Introduction to Astrophysical Fluids
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Lecture - 55
Turbulence in MHD fluids

Hello and welcome to another lecture session of Introduction to Astrophysical Fluids. In this lecture, we will discuss a very interesting topic that is the Turbulence in MHD fluids. So, MHD or magnetohydrodynamics is the monofluid model of a plasma. So, it is true that in astrophysical or space physical fluids we always see clear signature of turbulent motion.

Now, how to categorize this or how to classify this or actually how to perform a quantitative study of this type of turbulence is a great question and this question is not easy to answer. So, depending on our interests in different frameworks we would use different type of models. Sometimes we are happy with the hydrodynamic turbulence model, sometimes we have to take the plasma property into account. Sometimes just the monofluid model of MHD is sufficient, sometimes we have to go beyond.

Then some sophisticated plasma fluid models and MHD is the simplest plasma fluid model, as you know that the plasma fluids can be also described in the framework of multi fluids. So, this type of turbulence can also be very much interesting but of course, given our interest okay.

So, if certain questions are still not answered then even, we have to go beyond. So, we have to take the kinetic effects into account but here, just for this course or just for, I mean just to initiate you to the turbulence in plasma, we will simply give you a very introductory overview of MHD turbulence, also very qualitatively. I will just enter into some mathematical details whenever it is necessary but once again the mathematical rigor is not really followed over here.

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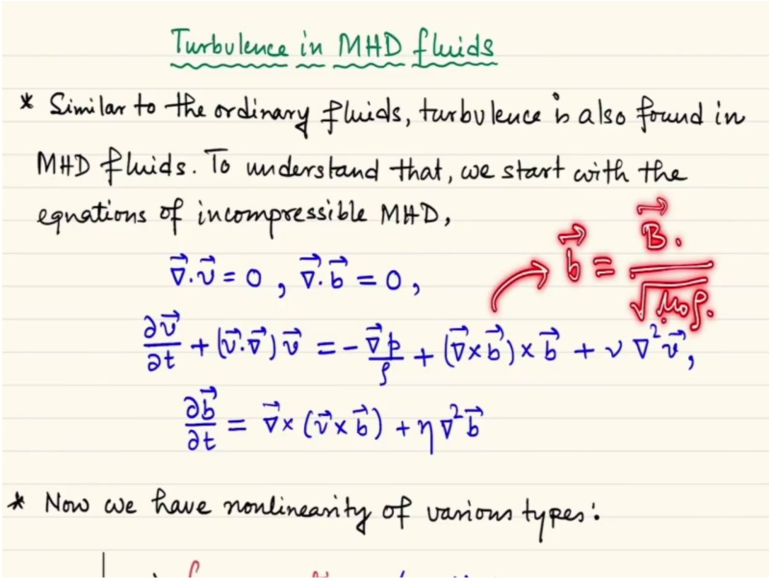

So, just as I said that similar to ordinary hydrodynamic fluids, turbulence is also found in MHD fluids also. Actually, to be very honest in plasmas, one of the easiest ways to model this plasma turbulence is through the turbulence in MHD fluids because we know the MHD equations.

So, how to understand the features of in MHD turbulence? we have to start from the basic equations of a MHD, again as we did in the case of hydrodynamic fluid, we will just consider an incompressible fluid. Because incompressible fluid turbulence is much easier to handle and much easier to see what is happening; the phenomenology is much easier or much more concrete to discuss okay.

So, in this framework, we will only be constraining ourselves into the discussion of incompressible hydrodynamic and MHD fluids. Although for your information, many cases in astrophysics, we have to consider compressible MHD turbulence or compressible hydrodynamic turbulence. Now, just to start with the incompressible MHD equations,

$$\boldsymbol{\nabla}.\boldsymbol{v} = 0\,, \boldsymbol{\nabla}.\boldsymbol{b} = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}.\boldsymbol{\nabla})\boldsymbol{v} = -\frac{\boldsymbol{\nabla} p}{\rho} + (\boldsymbol{\nabla}\times\boldsymbol{b}\,)\times\boldsymbol{b} + \nu\nabla^2\boldsymbol{v} \tag{2}$$

$$\frac{\partial \boldsymbol{b}}{\partial t} = \boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{b}\,) + \eta\nabla^2\boldsymbol{b} \tag{3}$$

The divergence of $\boldsymbol{v}$ and divergence of $\boldsymbol{b}$ both vanish. If you remember $\boldsymbol{b}$ is nothing but the magnetic field normalized to a velocity well this is given by

$$\boldsymbol{b} = \frac{\boldsymbol{B}}{\sqrt{\mu_0 \rho_0}} \tag{4}$$

ρ_0 is the constant density of the incompressible fluid okay. As you know that for ideal MHD, of course we just need this but here we are talking about turbulence in MHD fluids so, we do not need an ideal MHD equation but this is the normal resistive MHD equation. Whenever we have done this, we say the corresponding MHD to be the resistive MHD.

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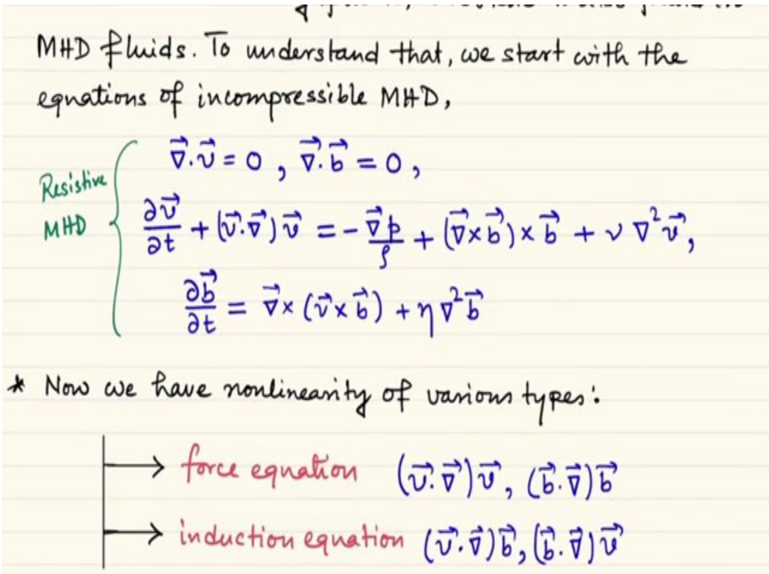

So, these are the equations of resistive MHD (equation (1)-(3)). Now, if you remember the normal hydrodynamic Navier-Stokes equation then the non-linear term which was present was this one $(\boldsymbol{v}.\boldsymbol{\nabla})\boldsymbol{v}$ and viscous term was this one $\nu\nabla^2\boldsymbol{v}$. Now, in this case we have a lot of terms and certain terms are actually subjected to give the non-linearity.

So, here the non-linearity can be of various types. So, in the force equation or the momentum evolution equation you have the non-linear terms like $(\boldsymbol{v}.\boldsymbol{\nabla})\boldsymbol{v}$ and from here $(\boldsymbol{\nabla} \times \boldsymbol{b}\,) \times \boldsymbol{b}$ if you remember, that there will be a term of magnetic tension which should look like this $(\boldsymbol{b}.\boldsymbol{\nabla})\boldsymbol{b}$

and this is also non-linear. So, that one $(\boldsymbol{v}.\nabla)\boldsymbol{v}$ is non-linear in $\boldsymbol{v}$ and this was $(\boldsymbol{b}.\nabla)\boldsymbol{b}$ is non-linear in $\boldsymbol{b}$.

Then comes the induction equation (3), so induction equation; in true sense; it is linear in $\boldsymbol{b}$ only when $\boldsymbol{v}$ has no feedback on $\boldsymbol{b}$ but for magnetohydrodynamic turbulence, for an MHD plasma $\boldsymbol{b}$ and $\boldsymbol{v}$ they affect each other. So, we cannot say that this is simply a linear equation in $\boldsymbol{b}$. So, actually $(\boldsymbol{v}.\nabla)\boldsymbol{b}$ and $(\boldsymbol{b}.\nabla)\boldsymbol{v}$ both are non-linear terms.

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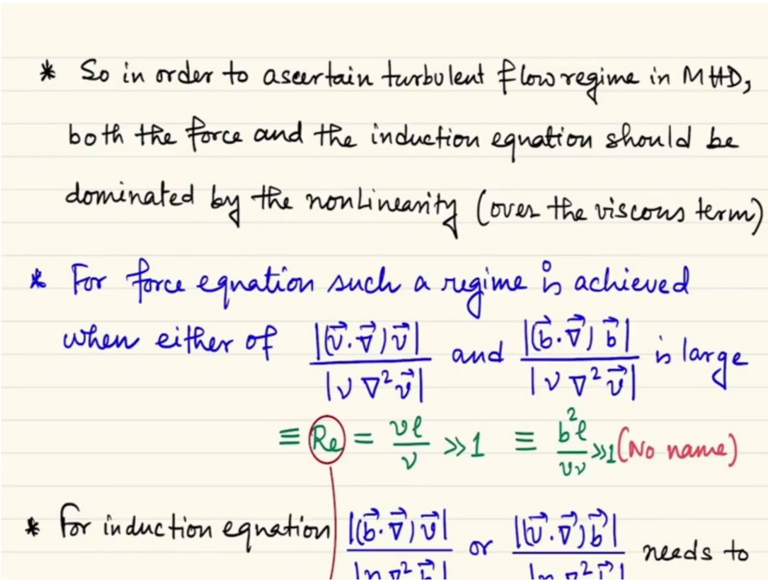

So, we have now all this type of non-linear terms and we know that in order to ascertain turbulent flow regime in case of an MHD fluid of course, both the force and the induction equation must be dominated by the non-linearity and this is the normal necessity that every equation should be dominated by the non-linearity.

Now, for force equation such a regime is achieved by having the Reynolds number we with a very large value. So, Reynolds number is the ratio of the kinetic non-linear term to the kinetic dissipation term.

$$Re = \frac{|(\boldsymbol{v}.\nabla)\boldsymbol{v}|}{|\nu\nabla^2\boldsymbol{v}|} \equiv \frac{vl}{\nu} \gg 1 \qquad (5)$$

There is another term which is the magnetic tension ($(\boldsymbol{b}.\nabla)\boldsymbol{b}$) by the kinetic dissipation term ($\nu\nabla^2\boldsymbol{v}$), that also should be large i.e., $\frac{|(\boldsymbol{b}.\nabla)\boldsymbol{b}|}{|\nu\nabla^2\boldsymbol{v}|} \gg 1$, but this term is not in general very much discussed only because if we say the fluid is very fast moving, then already this term $\frac{|(\boldsymbol{v}.\nabla)\boldsymbol{v}|}{|\nu\nabla^2\boldsymbol{v}|}$ is having a large Reynolds number and this basically makes the whole the momentum evolution equation dominated by the non-linearity. So, we really do not need to have a large value for this $\frac{|(\boldsymbol{b}.\nabla)\boldsymbol{b}\,|}{|\nu\nabla^2\boldsymbol{v}|}$; however, if your velocity is not very large, then you can have to have a look on this but then the problem is that if your velocity is not large, but your magnetic field is large enough then the problem is that velocity is in the denominator, so when it is small and magnetic field is let us say large, then there will be a non-trivial competition to look at and in certain cases that can be also interesting.

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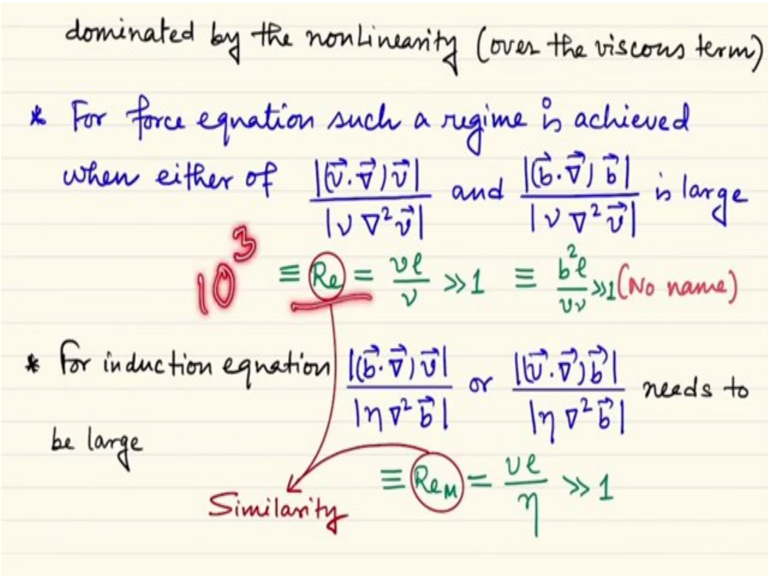

But here for our current case we will simply be talking about the large value of Reynolds number to be the necessary condition for the dominance of non-linearity in the force equation.

For induction equation, we have two types of non-linear terms as we have discussed, $(\boldsymbol{b}.\nabla)\boldsymbol{v}$ and $(\boldsymbol{v}.\nabla)\boldsymbol{b}$ and the ratio of any of the two things to the corresponding magnetic diffusion or magnetic viscosity term; magnetic viscosity is a very unusual term but magnetic diffusion term or the resistive term gives us another quantity; both of them, can lead us to some quantity which should look like this one

$$\frac{|(\boldsymbol{b}.\nabla)\boldsymbol{b}|}{|\boldsymbol{\eta}\nabla^2\boldsymbol{b}|}\ or\ \frac{|(\boldsymbol{v}.\nabla)\boldsymbol{b}|}{|\boldsymbol{\eta}\nabla^2\boldsymbol{b}|} \equiv Re_M = \frac{vl}{\eta} \qquad (6)$$

So, the numerator will exactly be the common Reynolds number but the dissipation or the viscosity coefficient will now be replaced by the magnetic diffusivity and this is known as magnetic Reynolds number. For the induction equation to be dominated by the non-linear effects the value of this magnetic Reynolds number should also be very greater than one, that means, it should have a very large value.

Now, at this point I have to make one point that it is true that this Reynolds number and magnetic Reynolds number they are introduced in our course as they are looking like just the parameters for deciding whether a fluid flow is turbulent or not. But, in general Reynolds number or even magnetic Reynolds number if you understand correctly the association, you will see that these two things are actually very important to represent or to manifest another property of fluid flow that is the similarity property.

Now just take the case of the normal Reynolds number, its expression is simply $\frac{vl}{\nu}$. Now, if the geometry of the flow field is same but the length scale is now twice and the velocity is also changed accordingly and ν is also changed accordingly. So, that finally, this ratio is simply having the same value as that of the previous value then one can simply say that the flows of the corresponding two system are actually similar in nature.

So, in some sense just by smartly choosing the velocity and the coefficient of viscosity you can actually change the large-scale dimension of the flow field but then the basic properties of the flow field will not be different, for example, the transition from laminar to turbulent flow. Suppose, when the flow field is basically 10 times larger, then just by choosing the corresponding velocity and the coefficient of viscosity, you can simulate or just by doing the inverse thing; that means, you can choose the length scale to be 10 times smaller but again by smart choice of velocity and the coefficient of viscosity, you can again simulate the fluid flow with the same type of property.

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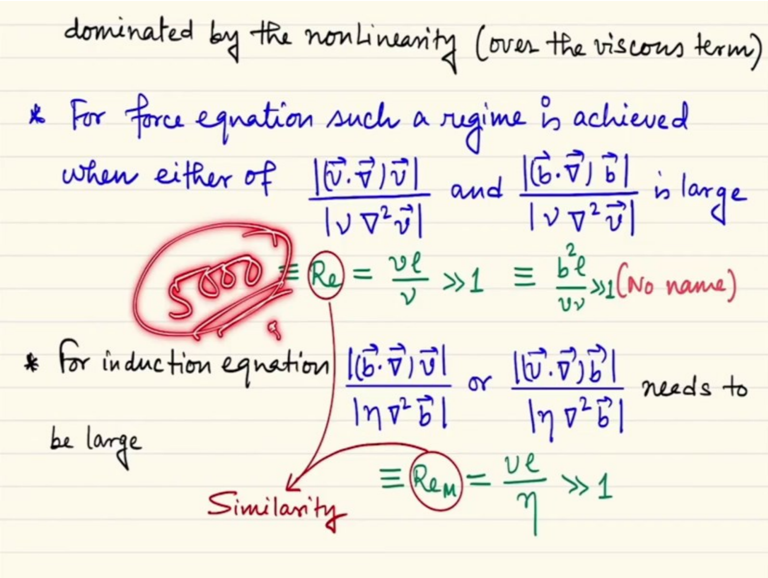

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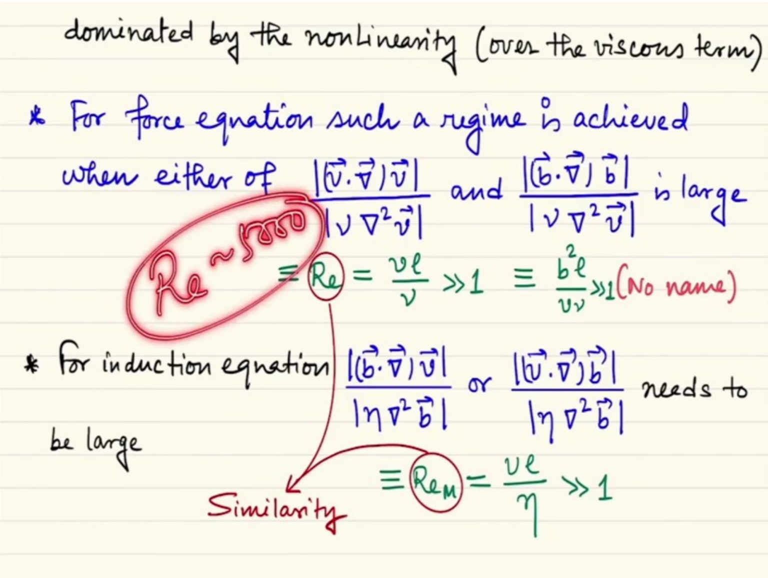

Then in that system you can produce turbulence directly from laminar flow, so the message is that the use of Reynolds number is very much useful that you can actually simulate a very big large astrophysical system or a galactic system just by smartly choosing or keeping the Reynolds number value intact.

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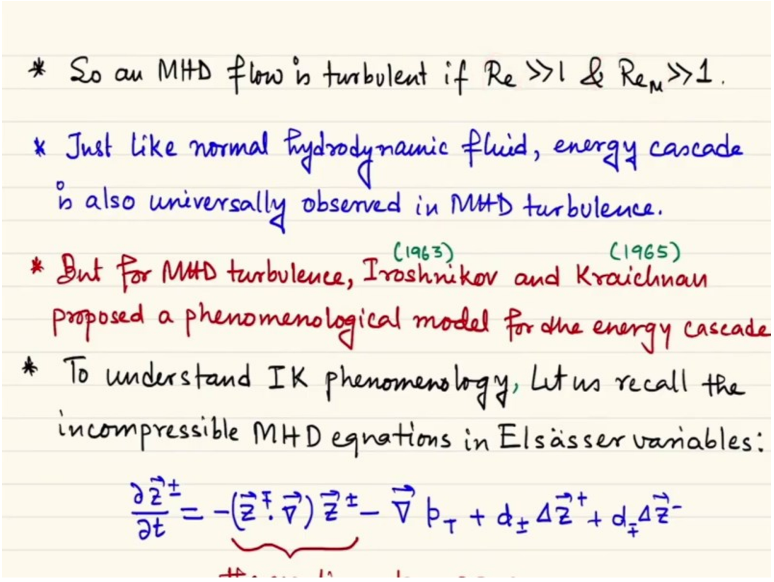

So now, coming to our main point that the MHD flow to be turbulent, the necessary condition is that the both the Reynolds number and the magnetic Reynolds number should be very greater than 1. Now, just like normal hydrodynamic fluid, energy cascade is also universally observed in MHD turbulence. Something which is already been studied in several research and also been founded in several observation or in experiments. Now, it is true that for hydrodynamic turbulence we had the phenomenology of Kolmogorov, so, Iroshnikov and Kraichnan they said in a different way which we will discuss here. So, to understand Iroshnikov and Kraichnan phenomenology, or IK phenomenology, let us recall the incompressible MHD equations in terms of Elsässer variables.

Just remember when I talked or discussed about Elsässer variables, I said that expressing the incompressible MHD equations in terms of Elsässer variables, can actually be useful when we will study turbulence in MHD fluids and that is exactly the case which we are now discussing.

So, to understand IK phenomenology, let us recall the incompressible MHD equations in terms of Elsässer variables. So,

$$\frac{\partial \boldsymbol{z}^{\pm}}{\partial t} = -(\boldsymbol{z}^{\mp}.\boldsymbol{\nabla})\boldsymbol{z}^{\pm} - \boldsymbol{\nabla} p_T + d_{\pm}\Delta\boldsymbol{z}^{+} + d_{\mp}\Delta\boldsymbol{z}^{-} \quad (7)$$

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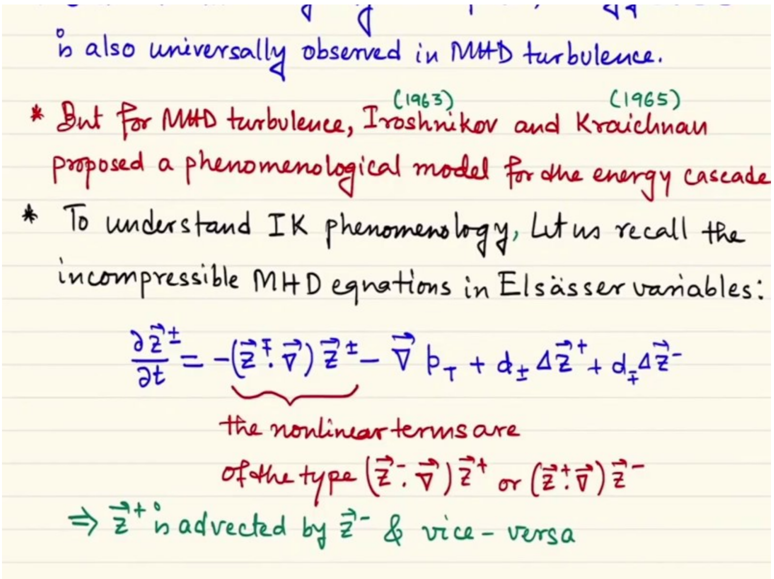

So, these two terms $d_{\pm}\Delta\boldsymbol{z}^{+}$ *and* $d_{\mp}\Delta\boldsymbol{z}^{-}$are the dissipative terms. So, for instance we are not really interested in those dissipative terms because in turbulence we are mostly interested in the non-linear terms which are actually dominating than dissipating terms.

So, here you again see, in case you have forgotten that the non-linear terms are of the type of $(\boldsymbol{z}^{-}.\boldsymbol{\nabla})\boldsymbol{z}^{+}$or $(\boldsymbol{z}^{+}.\boldsymbol{\nabla})\boldsymbol{z}^{-}$, but there are no such terms like $(\boldsymbol{z}^{+}.\boldsymbol{\nabla})\boldsymbol{z}^{+}$or $(\boldsymbol{z}^{-}.\boldsymbol{\nabla})\boldsymbol{z}^{-}$. So, they have a cross type of non-linear terms. So, it simply says that $\boldsymbol{z}^{+}$ is advected by $\boldsymbol{z}^{-}$ and vice versa.

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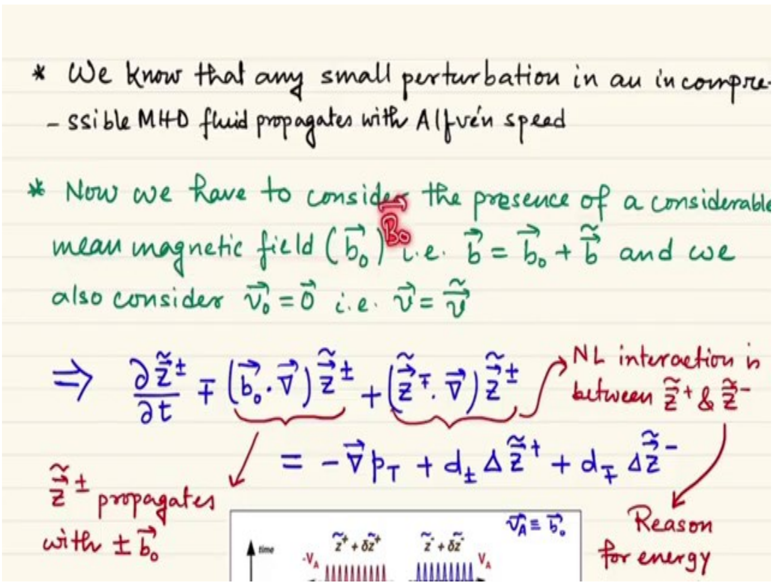

Now, we know that any small perturbation (first order perturbation); when we talked about the wave modes in MHD; propagates with the speed which is equal to the Alfvén speed and Alfvén speed is corresponding to the phase speed of the Alfvén mode, which is the only incompressible mode of an MHD fluid.

So, in normal hydrodynamic fluid we do not have any wave mode for incompressible fluid, but for incompressible MHD we have one mode which is called the Alfvén mode. Now, in this case we have to actually consider; in order to proceed; the presence of a considerable mean magnetic field $\boldsymbol{b_0}$. So, $\boldsymbol{b_0}$ is the $\boldsymbol{B_0}$ normalized by $\sqrt{(\mu_0\rho_0)}$. Then the total magnetic field ($\boldsymbol{b}$) can be written as the mean magnetic field ($\boldsymbol{b_0}$) or the background magnetic field plus the fluctuation with the tilde sign ($\widetilde{\boldsymbol{b}}$), i.e., $\boldsymbol{b} = \boldsymbol{b_0} + \widetilde{\boldsymbol{b}}$.

Also consider, for velocity, the background velocity ($\boldsymbol{v_0}$) is 0, so the velocity field is nothing but the fluctuation $\boldsymbol{v} = \widetilde{\boldsymbol{v}}$, that we can always do just by sitting to a reference frame which is moving with the mean velocity of the system and that is possible because the fluid equations are Galilean invariant okay.

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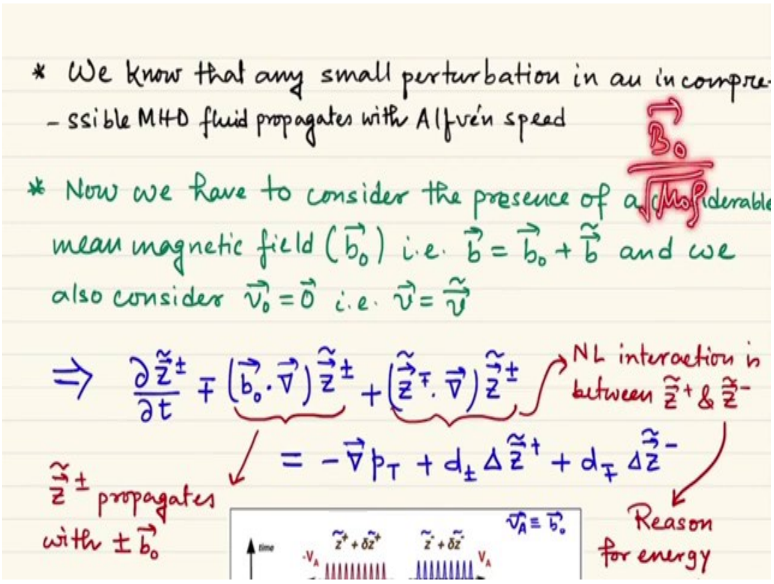

Now, if you just decompose every magnetic field as $\boldsymbol{b} = \boldsymbol{b_0} + \widetilde{\boldsymbol{b}}$, then actually you will be obtaining an equation which should look like equation

$$\frac{\partial \tilde{\boldsymbol{z}}^{\pm}}{\partial t} \mp (\boldsymbol{b_0}.\boldsymbol{\nabla})\tilde{\boldsymbol{z}}^{\pm} + (\tilde{\boldsymbol{z}}^{\mp}.\boldsymbol{\nabla})\tilde{\boldsymbol{z}}^{\pm} = -\boldsymbol{\nabla} p_T + d_{\pm}\Delta\tilde{\boldsymbol{z}}^{+} + d_{\mp}\Delta\tilde{\boldsymbol{z}}^{-} \qquad (\mathbf{8})$$

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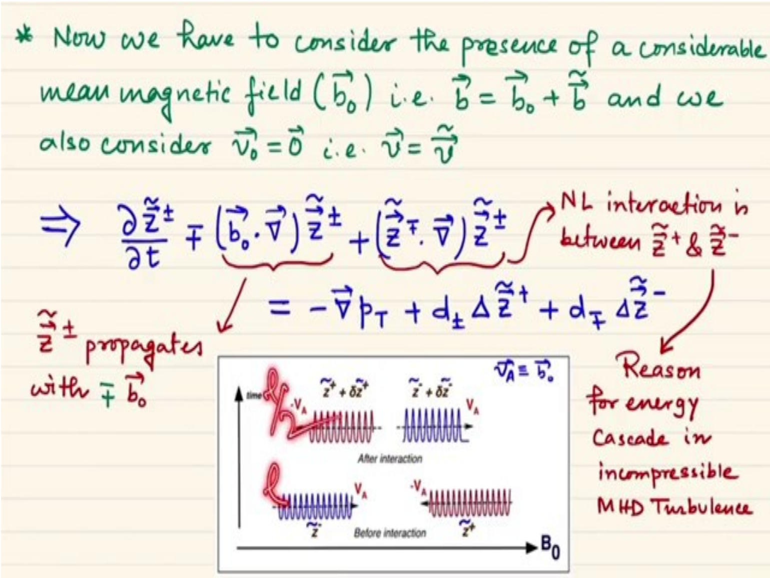

Now, there are two things, one thing is that what is the meaning of this term $(\boldsymbol{b_0}.\boldsymbol{\nabla})\tilde{\boldsymbol{z}}^{\pm}$? This term simply says that both the $\tilde{\boldsymbol{z}}^{+}$ and $\tilde{\boldsymbol{z}}^{-}$, they are advected with a speed which is equal to the Alfvén speed but in the opposite direction. So, $\tilde{\boldsymbol{z}}^{+}$ is advected in the negative direction. So, it actually moves with a $\boldsymbol{b_0}$ velocity. So, $\boldsymbol{b_0}$ is the equivalent to a mean magnetic field normalized to a velocity, but here in this case, this is also equal to the Alfvén speed.

So, $\tilde{\boldsymbol{z}}^{+}$ basically propagates with velocity $-\boldsymbol{b_0}$ and $\tilde{\boldsymbol{z}}^{-}$ propagates with a velocity $\boldsymbol{b_0}$. Then this one $(\tilde{\boldsymbol{z}}^{\mp}.\boldsymbol{\nabla})\tilde{\boldsymbol{z}}^{\pm}$ simply says; so, $(\boldsymbol{b_0}.\boldsymbol{\nabla})\tilde{\boldsymbol{z}}^{\pm}$ is not a non-linear thing because $\boldsymbol{b_0}$ is uniform background magnetic field and one $(\tilde{\boldsymbol{z}}^{\mp}.\boldsymbol{\nabla})\tilde{\boldsymbol{z}}^{\pm}$ gives us an account of the non-linear interaction of the perturbations; the perturbation in $\tilde{\boldsymbol{z}}^{+}$ is advected non-linearly by the perturbation of $\tilde{\boldsymbol{z}}^{-}$ and vice versa.

So, this non-linear interaction between $\tilde{\boldsymbol{z}}^{+}$ and $\tilde{\boldsymbol{z}}^{-}$ is basically the reason for energy cascade in incompressible MHD turbulence and that is the essence of Iroshnikov and Kraichnan phenomenology which simply says that if you have two wave packets, $\tilde{\boldsymbol{z}}^{-}$ and $\tilde{\boldsymbol{z}}^{+}$. So, $\tilde{\boldsymbol{z}}^{-}$ will move to the positive direction with some velocity $\boldsymbol{V_A}$ which is equivalent to our $\boldsymbol{b_0}$ okay.

So, I took this picture from my thesis where I use the Alfvén velocity to be equal to $\boldsymbol{V_A}$ and $\tilde{\boldsymbol{z}}^{+}$ would move with a velocity $-\boldsymbol{V_A}$, then just after interaction this perturbation $\tilde{\boldsymbol{z}}^{+}$ has now deformed to $\tilde{\boldsymbol{z}}^{+} + \boldsymbol{\delta}\tilde{\boldsymbol{z}}^{+}$ and the same thing will be for $\tilde{\boldsymbol{z}}^{-}$. Now, $\tilde{\boldsymbol{z}}^{+} + \boldsymbol{\delta}\tilde{\boldsymbol{z}}^{+}$ is still moving with a velocity $-\boldsymbol{V}_{\boldsymbol{A}}$ and $\tilde{\boldsymbol{z}}^{-} + \boldsymbol{\delta}\tilde{\boldsymbol{z}}^{-}$ is still moving with the velocity $\boldsymbol{V}_{\boldsymbol{A}}$ or maybe a slight change in velocity.

But at least they are deformed and this is exactly equivalent to the fragmentation of eddies from one size to the other. Because after deformation, these $(\tilde{\boldsymbol{z}}^{-} + \boldsymbol{\delta}\tilde{\boldsymbol{z}}^{-}, \tilde{\boldsymbol{z}}^{+} + \boldsymbol{\delta}\tilde{\boldsymbol{z}}^{+})$ are of different size and as people have seen that the energy cascades both in incompressible MHD and incompressible HD in 3D, energy cascades from larger scales to smaller scales then we can say that after deformation both of them get shorter okay.

So, according to Iroshnikov and Kraichnan phenomenology, energy is cascaded from larger to smaller length scales due to the non-linear interaction and subsequent deformation between two counter propagating Alfvén wave packets. So, $\tilde{\boldsymbol{z}}^{\pm}$, we just call them Alfvén wave packets because they are small perturbations moving with Alfvén speed. Now, Iroshnikov and Kraichnan, they considered a specific case of isotropic MHD turbulence with a considerable $\boldsymbol{b}_{\boldsymbol{0}}$ to obtain some spectral prediction like Kolmogorov's $-5/3$ prediction for MHD turbulence.

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* So, according to IK phenomenology, energy is cascaded from larger to smaller length scales due to the NL interaction and subsequent deformation between two counter propagating Alfvén wave packets.

* IK considered a specific case of isotropic MHD turbulence with a considerable $\vec{b}_0$ (Not really practical) and they also considered that for an adequate deformation of $\vec{z}^{\pm}$, a large number of collisions are required $\Rightarrow$ The characteristic time for NL

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interaction and subsequent deformation between two counterpropagating Alfvén wave packets.

* IK considered a specific case of isotropic MHD turbulence with a considerable $\vec{b}_0$ (Not really practical) and they also considered that for an adequate deformation of $\tilde{\vec{z}}^{\pm}$, a large number of collisions are required $\Rightarrow$ The characteristic time for NL transfer of energy is neither $\underbrace{l/\tilde{z}}_{\tau_l}$ nor $\frac{l}{b_0}$ but $\frac{\tau_l^2}{\tau_{Al}}$

$\Rightarrow$ $\boxed{E(k) \sim k^{-3/2}}$

So, they said that, our system has a considerable mean magnetic field but the turbulence is isotropic which is not really practical as you know that a strong mean magnetic field always like makes the system axisymmetric; that means, the systems property along the mean magnetic field and in the plane perpendicular to the mean magnetic field are quite different in general. They also considered that for an adequate deformation of $\tilde{\boldsymbol{z}}^{\pm}$, a large number of collisions are required. So, this is the fundamental difference between the hydrodynamic turbulence and MHD turbulence.

In hydrodynamic turbulence one eddy; so, the eddy got fragmented into two as I said, so the eddy gets elongated in one side and then in order to keep the incompressibility and it gets compressed in the perpendicular direction and then at one point it gets fragmented. But the fragmentation process is just once. So, this is a one soht process.

On the other hand, here they considered that one collision between $\tilde{\boldsymbol{z}}^{-}$ and $\tilde{\boldsymbol{z}}^{+}$ is not sufficient for the energy cascade and for that they assumed that actually a large number of collisions is necessary and then what happened after some very heuristic arguments one can show that the characteristic time for the non-linear transfer is neither $\frac{l}{\tilde{z}}$ $(= \tau_l)$; which is the case for normal hydrodynamics, here it should be $\frac{l}{\tilde{z}^{\pm}}$ whatever nor $\frac{l}{b_0}(= \tau_{Al})$ because $\boldsymbol{b_0}$ is another characteristic velocity in this system, but $\frac{\tau_l^2}{\tau_{Al}}$.

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* IK considered a specific case of isotropic MHD turbulence with a considerable $\vec{b}_0$ (Not really practical) and they also considered that for an adequate deformation of $\tilde{z}^{\pm}$, a large number of collisions are required $\Rightarrow$ The characteristic time for NL transfer of energy is neither $\underbrace{l/\tilde{z}}_{\tau_l}$ nor $\underbrace{\frac{l}{b_0}}_{\tau_{Al}}$ but $\frac{\tau_l^2}{\tau_{Al}}$

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So, if you call this $\frac{l}{\tilde{z}}$ as τ_l and you call this $\frac{l}{b_0}$ as τ_{Al}, then the resultant transfer time from one scale to the other is actually $\frac{\tau_l^2}{\tau_{Al}}$. This simply leads to another spectral prediction which is for isotropic MHD turbulence is not $-\frac{5}{3}$ but $E(k)$ should then scale as $k^{-\frac{3}{2}}$.

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* However, IK phenomenology does not quite represent the reality as all the incompressible MHD flows become anisotropic or rather axisymmetric in the presence of a considerable $\vec{b}_0$

* In fact, one can argue that, in the presence of $\vec{b}_0$, MHD turbulence follows Kolmogorov phenomenology in the plane perpendicular to $\vec{b}_0$.

* In Solar wind, a $-5/3$ spectrum is often found for

However, IK phenomenology does not quite represent the reality as I just said as all the incompressible MHD flows become anisotropic or rather axisymmetric in the presence of a considerable mean magnetic field.

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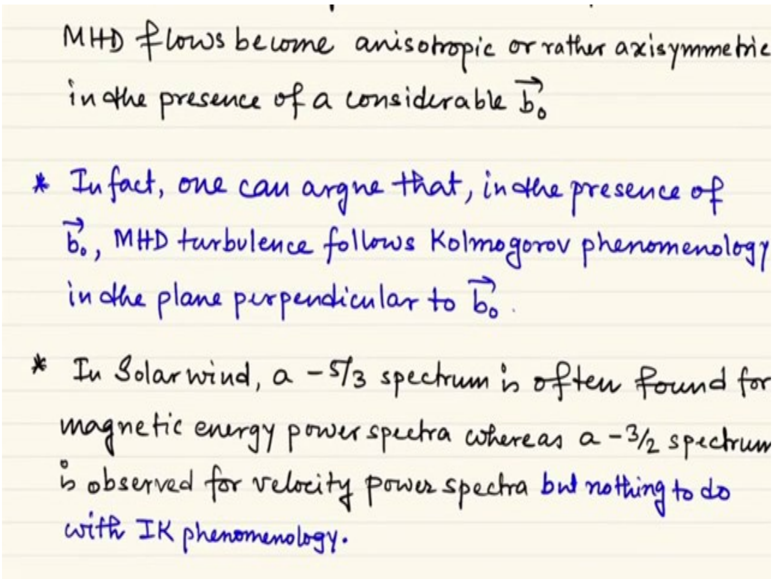

And in fact, one can argue that, in the presence of $\boldsymbol{b_0}$, MHD turbulence follows Kolmogorov phenomenology in the plane perpendicular to the $\boldsymbol{b_0}$. So, in the plane perpendicular to the $\boldsymbol{b_0}$, one actually can expect a $-\frac{5}{3}$ spectrum rather than a $-\frac{3}{2}$ spectrum.

Now, in solar wind a $-\frac{5}{3}$ spectrum is often found for magnetic energy power spectra; that is just for your information; and a minus $-\frac{3}{2}$ spectrum is observed for velocity power spectra but that can actually look like the velocity power spectra follows maybe the IK phenomenology, well, nothing to do with that they have some plausible explanation but it is true that this $-\frac{3}{2}$ spectrum has nothing to do with IK phenomenology okay. Now, finally, what I am trying to emphasize that whatever your $-\frac{5}{3}$ or $-\frac{3}{2}$, in practice when you do some simulation or some observational analysis most of the cases your spectral indices are in such a range that whether this is $-\frac{5}{3}$ or $-\frac{3}{2}$ is not really easy to understand. Actually, there are several variations, most of the cases you can see that it can be -1.55 sometimes it can be -1.6 and then the whole range of variation can lie within the range of errors. Finally, for the other scales which are lower or smaller than the normal MHD scales, people actually use much more sophisticated plasma fluid models or even kinetic models to describe or to explain the corresponding spectral slopes.

And finally, last but not the least, in case of describing various type of spectra, power spectra, energy power spectra or kinetic energy power spectra, magnetic energy power spectra or the total energy power spectra, actually in the case of astrophysics and even for the solar wind, it is seen that sometimes the consideration of the compressibility that is the fluctuation of density is of prior importance as well. I personally interested in the in studying the compressible effects in space and astrophysical plasma turbulence okay.

So, this was I mean somehow a very brief overview of turbulence in normal fluids and plasmas. Just to tell you once again that turbulence is something which is of potential importance in explaining several phenomena starting from the star formation. I have not really attacked that point because the role of turbulence in star formation; how it regulates the star formation is totally possible, if we try to if we introduce the compressible turbulence; but, in this framework of incompressible turbulence what I was just tried to convey that starting from solar wind plasma there are magnetospheric plasmas, all these plasmas give a clear signature of turbulence and according to our interest and the concerned length scale we try to study using different type of turbulent models, I mean fluid models okay. when we are interested once again in the length scales which are much larger than the ion inertial length scale then we try to feed this in the framework of MHD turbulence okay.

Thank you very much.