Introduction to Astrophysical Fluids
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Lecture - 50
MHD in space plasmas

Hello and welcome to another lecture session of Introduction to Astrophysical Fluids. In this session, we will discuss very briefly some interesting aspect of MHD equations.

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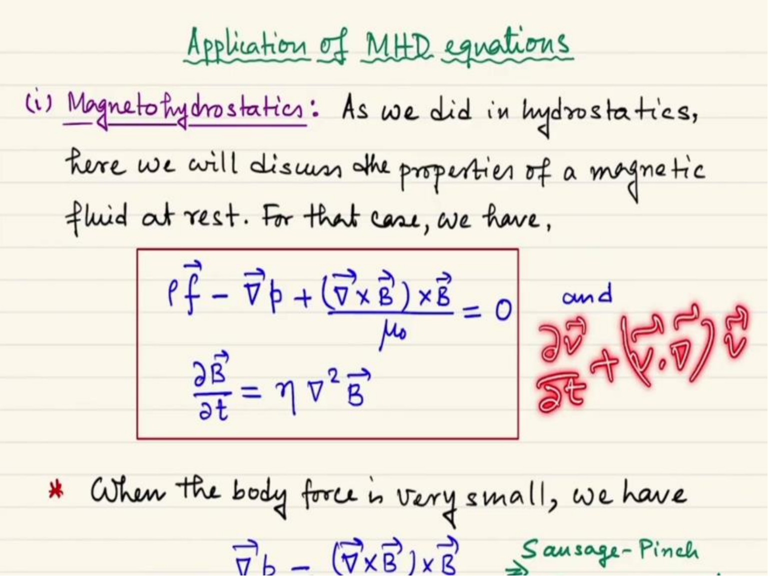

So, Magnetohydrodynamics, as we have seen that it consists of the continuity equation, the momentum evolution equation of a plasma mono-fluid, then comes the induction equation, and then we have also a generalized Ohm's law. And finally, we have a closure which for instance we use the polytropic closure or isothermal closure.

Now, we want to see, in different limits, how these equations would behave. So, the first one as we did for hydrodynamic case, we discussed the fluid property when it is at rest, that was the case of hydrostatics, here similarly we will discuss the properties of a magnetic fluid at rest, and this is known as magnetohydrostatics.

So, for magnetohydrostatics, of course, the fluid velocity $\boldsymbol{v}$ is equal to 0. So, if you just replace that in the momentum evolution equation, you will simply have

$$\rho \boldsymbol{f} - \nabla p + \frac{(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}}{\mu_0} = \boldsymbol{0}.$$

Where $\boldsymbol{f}$ is the body-force density.

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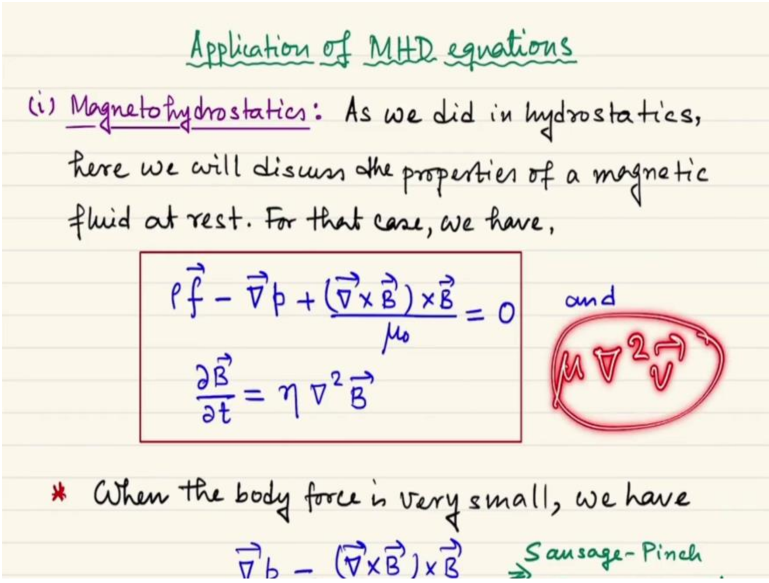

So, both the $\frac{\partial \boldsymbol{v}}{\partial t}$ term and the inertial term and the viscosity term, all are vanishing because the $\boldsymbol{v}$ is vanishing.

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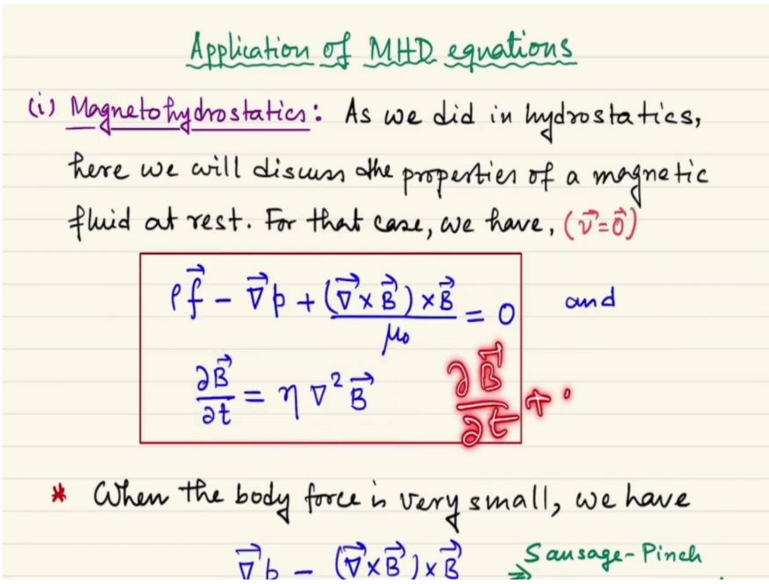

And for the magnetic field evolution equation, we in general have

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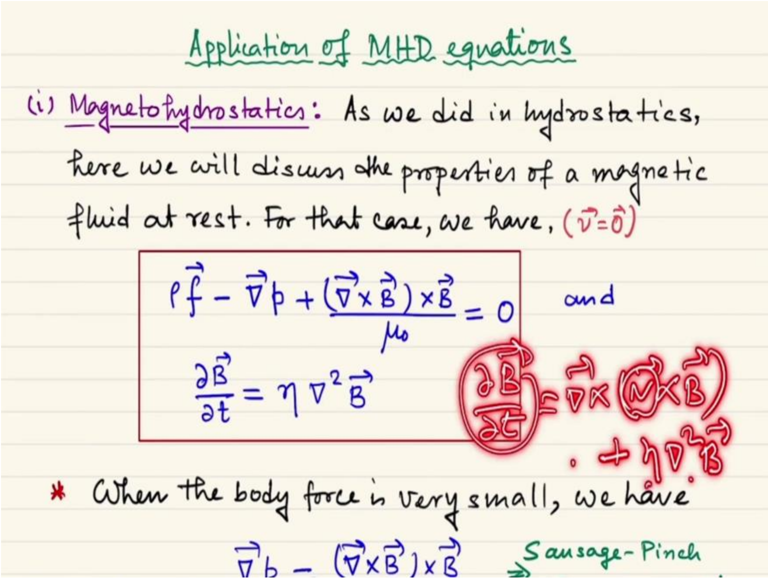

$$\frac{\partial \boldsymbol{B}}{\partial t} = \eta \boldsymbol{\nabla}^2 \boldsymbol{B},$$

which is nothing but an equation of a simple diffusion.

So, when the fluid is at rest, then the magnetic field would simply diffuse. And all the three forces – body force, pressure gradient force, and the Lorentz force, they balance each other just by giving a net contribution of zero.

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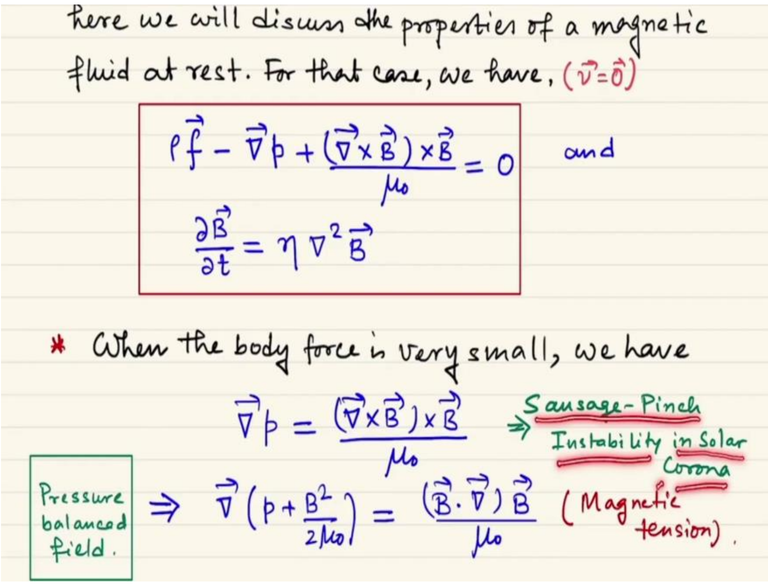

Now, when the body force is very small, for example, in most of the cases in space and astrophysical cases, if the gravitational force is very small with respect to the pressure gradient force and the Lorentz force for a system, then for example, in a solar surface this is a very good example for that, then $\rho \boldsymbol{f}$ can be neglected. And we simply have a balance between Lorentz force and the pressure gradient.

$$\boldsymbol{\nabla} p = \frac{(\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B}}{\mu_0} .$$

Now, I can divide the total Lorentz force term into two parts. One part, $\boldsymbol{\nabla}\left(\frac{B^2}{2\mu_0}\right)$, can be absorbed in the pressure term. So, this is the magnetic pressure part and the term $\frac{1}{\mu_0}(\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{B}$ is nothing but the magnetic tension. So, here we can see that in this situation, the total pressure gradient force, $\boldsymbol{\nabla}\left(p + \frac{B^2}{2\mu_0}\right)$ is exactly balanced by the force due to magnetic tension. So, in this situation, the magnetic field is known as pressure balanced field.

And if you just follow some steps, actually one can show that to solve this type of equation for cylindrical symmetric case, you can actually be led to some instability which is known as Sausage-Pinch instability and that is a very familiar instability in some places of solar coronal plasma.

So, here I am just mentioning a number of cases where you can simply see the direct application of magnetohydrodynamics, of course in limits of magnetohydrodynamic equations for example, here we are in the static case. And then if you are interested furthermore you can actually search over internet or in various books to go into the detail. And if you have any question, you can ask me always.

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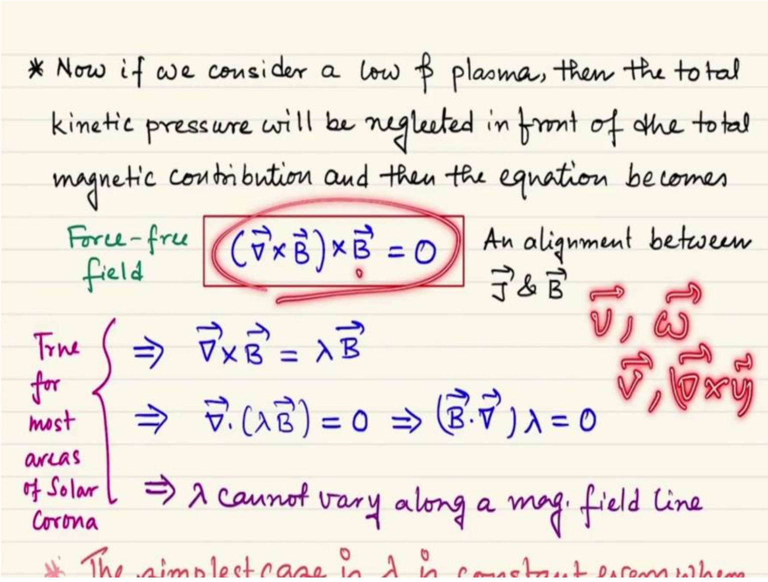

So, after this step, you can simply see that there is finally, a grouping of the terms in a pressure type of term and the tension term. So, inside the pressure type of term, finally, you can again say that if we have a plasma whose β-parameter is very small, that means, the magnetic pressure part dominates largely the kinetic pressure part, then you can simply neglect the pressure term with respect to magnetic pressure.

So, finally, you have $\boldsymbol{\nabla}\left(\frac{B^2}{2\mu_0}\right) = \frac{1}{\mu_0}(\boldsymbol{B}\cdot\boldsymbol{\nabla})\boldsymbol{B}$ and which is nothing but equivalent to writing $(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B} = \boldsymbol{0}$. And that simply means that $\boldsymbol{\nabla}\times\boldsymbol{B}$ and $\boldsymbol{B}$ they are parallel. And this is just nothing but an alignment between $\boldsymbol{J}$ and $\boldsymbol{B}$.

If you are familiar with Beltrami alignment you can easily understand, this is also a type of alignment. So, Beltrami alignment is an alignment between $\boldsymbol{v}$ and $\boldsymbol{\omega}$, that is $\boldsymbol{v}$ and $\boldsymbol{\nabla}\times\boldsymbol{v}$. So, here exactly we have Beltrami type of alignment for $\boldsymbol{B}$ and $\boldsymbol{\nabla}\times\boldsymbol{B}$. And what is the meaning of that? That means $\boldsymbol{J}\times\boldsymbol{B} = \boldsymbol{0}$.

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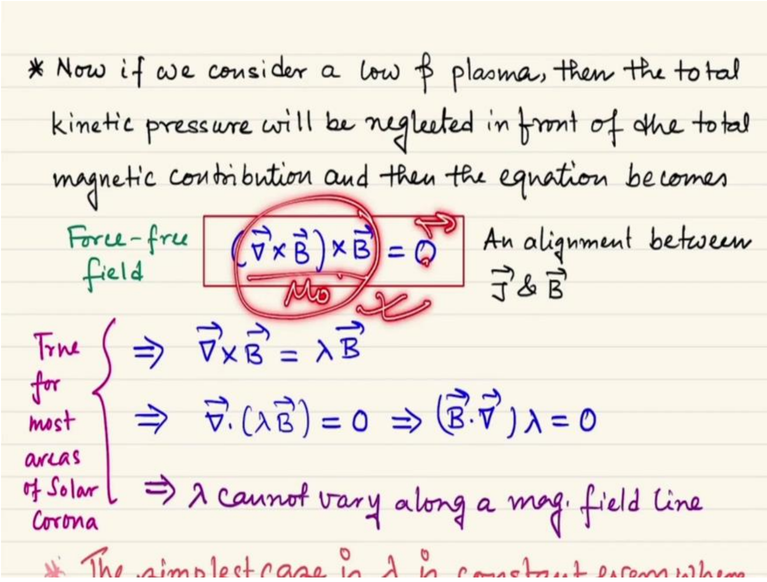

Because when this is 0, the Lorentz force term basically vanishes and that is why we call this as force free field, the corresponding magnetic field is known as force free field.

Now, if we have this situation, then of course we can say that $\boldsymbol{\nabla} \times \boldsymbol{B}$ can be any scalar times $\boldsymbol{B}$. So, because they are two collinear vectors now, so one vector can be written as a scalar function times other, $\boldsymbol{\nabla} \times \boldsymbol{B} = \lambda \boldsymbol{B}$. Then you take the divergence of both sides. So, in the LHS, it is a divergence of a curl which is 0 identically, in the RHS part that it will be $\boldsymbol{\nabla} \cdot (\lambda \boldsymbol{B}) = 0$.

Now, remember λ is a scalar. So, it can be a function of space as well. I am not saying λ is a constant, just that λ is a scalar. So, $\boldsymbol{\nabla} \cdot (\lambda \boldsymbol{B}) = 0$, since $\boldsymbol{\nabla} \cdot \boldsymbol{B}$ is always 0, then this is equivalent to writing $(\boldsymbol{B} \cdot \boldsymbol{\nabla})\lambda = 0$.

So, that means, if you are following one specific magnetic lines of force, then along these magnetic lines of force in space, value of λ's do not change. So, every magnetic lines of force, you have a specific value of λ. So, λ cannot vary along a magnetic field line. Of course, from one magnetic field line or lines of force to another magnetic line of force, it can change.

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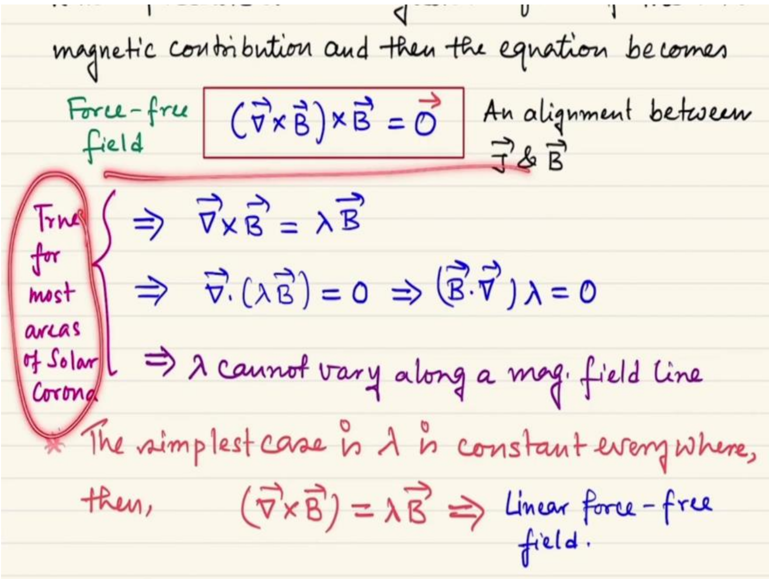

Now, there can be a very simple case, let us say we are in such a system that λ itself is an absolute constant of the system, so that means, it is constant everywhere. It is constant over all the magnetic lines of force. In that case, we can simply say that $\boldsymbol{\nabla} \times \boldsymbol{B}$ is equal to some constant times $\boldsymbol{B}$, $\boldsymbol{\nabla} \times \boldsymbol{B} = \lambda \boldsymbol{B}$. And this specific case leads to the magnetic field which are then known as linear force free field.

So why these are important? Because this type of alignments are very much true for most areas in the solar corona. So, in some areas in the solar corona we have pressure balance, but in the most area we have this force free field.

And you can see that this is a domain of active research. So, very recent papers are on that. So, you can have a look. You just have to search just by typing that force free magnetic field regions in solar coronal plasma or in solar corona.

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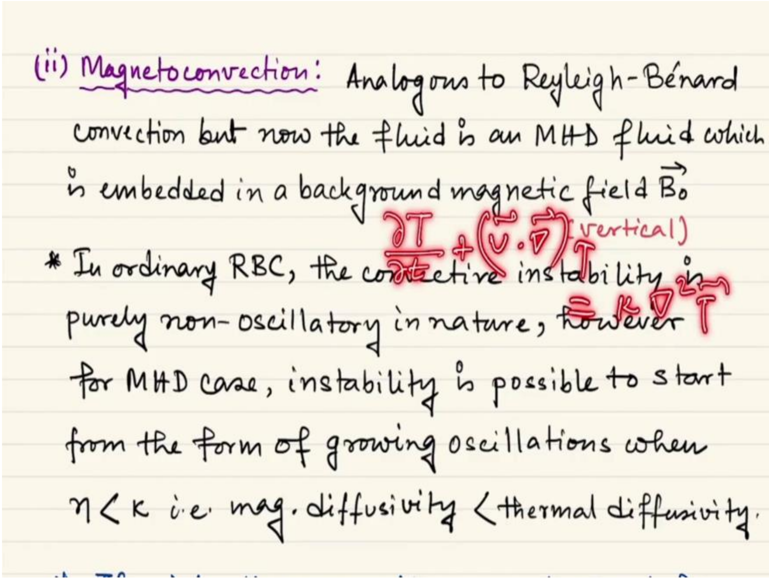

Now, coming to another point about a very interesting phenomenon, called magneto-convection. As we did for normal hydrodynamic fluid Rayleigh-Benard convection (RBC), so this is nothing but a generalization of Rayleigh-Benard convection for magnetic fluid, more specifically for an MHD fluid, which is embedded in a background magnetic field $\boldsymbol{B_0}$ that was absent for a normal RBC.

So, in ordinary RBC, the convective instability is purely non-oscillatory in nature. If you remember the nature of the solutions, when the instability was setting in, then the system was totally non-oscillated.

But in MHD case, actually if we are in such a situation that the magnetic diffusivity is less than the thermal diffusivity, so if you remember thermal diffusivity comes into play when we are talking about the evolution equation of the temperature. So, there will be an evolution equation of the temperature, $\frac{\partial T}{\partial t} + (\boldsymbol{v} \cdot \boldsymbol{\nabla})T = \kappa \nabla^2 T$.

So, if we have such type of equation and we have two diffusivities, one is magnetic diffusivity, another is thermal diffusivity. So, if thermal diffusivity dominates over magnetic diffusivity, instability is possible to start, in the form of growing oscillations. So, that means, that even at the very outset of the instability also we have oscillatory type of solution. But this oscillation actually grows in time which leads to the instability.

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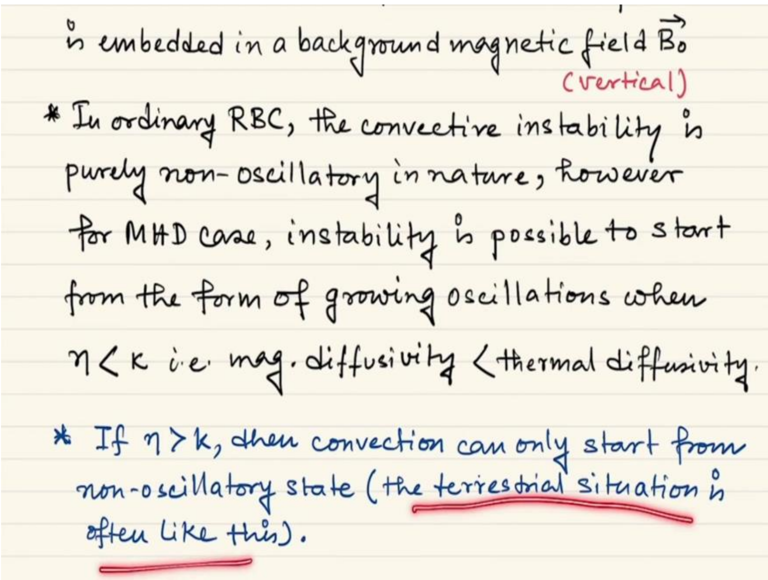

But, in the case where $\eta > \kappa$, that means the magnetic diffusivity now overcomes the thermal diffusivity, then the convection can only start from non-oscillatory state and that is exactly similar to the case of hydrodynamic RBC.

And most of the terrestrial situation is exactly like that because for mean most of the laboratory plasma and also very often for space plasmas as well, the magnetic diffusivity is actually much more larger than the thermal diffusivity.

Now, it is your task to find out some instances or at least one instance where $\eta < \kappa$. So, I want you to do some research on it.

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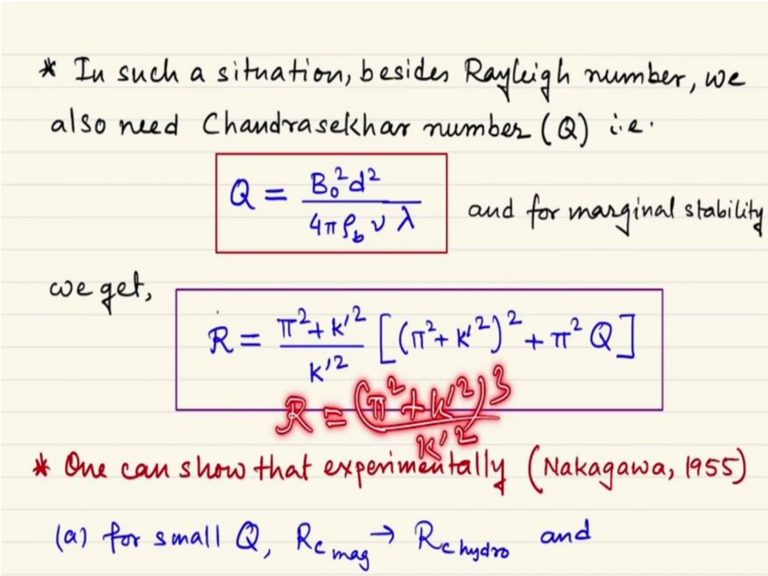

So, in such a situation of Rayleigh-Benard convection of an MHD fluid which we call now magneto-convection, we have two dimensionless numbers which are of prior importance. One is of course, the Rayleigh number which mainly talks about the importance of the temperature gradient. And we also have another number which is the Chandrasekhar number (Q). So, this number represents the importance of the external background magnetic field. So, it is a very small and elegant task to just verify that this is a dimensionless number. So, if this Q is 0, then of course, we have the normal RBC condition.

But if it is non-zero, then for marginal stability, the marginal Rayleigh number is related to the dimensionless wave number (k') by the equation

$$R = \frac{\pi^2 + k'^2}{k'^2}\left[(\pi^2 + k'^2)^2 + \pi^2 Q\right].$$

Now, if Q is equal to 0, we simply have $R = \frac{(\pi^2 + k'^2)^3}{k'^2}$.

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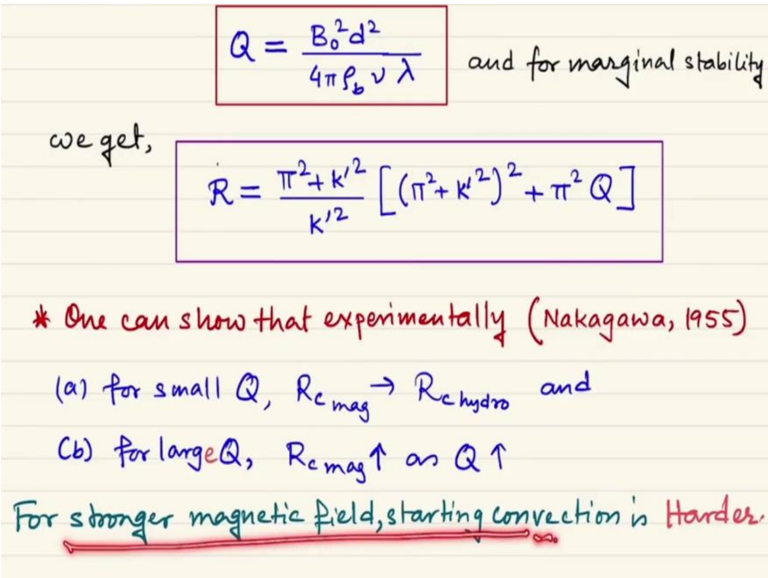

And then one can show experimentally (Nakagawa, 1955) that for small Chandrasekhar number, what happens that the critical Rayleigh number for the magneto-convection, is the minimum value of the marginal Rayleigh numbers. Actually critical Rayleigh number tends towards the critical Rayleigh number for hydrodynamic case.

But for large value of Chandrasekhar number, the magnetic critical number also increases in a monotonic way. What is the meaning of that?

The meaning is simply that if your Q increases, that increases your critical Rayleigh number, that simply says that for stronger magnetic field you have to arrange for a greater temperature gradient to have a convective instability, that means starting convection is comparatively harder.

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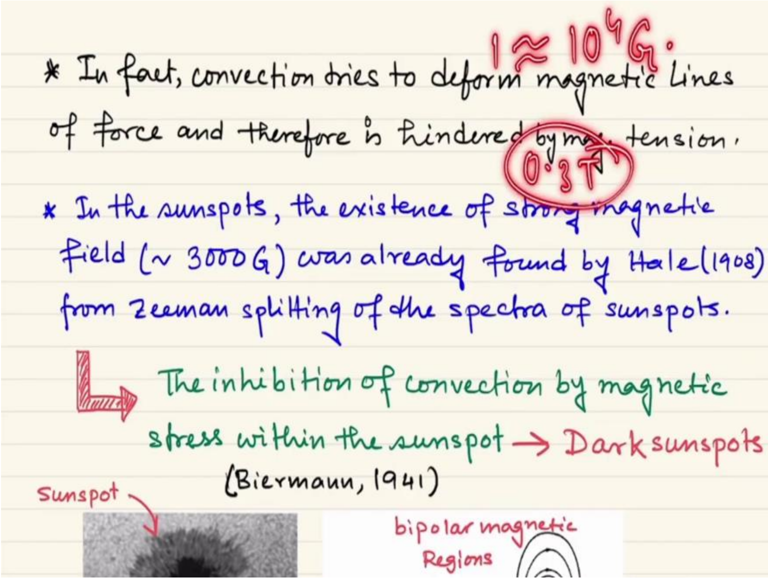

Why is this? Physically, it is because the convection tries to deform the magnetic lines of force, and therefore the magnetic tubes of force. And therefore, this is hindered by the magnetic tension, because magnetic tension does not entertain any type of sudden deformation of the magnetic lines of force or tubes of force.

So, of course, now if your temperature gradient is so important that this magnetic tension effect is getting over dominated by the temperature gradient force, then you can have convection there.

Now, why this is important? because we all know that Sun contains dark spots on the surface, and they are called sunspots. And one very popular question in space and astrophysics is why the sunspots are dark? So, it is true that the sunspots are the place where you can see very strong magnetic field of the order of 3000 Gauss. So, 1 Tesla is 10^4 Gauss. So, it simply says that it is nearly 0.3 Tesla, it is very very strong. Whereas, in the solar wind, the magnetic field is of the order of nano tesla (nT).

Before that I have to mention one thing, this was for the first time found by Hale in the year 1908, and he found that from Zeeman splitting of the spectra of sunspots.

And then he said that the inhibition of the thermal convection by magnetic stress within the sunspot makes them colder, because the thermal convection is not efficient. So, from the convection region to the surface, the energy is not transported efficiently, and that gives sunspots a lower temperature and that is why you see them as dark. And that was proposed for the first time by Biermann in the year 1941.

Now, just before going to another topic, let me just tell you the dimension of the sunspot, just for you to have an idea.

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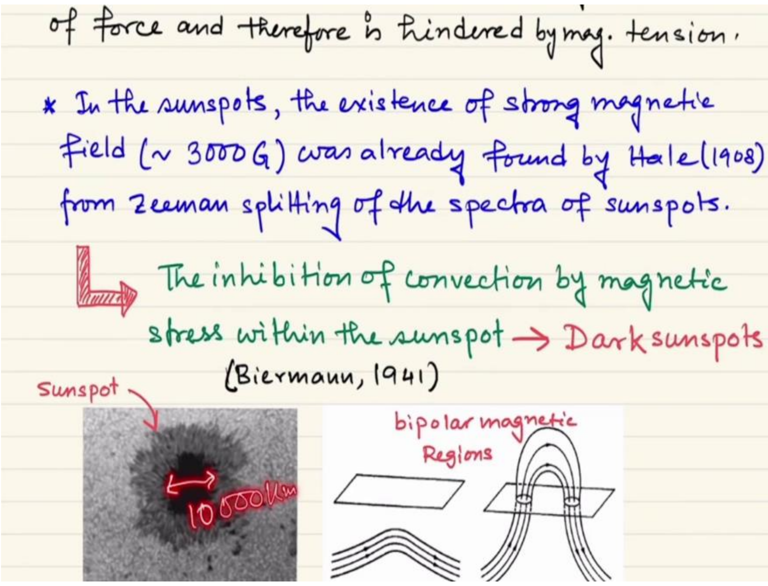

So, the sunspots are of course nearly 10,000 kilometers in size. And actually you can see that there are sunspots in which the total Earth can actually go in.

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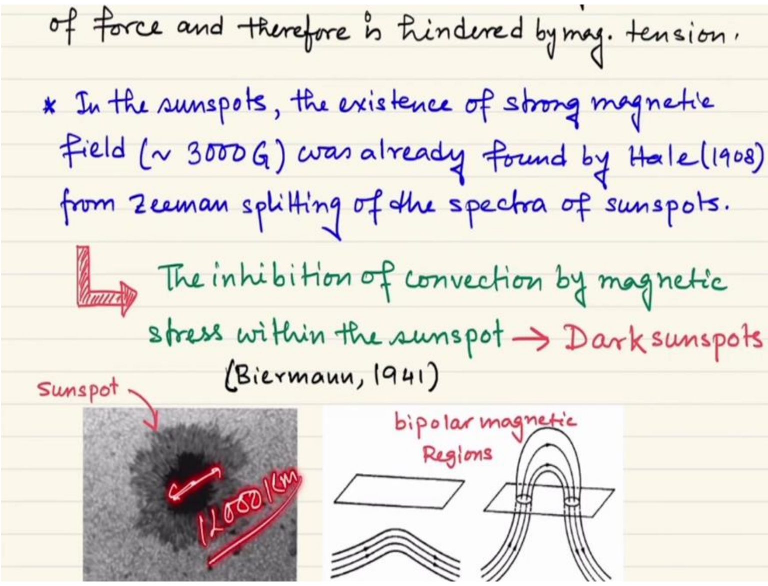

So, Earth has an average diameter of 12000 kilometer, so it can enter some sunspots. Now, that was the reason for the darkness of the sunspot. There is another very interesting thing that when many astronomers observed, but Hale was the first to understand and to explain the reason, that the sunspots are not situating in an isolated manner.

So, they are very frequently found in couples, located very close to each other.

And then in most of the cases, you can see that the flow of plasma which of course representing the magnetic lines of force type of thing is emanating from one of the sunspot and is just entering to the other sunspots, that means, that this two sunspots are of different polarity.

Now, these type of regions are known as bipolar magnetic regions. And they are very frequently seen. Now, bipolar magnetic regions, why are they formed, and why some plasma actually comes out from one side and to go to the other side? So, the reason is very simple, in general what happens that just above the surface, you have some environment where the magnetic field is very weak. Now, in the sunspots, the magnetic field is very strong. So, at the surface you should have a pressure balance that is very common sense.

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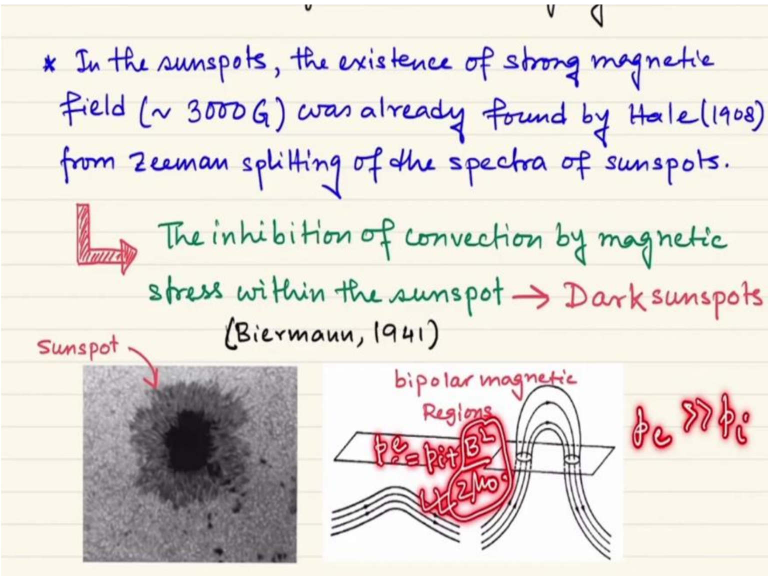

And now if you see that above surface you have the pressure external (p_e) is equal to pressure internal (p_i) plus the magnetic pressure $\frac{B^2}{2\mu_0}$. So, since $\frac{B^2}{2\mu_0}$ is very large inside, then $p_e \gg p_i$, so the internal pressure is low.

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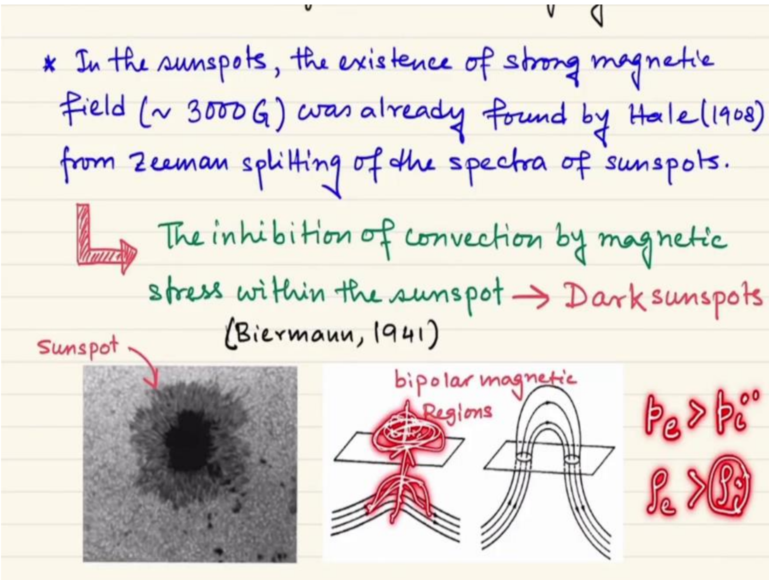

And if you simply just take, for simplicity, isothermal region or polytropic region, you can roughly say $\rho_e > \rho_i$.

So, it simply says that you have a lighter mass of plasma which is under the mass of plasma which is of greater density. Then what will happen? Then the plasma from below will try to get out due to buoyancy. And this is known as magnetic buoyancy.

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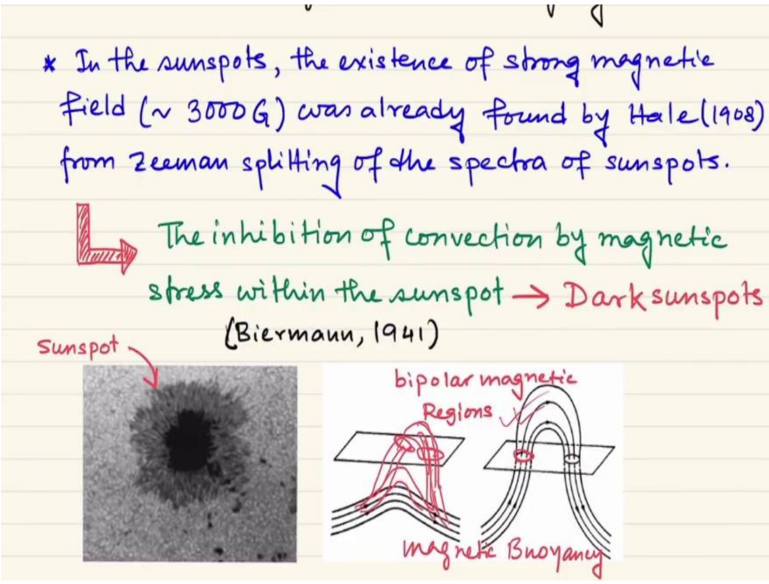

And the plasma simply pulls the magnetic field lines with it out of the surface, so then they simply gets out of a sunspot. And then of course, in order to close the loop of the magnetic fields, they enter through the other sunspot.

Now, that was only a very schematic explanation of what bipolar magnetic regions are, and how are they forming. But it is true that the true story is much more complicated.

And really it is still a matter of active or ongoing research that, what is the relation between the characteristic size of a sunspot to the corresponding magnetic strength of that region, and why exactly the bipolar magnetic regions are formed in a much more details. So, all these things are still ongoing and there are settle things as well.

So, if you are interested, I always will encourage you to go to any of the papers – recent papers on that. You have to just search. So, mostly all these things are taking place in the solar surface. So, you have to just search that magnetic buoyancy, the bipolar magnetic regions and then magnetic buoyancy in the sunspots, bipolar sunspots, all these things.

So, just start with different type of keywords. This is a trial and error method, and you will come across a bunch of papers. And you can see how people are trying to advance the understanding with time.

So, within the scope of this course this is all about the brief discussion of the application or the importance of MHD equations. Of course, MHD equations are not that much used for galactic fluids or other accretion disks.

Still now of course there are works on the role of magneto-convection for the effective transport in the accretion disk, but for the lack of time I am not going into that. So, the role of magnetic field is in the efficient angular momentum transport in an accretion disk.

And there is also another thing, if you remember that when we are talking about the Parker's model of solar wind, we took simply a hydrodynamic model. And then you can simply modify that model just by injecting magnetic field, and you will see that there will be something very interesting called Parker's stability, and you will have the spiral type of structures which are forming.

So, all these things you can see in any standard book. So, I am just telling you the names. So, within the scope of this course, I am just ending this discussion here. And from the next lecture onwards, I will try to introduce turbulence both in normal hydrodynamic fluids and plasmas.

Thank you very much.