

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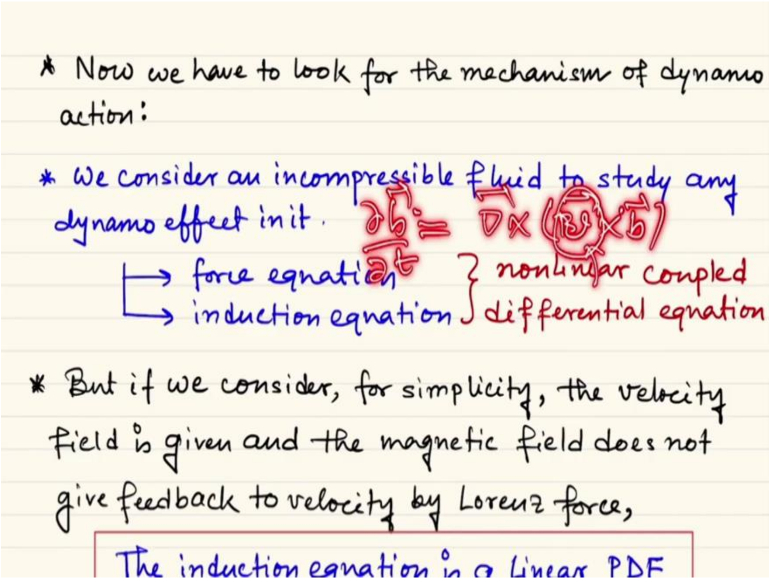

Now, we have to look for the mechanism of the dynamo action. Of course, we consider for simplicity an incompressible fluid because compressibility always makes the life a bit complicated. So, for incompressible fluid, we will just see that how in a very simplistic way, we can study the dynamo effect in it.

That means, once some considerable magnetic field is generated from some seed field, how can the system sustain it and we have two equations in hand, force equation and the induction equation. Because for incompressible fluid, we do not care about divergence of $\boldsymbol{v}$, divergence of $\boldsymbol{B}$, which equal to 0 and then we also do not care about the energy equation.

But these equations are also not easy to be solved because they are non-linear coupled differential equation, as I just said maybe in the last lecture that in general the induction equation has this term $\nabla \times (\boldsymbol{v} \times \boldsymbol{B})$ which is equal to $\frac{\partial \boldsymbol{B}}{\partial t}$. So, although it may look like this is linear in $\boldsymbol{B}$. But $\boldsymbol{v}$ has a feedback of $\boldsymbol{B}$ and $\boldsymbol{B}$ has a feedback of $\boldsymbol{v}$. So, that is why the system is actually non-linear.

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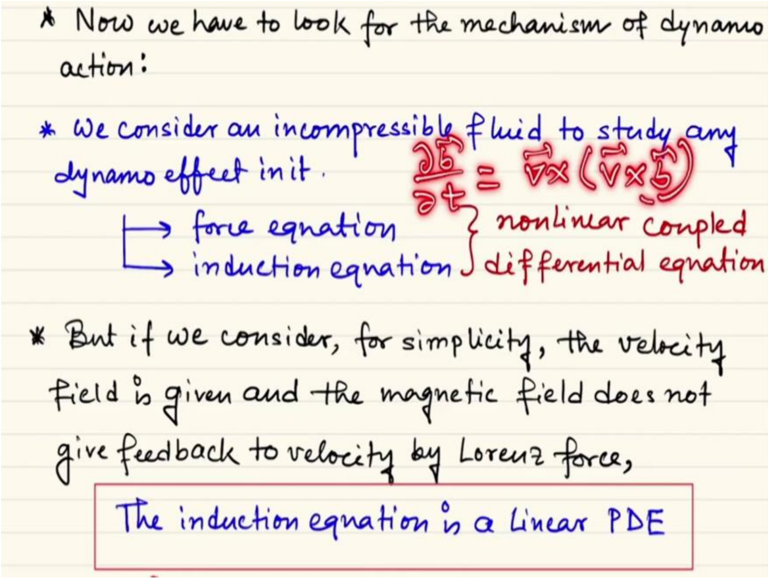

But if we consider for the time being to start with a velocity field is given by some process, which we are not interested in and the magnetic field is just affected by that velocity field, and the magnetic fields evolution does not give any feedback to the velocity field, then the induction equation is effectively linear in $\boldsymbol{B}$ and then it is much more easy to solve.

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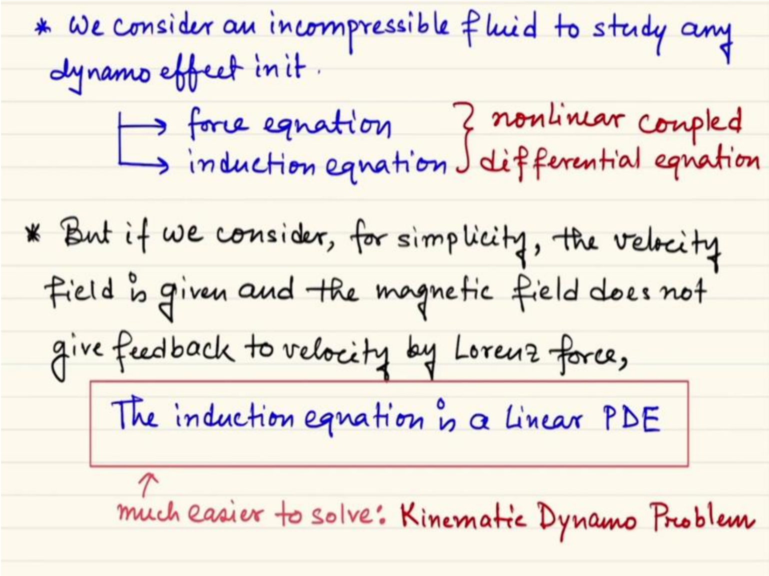

And so, then this induction equation is known as the induction equation for a kinematic dynamo. The corresponding problem is known as a kinematic dynamo problem.

Now, for historical reasons and for the sake of simplicity, we in general talk for academic purpose in almost all the time in general in the introductory courses at least the kinematic dynamo problem, where the induction equation is totally decoupled from the momentum equation and the magnetic field does not give any feedback to the velocity through the Lorentz force.

So, in the next discussion we will discuss very briefly one theorem and one qualitative idea of turbulent dynamo to see how in a very simplistic way, we can try to solve the problem. Of course, this will be also done in a very superficial manner.

Thank you very much.