Introduction to Astrophysical Fluids

Prof. Supratik Banerjee
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 53
Turbulent diffusion

Hello and welcome to another lecture session of Introduction to Astrophysical Fluids. This week is going to be the 12th and the last week of this course and I really hope that you have been enjoying the course throughout. Of course, the course consisted of many new and very interesting topics of astrophysics and space physics, but it also consisted of some non-trivial mathematical techniques.

Sometimes I followed step by step derivations and sometimes I just deliberately skip those, but the main objective of this course was to give you an exposure to the ongoing research on this domain. That means the astrophysics and of corresponding fluids, I mean starting from the solar physics, planetary physics, stellar physics etcetera okay.

We did not really cover planetology part, but at least the solar physics and the other stellar bodies, we tried to cover as much as possible, although most of the time in a superficial manner. In this lecture we will continue our discussion on turbulence in fluids. Still, we will just talk about normal hydrodynamic fluid turbulence and also for the incompressible fluids.

So, in the previous lecture, we already introduced the definition of turbulence. Practically, how to characterize the turbulent flow, the corresponding important concepts like the energy cascade, the universality, Kolmogorov's minus five-third law, exact relations these types of things.

So, today we will mostly discuss a very practically important topic that is the turbulent diffusion. Why this topic is important both for everyday life and for astrophysics? We will discuss when I will elaborate several aspects of this.

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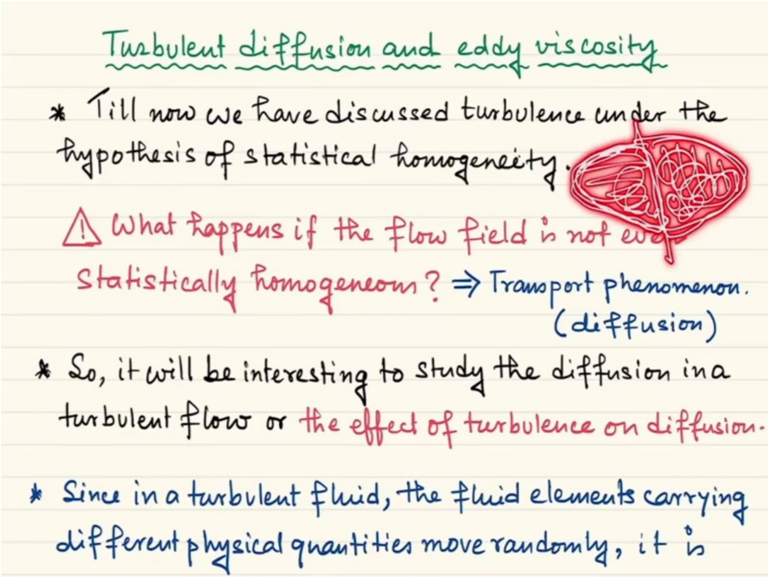

So, if you really followed the previous discussion on turbulence, you will easily understand that till now our basic assumption was that the turbulent flow is considered to be in a field which is statistically homogeneous and also isotropic. But here we have just taken for convenience the assumption of statistical homogeneity.

Now let us say, we have a system where even the statistical homogeneity cannot be applied; that means, the system has a very strong local gradient. For example, if let us say if you have a flow field in which this part and this part, they cannot be compared with each other.

So, in that case you can say that the turbulence over here cannot be just put with the turbulence on here like in a global homogenous framework right. So, let us say then you have to take the sub domains like this and this and within each sub domain you can think of homogeneous turbulence.

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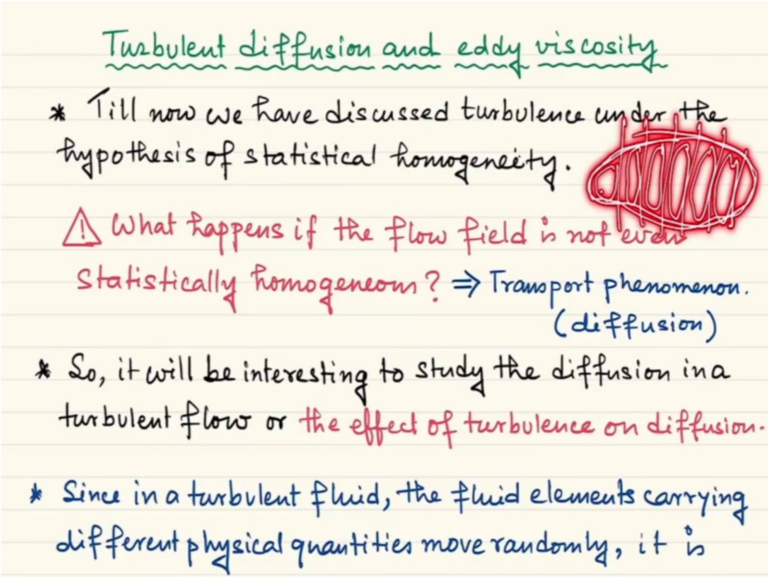

Now, let us say instead of just these two types of regions, you have several regions and then actually for every sub domain you can still apply the assumption of statistical homogeneity. But the only thing is that if you consider the system globally, you should understand one thing that this type of system should permit transport phenomena because there is a gradient.

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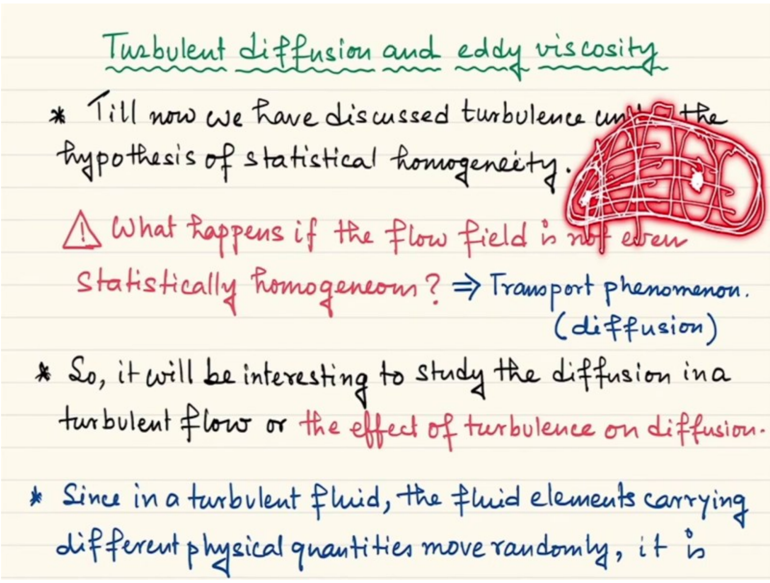

So, for example, one simple gradient is the gradient of density. Let us say your system is such that here the concentration of some fluid is very high and the concentration of the fluid is a bit lower here, a bit lower and here this is very low and here this is almost nothing. That is because maybe in this point you have a source of the fluid. So, it is true that after some certain time, there can have some homogeneity. But let us say you have a sink at this position, then although from the source the fluid is coming, this fluid will again try to vanish by this sink at that point and that is why; to be very honest; in your system there will always be a steady gradient of the density of the fluid.

Now in this type of situation what you can really expect from your previous knowledge of transport phenomena? You can expect some phenomena like diffusion and this will be then very interesting to study the diffusion in a turbulent flow. So, I mean we are now assuming that the flow is turbulent because the Reynolds number is very high. So, the non-linearity in the flow field is totally dominating and in addition, we have a transport phenomenon like diffusion.

So, in this case, we have to consider two things side by side; diffusion and turbulence. So, basically, we are interested to study the effect of turbulence on diffusion or you can say to study the diffusion in a turbulent flow.

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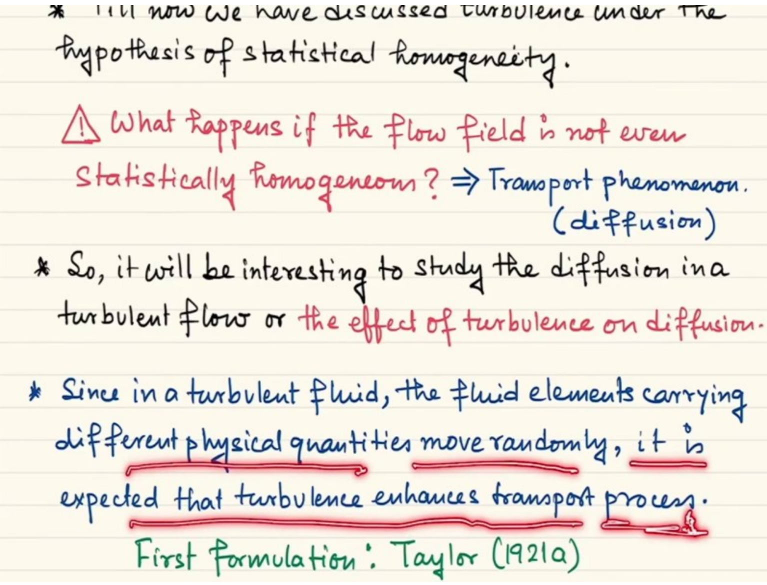

Now, it is true that since in a turbulent fluid, the fluid elements carrying different physical properties, physical quantities for example; velocity, density, pressure whatever move randomly; then it is somehow expected that turbulence enhances transport process.

Now, of course, this is true that turbulence will help in mixing right! So, just take the very famous example of adding some sugar or milk in a cup of coffee or tea. Let us say you just added the milk drop by drop and you didn't stir or you added some sugar in the cup of coffee or tea and you did not stir. Of course, they will mix, but they will mix by normal diffusion process and that is a very slow process.

What you in general do? We stir the system by a spoon or a stirrer right, then it gets mixed. So, this is something very intuitive that turbulence can actually make an efficient mixing that is only because for a good mixing what you have to have? You have to have a very randomized motion of the fluid blobs or fluid packets and turbulence exactly does that because of the non-linearity.

Now; well, it is easy to say in words what turbulence does for efficient mixing but for the first time; the formal formulation was given by Taylor in the year of 1920, is the same Taylor who talked about the importance of two point correlations in turbulence.

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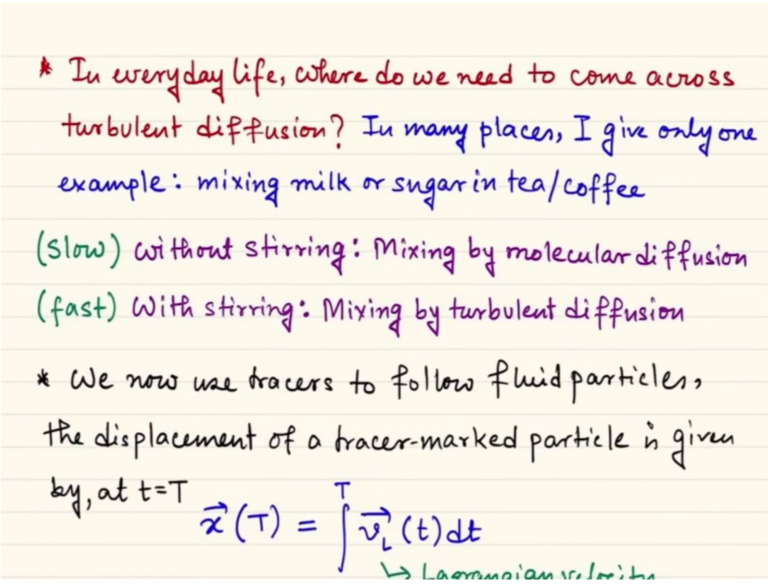

So, in a sense Taylor can be thought to be the father of the so-called modern turbulence or modern fluid mechanics. Now, as I just said that in everyday life, we need this turbulent diffusion for the mixing of sugar in tea or coffee and what happens that without stirring the process, mixing is very slow by molecular diffusion; may take hours or days, but would you wait for that?

Of course, no you need some fast process so that you can have your tea or coffee sweet in some seconds, then you just stir the system and then it is mixing by turbulent diffusion. So, we understood phenomenologically that what is turbulent diffusion. This is something where the diffusion or the mixing or the transport due to the gradient of density is done in the presence of turbulence. But the question is that how to do some quantitative thing out of that?

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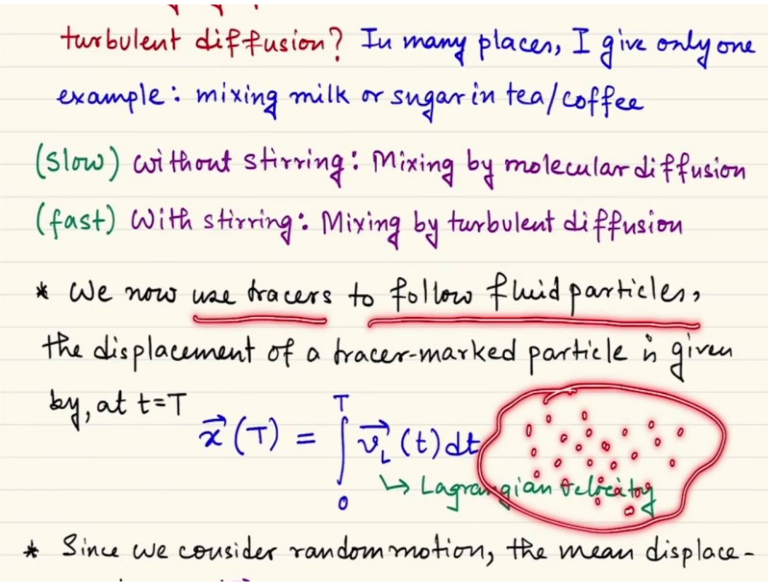

So, in order to do that let us start by doing say some tracing. So, first we use some tracers to follow some fluid particle; that means, in the container of fluid, you just use some fluorescent elements so that you just drop some fluorescent type of color points and then you would like to follow the trajectory of those fluorescent points. So, here I have drawn a lot of fluorescent points.

What you will do? You will just put something like this and then you just follow this trajectory. You will see one trajectory will go there, one trajectory will go there, one will be go there, one will be going there, one will be going there and so on. It is a totally abrupt process. it is a totally random process disordered process. Now if we do that, then the displacement of a tracer marked particle is given by after some.

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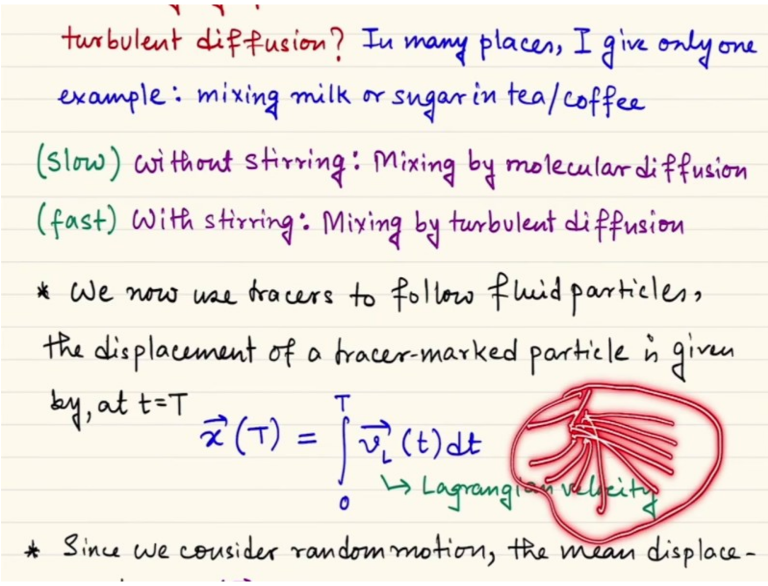

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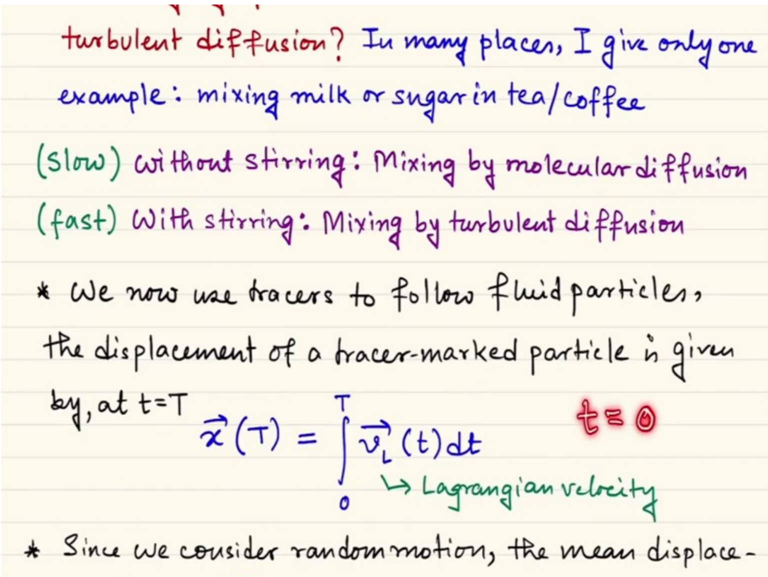

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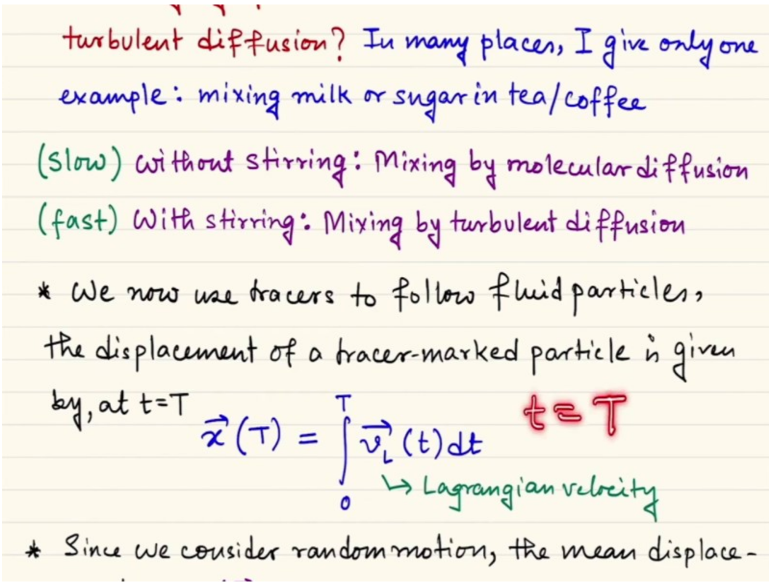

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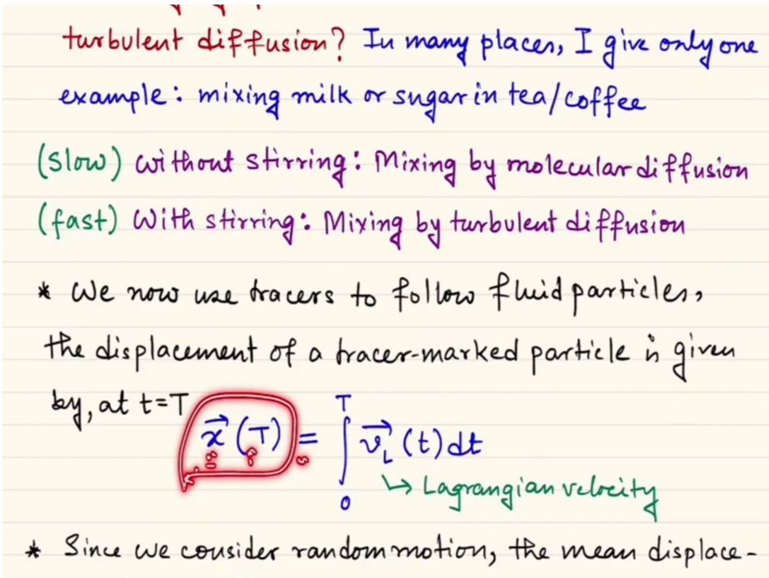

So, if you just start your watch at $t = 0$ (let us say) and then you stop your watch at some small $t = T$. Then you will say that my displacement vector at small $t = T$ is given by $\boldsymbol{x}(T)$; which is the function of capital T; will be simply equal to

$$\boldsymbol{x}(T) = \int_0^T \boldsymbol{v}_L(t)dt \qquad (1)$$

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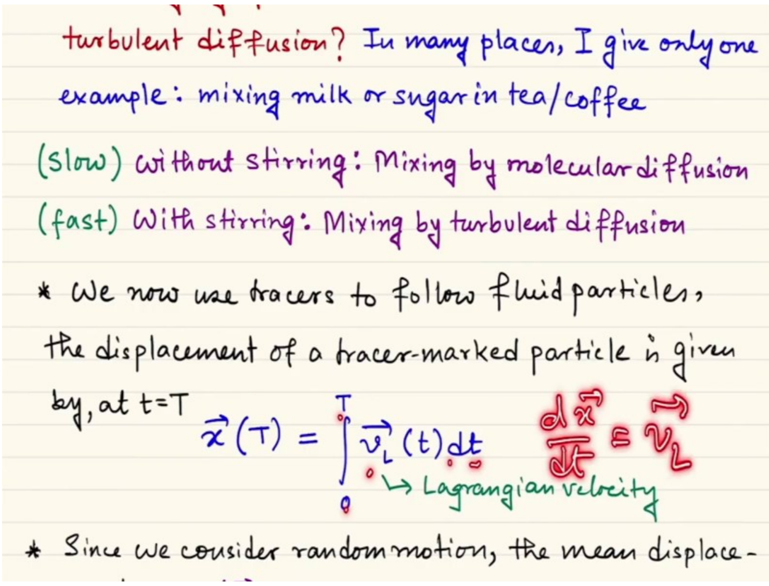

That is because simply $\frac{dx}{dt}$ is nothing but $\boldsymbol{v}_L$, then you just need to integrate to get $\boldsymbol{x}$ ok; as simple as that so, exactly just like as we are trying to track Newtonian particles, here we are just doing the same thing for Lagrangian particles or fluid particles okay.

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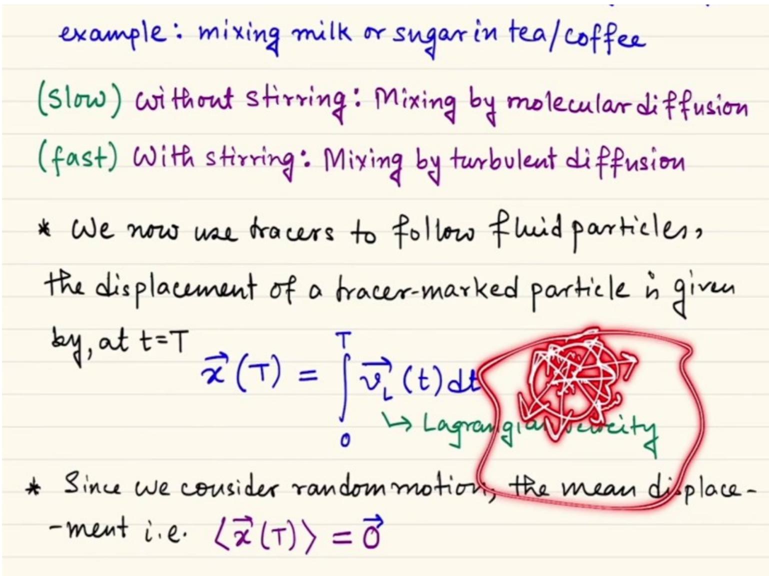

Since we consider a totally random or disordered motion, one can intuitively easily assume and which is a quite reasonable assumption that the mean displacement of all these displacement vectors will be equal to 0 .

$$\langle\, \boldsymbol{x}(T) \rangle = \boldsymbol{0} \qquad (2)$$

Of course, there will be a vector sign here in 0 ok. Frequently, I can see people do not make this vector sign, but this is the null vector.

So, the average of the displacement vector is 0, that is simply saying that if you take number of such fluid particles and as I just said that in a container, you start by saying okay I have a number of fluid particles where we are localized and then if you just track, you will see that this one will go there, this one will go there, this one will go there, this one will go there, this one will go there, this one will go there and so on.

Oh my god! this is the total mess and finally, if you just calculate them, the vector sum of all these things with respect to some specified origin, it will be very close to 0.

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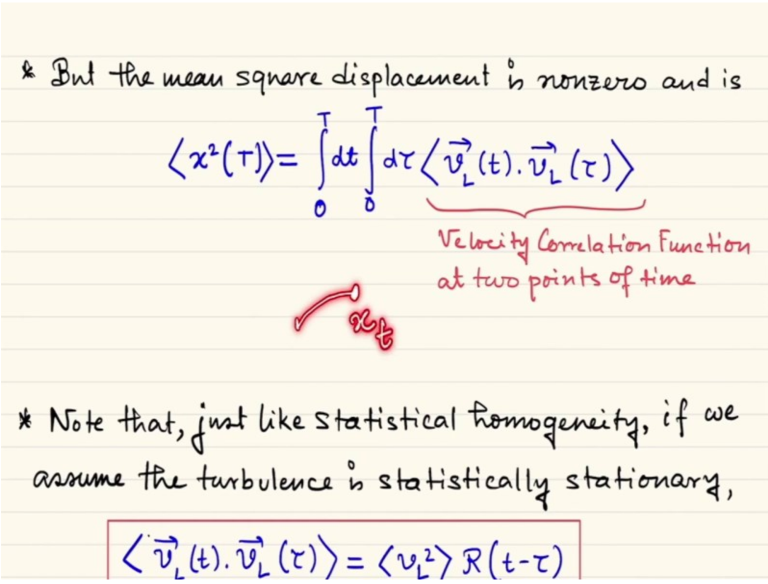

But of course, the mean square displacement vector is non-zero, I mean, mean square displacement is not a vector in general because this is taken as a square. So, the mean square displacement is non-zero and this is given by $\langle\, x^2(T) \rangle$, which is also a function of capital T and that will be simply equal to

$$\langle\, x^2(T) \rangle = \int_0^T dt \int_0^T \langle \boldsymbol{v}_L(t).\boldsymbol{v}_L(\tau) \rangle \qquad (3)$$

This double integration will be done on the two-point velocity correlation function $\langle \boldsymbol{v}_L(t).\boldsymbol{v}_L(\tau)\rangle$ of the Lagrangian velocities of two different time points one at t and one at τ . If you integrate the Lagrangian velocity at a given time point over time, you will get the displacement vector at that time point, now if you take the two-point velocity correlation function of the Lagrangian velocities and then you integrate them in time; double integration; then what we will get? We will simply get the mean squared displacement okay. Now, once again you have to understand that these things ($\boldsymbol{v}_L(t)$) are the velocity of some fluid particle at different point of time not different point of space and due to this different point of time actually, they will be at different point in space okay.

That means, here the space position is explicitly changing with time, is this part clear? So, this is like you are just tracking on particle. So, the particle has a $\boldsymbol{v}_L(t)$ at let us say some time instant t and is at $\boldsymbol{x}(t)$, after some time τ, the particle let us say at position $\boldsymbol{x}(\tau)$ okay.

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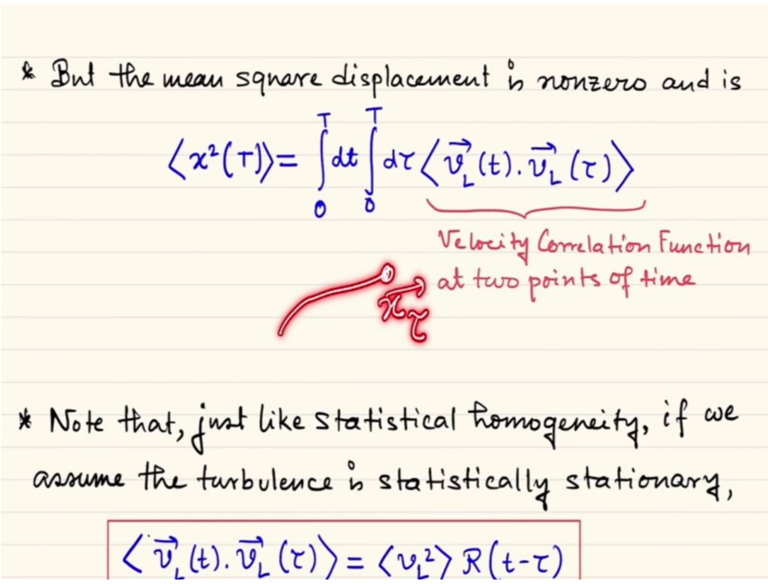

So, you will see that when we are talking about the two different time points, it is actually synonymous of saying that they are of two different positions. Here simply the position is explicitly depending on time, that is why we just write the functional dependence of the $\boldsymbol{v}_L's$ directly on time.

Now note that just like statistical homogeneity, if we assume here a new assumption which is the assumption of statistical stationarity, then what happens? So, statistical stationarity is nothing but a homogeneity in time.

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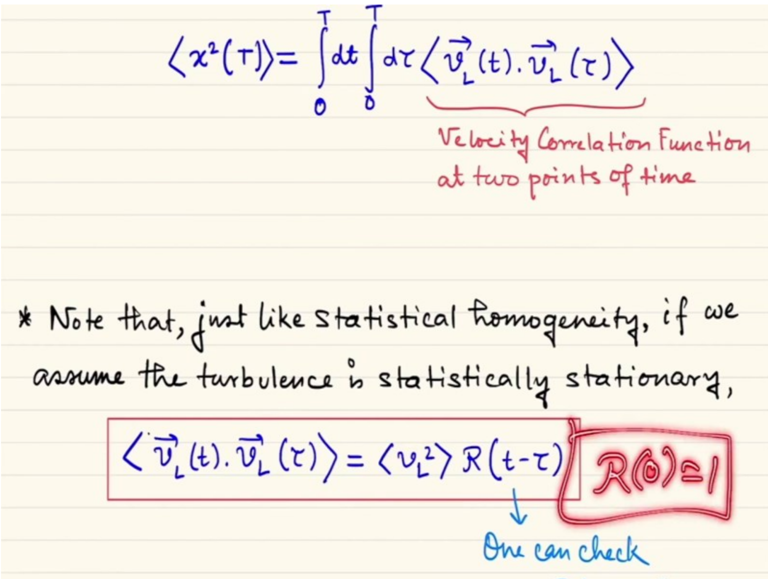

Then what happens? The two-point correlation function $\langle \boldsymbol{v}_L(t).\boldsymbol{v}_L(\tau)\rangle$ will simply be given by the normalization constant which is $\langle v_L^2\rangle$ times the Pearsonian correlation coefficient $R(t-\tau)$.

$$\langle \boldsymbol{v}_L(t).\boldsymbol{v}_L(\tau)\rangle = \langle v_L^2\rangle\, R(t-\tau) \qquad (4)$$

this correlation coefficient will then simply be a function of neither t nor τ, but the distance between t and τ . So, it will be a function of $(t-\tau)$. So, one can simply check where at $t=\tau$, $R(t-\tau)=R(0)$ and we will have,

$$\langle \boldsymbol{v}_L(\tau).\boldsymbol{v}_L(\tau)\rangle = \langle v_L^2\rangle\, R(t-\tau) \Rightarrow\ R(0)=1 \qquad (5)$$

Now, one thing, here I would like to say that here we are just talking about stationarity because this is something very important for our case. Remember in the previous discussion, I mentioned that Kolmogorov in the year 1941; he obtained his famous exact relation which was relating the third order structure functions or the third order moments of the two-point increments along with the length scales then specifically he considered stationary turbulence okay.

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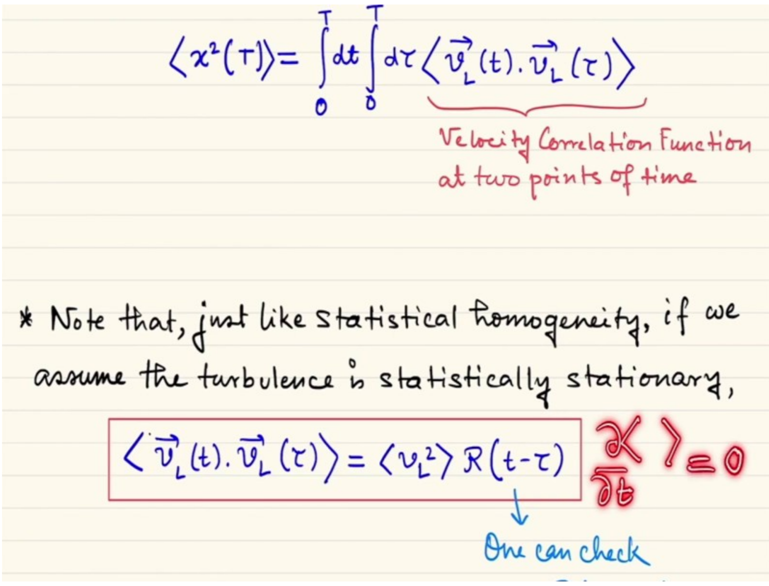

By statistical stationarity we mean that any statistical average is explicitly independent of time, I mean, there is no explicit dependence on time i.e., $\frac{\partial}{\partial t}\langle\ \rangle = 0$. his type of time this type of correlation functions $\langle \boldsymbol{v}_L(t).\boldsymbol{v}_L(\tau)\rangle$, two-point correlation functions in time will simply be a function of the difference between the two time points.

Just one point to remember, by definition this $\boldsymbol{v}_L(t)$, which we said, they are not really the total Lagrangian velocity they are the fluctuating Lagrangian velocity with respect to the mean value or some background value.

Now here $\boldsymbol{v}_L(t)$ is equal to the total velocity only when we just make the mean value of $\boldsymbol{v}_L(t)$ to be 0 and that is exactly what we are doing here okay. So, $\boldsymbol{v}_L(t)$ should be such that the $\boldsymbol{v}_L(t)$ mean is also 0.

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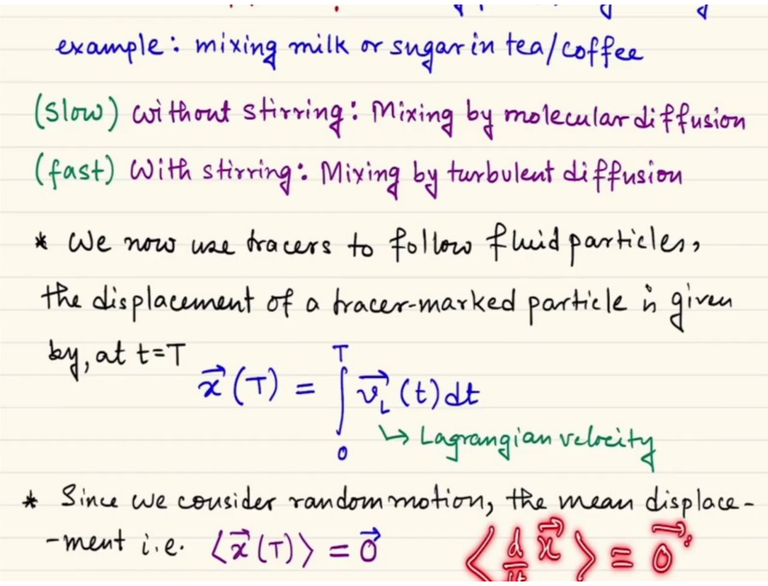

That is not a very difficult thing to understand from here because this $\langle \boldsymbol{x}(T) \rangle$ is 0, we can actually say that $\left\langle \frac{d}{dx} \boldsymbol{x}(T) \right\rangle$ this is also 0 in some way i.e., $\left\langle \frac{d}{dx} \boldsymbol{x}(T) \right\rangle = 0$. I mean just think okay. So, $\boldsymbol{v_L}(t)$ is also randomly distributed so average of $\boldsymbol{v_L}(t)$ is also 0.

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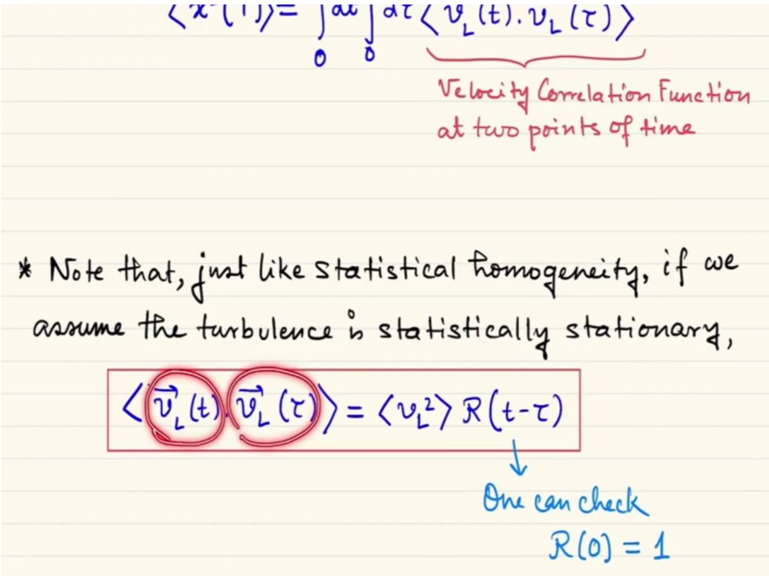

And that is why just by writing the total velocities are equivalent to writing their fluctuation with respect to the mean because the mean is 0.

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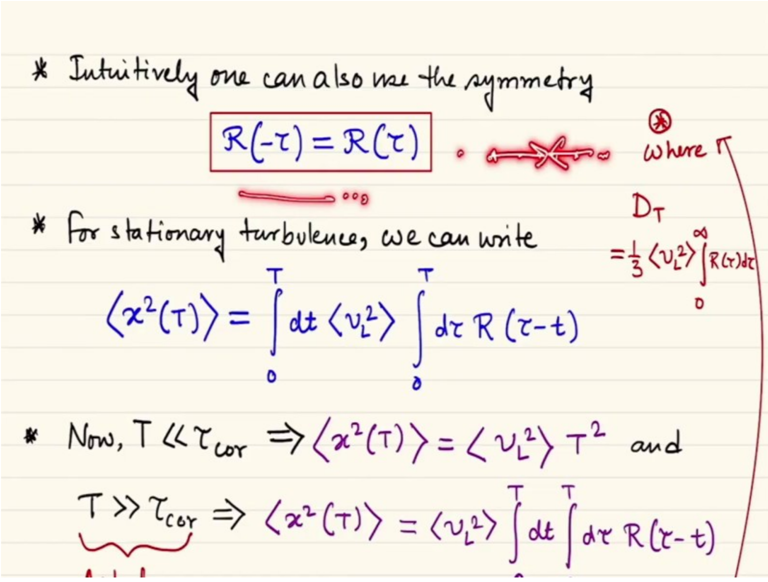

Now, here one point to note, not directly related to what I have just talked with, but one point that here very often we also use another symmetry which is called the mirror symmetry in time and that is very easy to understand. Because in time, homogeneity simply means that some property; some physical properties will be invariant in time translation. So, the time translation in positive direction or negative direction should be made equivalent, if we are talking about two-point quantities, right.

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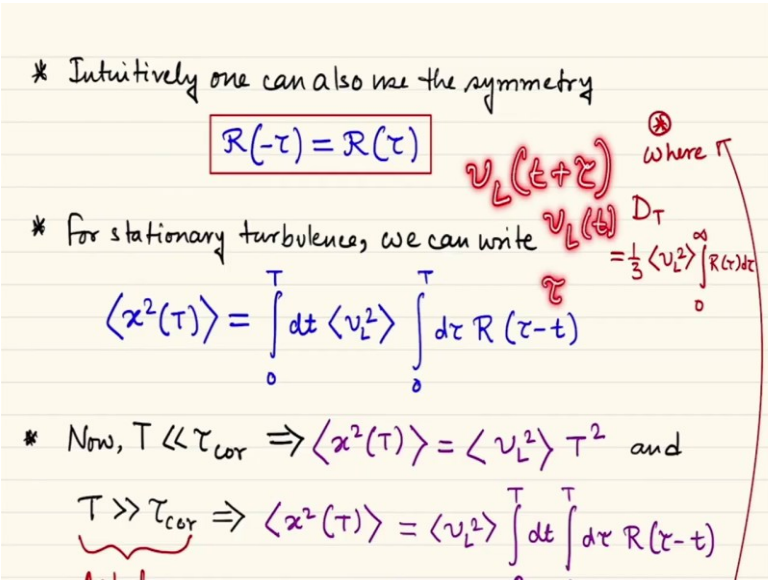

That means some so, let us say $\langle \boldsymbol{v}_L(t).\boldsymbol{v}_L(\tau)\rangle$, if this is the function of τ only, i.e.,

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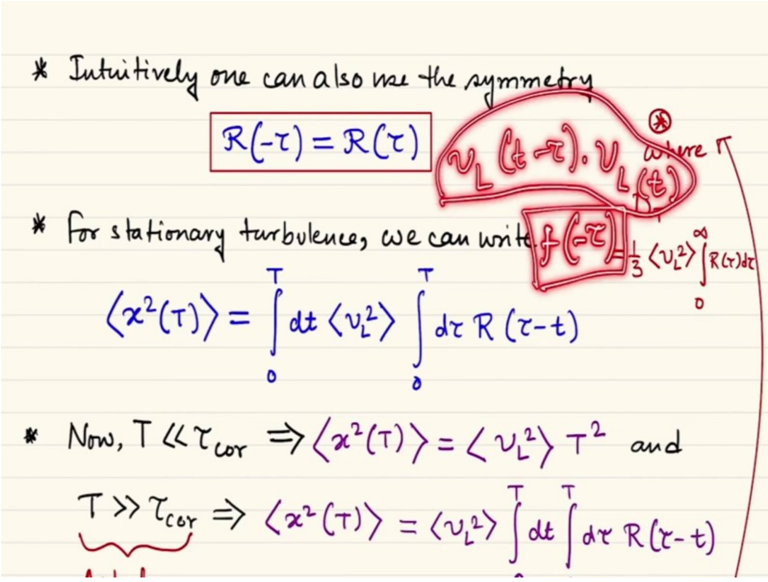

$\langle \boldsymbol{v}_L(t+\tau).\boldsymbol{v}_L(t)\rangle = f(\tau)$. Then of course, $\langle \boldsymbol{v}_L(t-\tau).\boldsymbol{v}_L(t)\rangle$ will be a function of some function $f(-\tau)$ and there is no reason to say that this $\langle \boldsymbol{v}_L(t-\tau).\boldsymbol{v}_L(t)\rangle$ will be in any case different from $\langle \boldsymbol{v}_L(t+\tau).\boldsymbol{v}_L(t)\rangle$ i.e., $\langle \boldsymbol{v}_L(t-\tau).\boldsymbol{v}_L(t)\rangle = \langle \boldsymbol{v}_L(t+\tau).\boldsymbol{v}_L(t)\rangle$, because this is totally statistically averaged, there is no reason that will be different from this thing.

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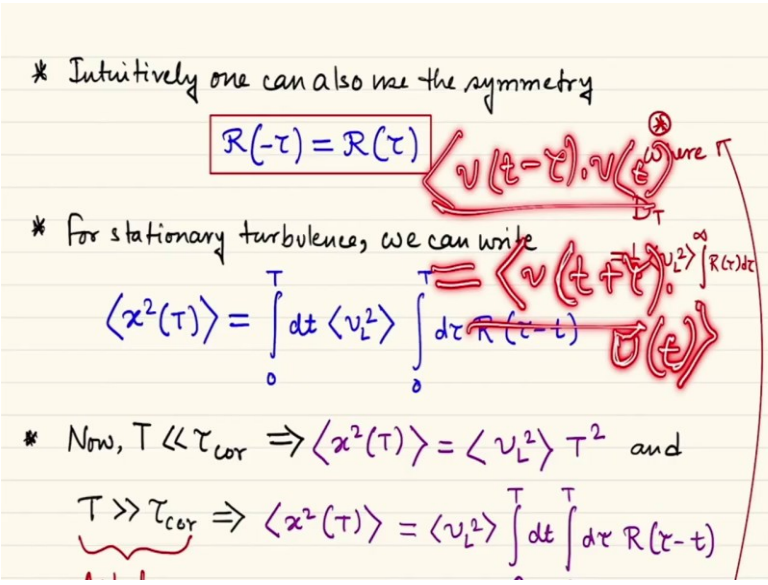

Because in both cases, you are just taking the averages of all the couples occurred τ distant apart and here actually in time you do not have a victorial property. So, you are on one axis; only one axis is there for time; so, this is trivially understood.

So, if we now consider the stationary turbulence, the mean square displacement; our final objective is to determine mean square displacement, why I am coming later; so, the mean square displacement $\langle x^2(T)\rangle$, which is averaged is equal to,

$$\langle x^2(T)\rangle = \int_0^T dt\, \langle v_L^2\rangle \int_0^T d\tau \langle R(\tau - t)\rangle \qquad (5)$$

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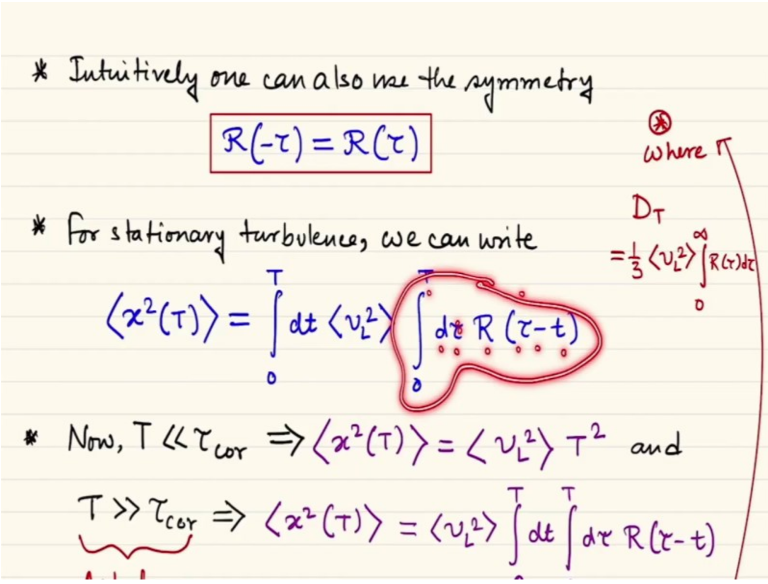

Now this thing $\int_0^T d\tau\langle R(\tau - t)\rangle$ when this will be integrated over τ, it will simply be a function of small t only and then this will be integrated over dt okay. This part we have just obtained by using the relation $\langle x^2(T)\rangle = \int_0^T dt \int_0^T \langle \boldsymbol{v_L}(t).\boldsymbol{v_L}(\tau)\rangle$ and then the expression $\langle \boldsymbol{v_L}(t).\boldsymbol{v_L}(\tau)\rangle = \langle v_L^2\rangle\, R(t-\tau)$ of the two-point correlation functions, this one for stationary turbulence.

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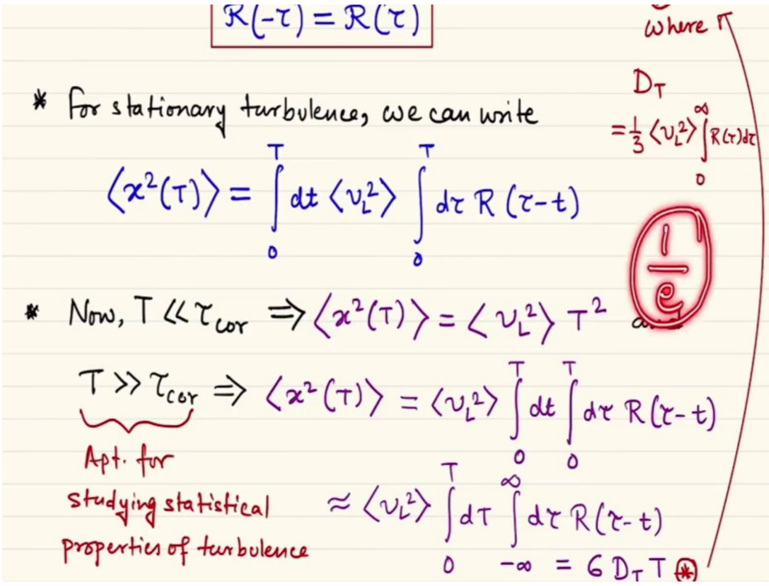

Now we have to understand what we are looking for. So, if we are looking for just to check that the systems behavior very near to the origin of the time; that means, you just start your clock and you just stop it okay. So, you are just checking the systems comportment or behavior during a very small time and that time is very small with respect to the correlation time τ_{corr}, i.e., $T \ll \tau_{corr}$.

Now correlation time is nothing, but the time beyond which the two-point velocity correlations are mean decreased by $\frac{1}{e}$ of the initial. Let us say at $R(0) = 1$, then after time $t = \tau_{corr}$, we will have, $R(\tau_{corr}) = \frac{R(0)}{e} = \frac{1}{e}$.

Now, if you want to be in such a time which is very small with respect to the correlation time, that means, that your correlation is almost one here so, no further no much decrease in the correlation coefficient so, then this can be assumed to be one i.e., $R(t-\tau) \sim 1$and then what else? this $\langle v_L^2 \rangle$ is constant, this $\int_0^T d\tau \langle R(\tau - t) \rangle$ will give you capital T, this $\int_0^T dt \, \langle v_L^2 \rangle$ will give you capital T so, you will simply have your mean square displacement will be equal to

$$\langle x^2(T) \rangle = \langle v_L^2 \rangle T^2$$

So, in very short time, the mean square displacement is proportional to T square, it will have some parabolic relationship with t.

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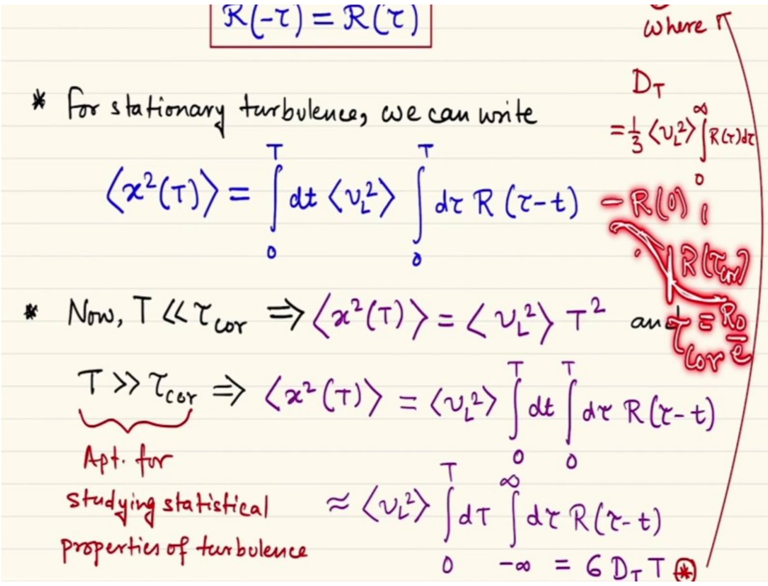

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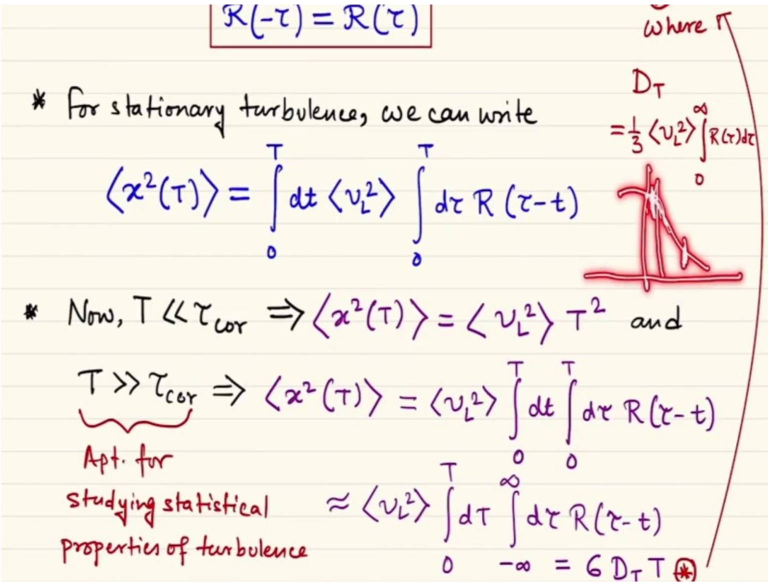

Now, let us say we want to check the statistical properties of turbulence, then what we have to do? We have to investigate the systems behavior in time which is very greater than the correlation time i.e., $T \gg \tau_{corr}$.

Then only we can check the statistical properties of the turbulence because if your time is long enough, then only the two points will be very weakly correlated and you can simply see the effect of the turbulence in the in the system.

That means, let us say, if you put your reference time which is very large with respect to the correlation time, then what happens is that inside that time interval you will simply have so many correlation times and that will give you a statistically very well trusted sampling.

So, if you your time is within the correlation time, it simply means that your system is not statistically very trustworthy. So, if you want bit more knowledge about the correlation time and this type of thing so, you can check any introductory text books on statistics or time series analysis type of thing.

Now, if we are in this $T \gg \tau_{corr}$ position, then we can say that this mean square displacement will simply be

$$\langle x^2(T)\rangle = \langle v_L^2\rangle \int_0^T dt \int_0^T d\tau \langle R(\tau - t)\rangle \qquad (6)$$

Now since we are interested in the time which is very greater than the correlation time, what happens? We can simply say that one can easily replace this integration $\int_0^T d\tau \langle R(\tau - t)\rangle$ with a changed boundary from minus infinity to plus infinity i.e., $\int_{-\infty}^{\infty} d\tau \langle R(\tau - t)\rangle$.

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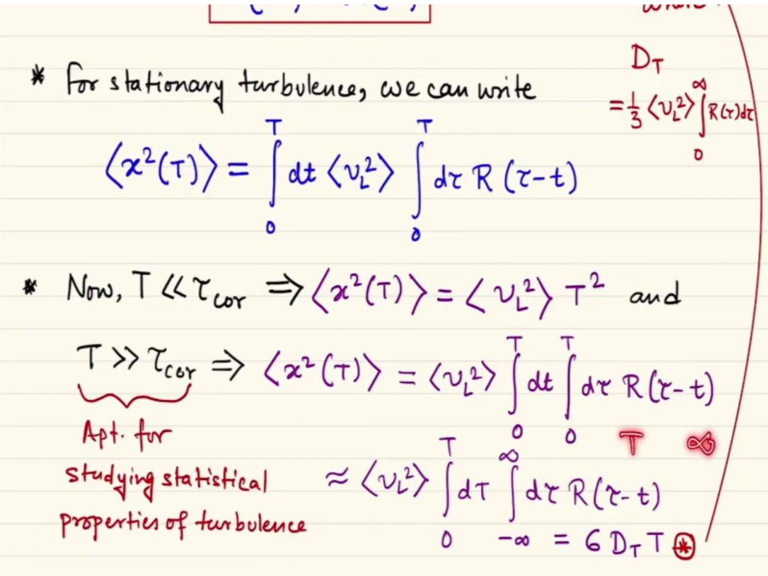

So, instead of 0, I just write $-\infty$ because before 0 the correlation does not have any contribution and if my time is very large, then I can say that instead of T, I can write ∞.

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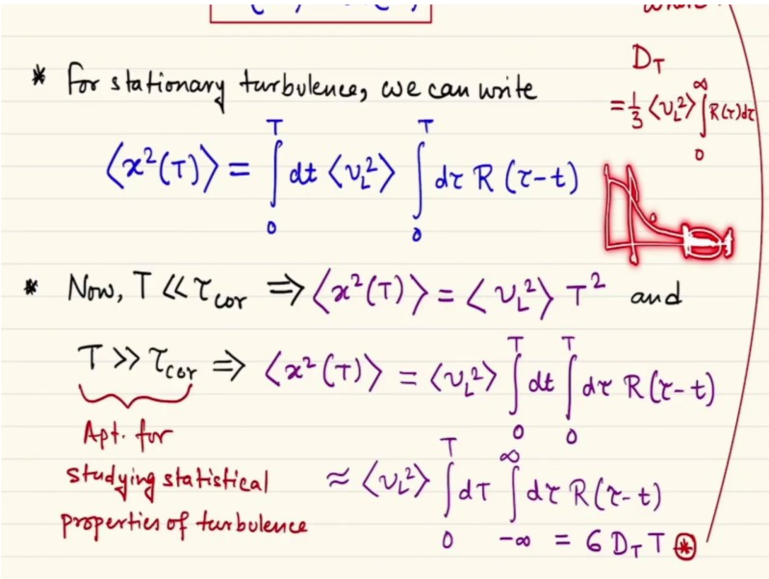

Because let us say if my correlation function is like that sorry; not a very good drawing and then I say that my t is here for example, then just by saying t here or here does not make any difference because the extra contribution which is made is perceived just due to this choice of t, is really negligibly small.

On the other end if my t is over here (within correlation time), then I cannot say that my t can be assumed to be at infinity, but when the t is very near to the end of the tail then you can say that my t is almost equal to infinity. So, if I just put $\int_{-\infty}^{\infty} d\tau\langle R(\tau-t)\rangle$ this there in (6), then just defining a coefficient called D_T which is equal to

$$D_T = \frac{1}{3}\langle v_L^2\rangle \int_0^\infty d\tau\langle R(\tau)\rangle \qquad (7)$$

we can simply write the whole expression to be equal to $6D_TT$ i.e.,

$$\langle x^2(T)\rangle = \langle v_L^2\rangle \int_0^T dt \int_{-\infty}^{\infty} d\tau\langle R(\tau-t)\rangle = 6D_TT \qquad (8)$$

Here, there is a small mathematical cheating that I did, after this integration $\int_{-\infty}^{\infty} d\tau\langle R(\tau-t)\rangle$ there should be some time dependence; to be very honest this thing $\int_{-\infty}^{\infty} d\tau\langle R(\tau-t)\rangle$ should have some t dependence; but this t dependence, we have just neglected over here and so that this integration $\int_0^T dt$ is done independently of this part $\int_{-\infty}^{\infty} d\tau \langle R(\tau - t) \rangle$. So, practically these two integrations become uncoupled here.

I am just saying that this integration $\int_{-\infty}^{\infty} d\tau \langle R(\tau - t) \rangle$ does not have a very strong t dependence. Once this is integrated over τ, this is already done. That is only because I am talking about very large τ then, what is happening at very large τ does not really matter for t and that is why that the dynamics of this part $\int_{-\infty}^{\infty} d\tau \langle R(\tau - t) \rangle$ can be very honestly to be thought of decoupled from this part $\int_0^T dt$. Once this $\int_{-\infty}^{\infty} d\tau \langle R(\tau - t) \rangle$ is simply integrated over, you can get almost a constant coefficient D_T. So, I am just saying that, I am here neglecting the explicit time dependence of this coefficient D_T and if I neglect that time dependence of that coefficient, then this $\langle x^2(T) \rangle$ mean square displacement simply becomes to be proportional to the t.

So, if we are at a time which is sufficiently greater than the correlation time, then the mean square displacement simply follows a linear relationship with time and that is exactly something which is very interesting because this has a striking similarity with diffusion.

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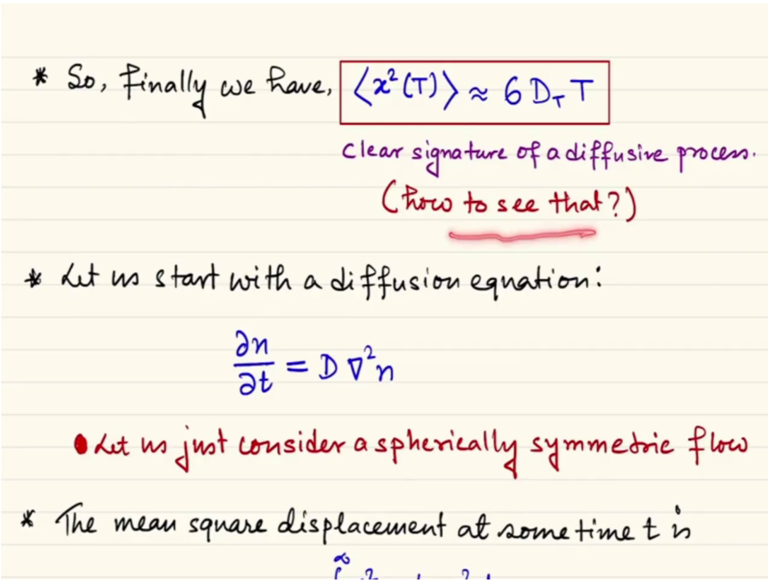

You see that here in the whole process, we have not talked anything of diffusion, we simply started by two-point correlation and stationary turbulence and we have arrived to some relation of the mean square displacement, where we saw some similarity with the diffusion, why?

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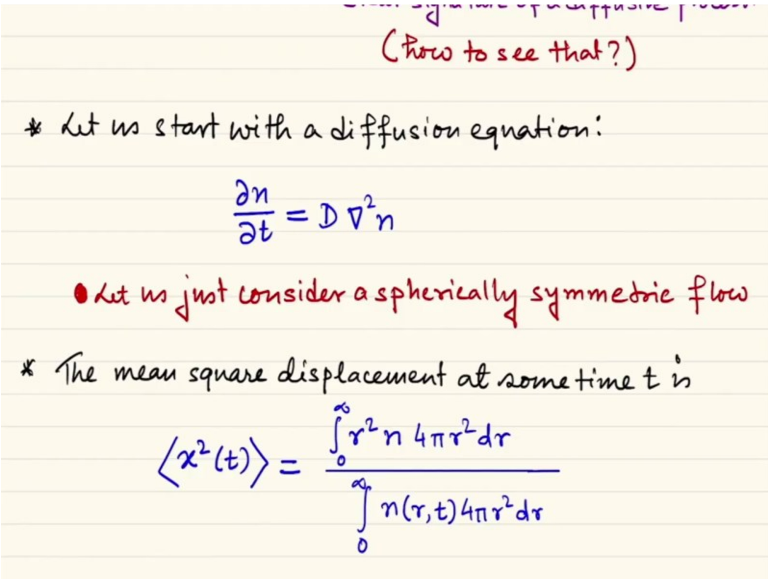

Because if you just now use your previous knowledge of diffusion, you can see that the diffusion equation for the density should be given by this

$$\frac{\partial n}{\partial t} = D\,\nabla^2 n \qquad (9)$$

n is the density and this is something which is the variable in the diffusion process.

Now, let us just consider a spherically symmetric flow in this case and then the mean square displacement term should simply be given by integration

$$\langle x^2(T) \rangle = \frac{\int_0^\infty r^2\, n\, 4\pi r^2 dr}{\int_0^\infty n(\boldsymbol{r},t)\, 4\pi r^2 dr} \qquad (10)$$

This is $\int_0^\infty n(\boldsymbol{r},t)\, 4\pi r^2 dr$ the whole number of the particles. So, this is used to normalize this number and this is here you see there is a r^2, this simply gives me finally the mean square displacement. So, this is (10) a second order moment in some sense for the displacements.

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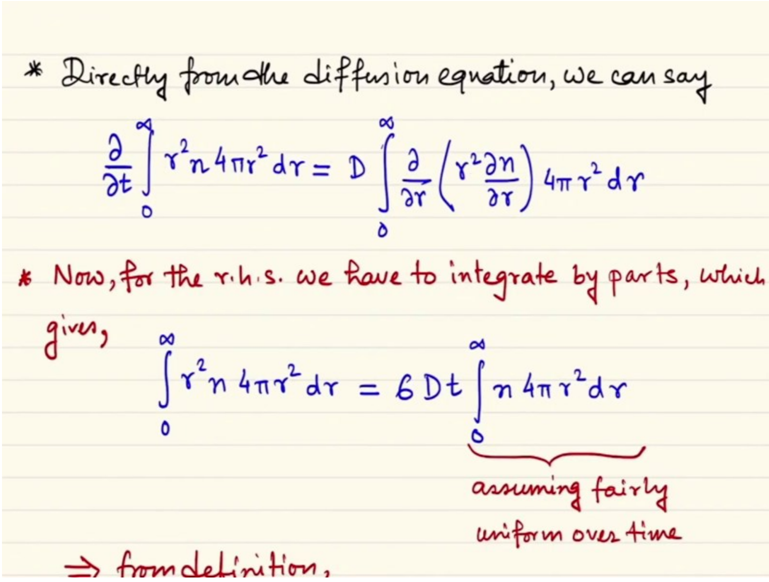

If you do that then, you have this expression (9) and again just using this equation (10) and integrating them again in space for a spherical symmetric system, you can again obtain this type of thing

$$\frac{\partial}{\partial t}\int_0^{\infty} r^2 n\, 4\pi r^2 dr = D\int_0^{\infty}\frac{\partial}{\partial r}\left(r^2\frac{\partial n}{\partial r}\right)4\pi r^2 dr \qquad (11)$$

This simply comes from the diffusion equation (9), if you integrate both sides just in space using spherical symmetry, this is $\frac{\partial}{\partial r}\left(r^2\frac{\partial n}{\partial r}\right)4\pi r^2$ nothing but the spherical symmetric form of the $\nabla^2 n$.

Now if you carefully calculate the right-hand side by integration by parts, you will see that this the part $D\int_0^{\infty}\frac{\partial}{\partial r}\left(r^2\frac{\partial n}{\partial r}\right)4\pi r^2 dr$ is equal to this $6Dt\int_0^{\infty} n 4\pi r^2 dr$. Now there is a $\frac{\partial}{\partial t}$ also, finally you have to integrate over time and you will have this term over here $D\int_0^{\infty}\frac{\partial}{\partial r}\left(r^2\frac{\partial n}{\partial r}\right)4\pi r^2 dr$ and then $\frac{\partial}{\partial t}$ goes in right hand side and then by integration, it will simply have $6DT$. Again, this thing assumed $D\int_0^{\infty}\frac{\partial}{\partial r}\left(r^2\frac{\partial n}{\partial r}\right)4\pi r^2 dr$ I have that to be independent of time; that is my assumption.

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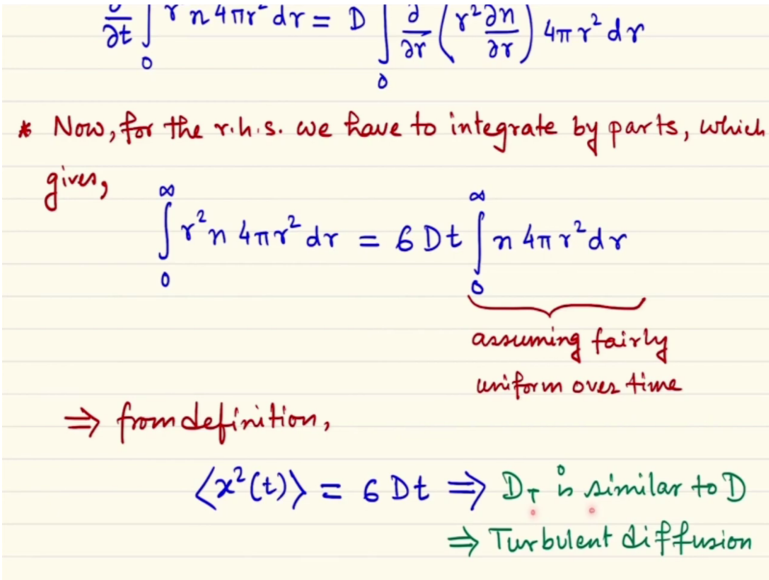

Then this in integration over time is simply the Dt and you have again something like this $6Dt\int_0^{\infty}\frac{\partial}{\partial r}\left(r^2\frac{\partial n}{\partial r}\right)4\pi r^2 dr$ and then you have by definition (10), your mean square displacement is nothing, but this one by this one right,

$$\langle x^2(T)\rangle = \frac{\int_0^{\infty} r^2\, n\, 4\pi r^2 dr}{\int_0^{\infty} n(\boldsymbol{r}, t)\, 4\pi r^2 dr}$$

Then finally, you will have this,

$$\langle x^2(T)\rangle = 6Dt \qquad (12)$$

So, you see so, that is the classical formula for mean square displacement in a diffusion process. As our D_T is similar to D , then we can say that what we find over here is nothing but the diffusion process and this process is known as turbulent diffusion. That is exactly the process which leads to a quick mixing of sugar or milk in a cup of tea or coffee.

Thank you very much.