Introduction to Astrophysical Fluids

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Lecture - 45
MHD fluids: magnetic pressure, magnetic tension and plasma beta

Hello, and welcome to another lecture session of Introduction to Astrophysical Fluids. In the last lecture, we qualitatively discussed how starting from the basic kinetic theory of a plasma we can reach to the fluid theory.

First, we described that for each species for example, the ions, the electrons we can describe or rather we can consider separate fluids so, then the plasma can be considered as a multi fluid. It is a combination of an ionic fluid; it is electronic fluid and if the plasma is very weakly ionized then neutral fluids; the neutral fluid that will also come into play.

Now, we also said that finally, if we are interested beyond a certain scale both in length and I mean both in space and time then simply the total I mean the charge quasi neutrality that is actually satisfied almost at each point of the flow field, so, very locally and then we can describe the plasma as just one single fluid, which is neither the pure ionic fluid nor the pure electronic fluid, but it is a combination of the two fluids.

This is known as popularly the mono fluid model of plasma or magneto hydrodynamics which is also sometimes called with its abbreviation MHD.

So, in today's discussion we will mostly be trying to throw some light on different interesting properties of such MHD fluids.

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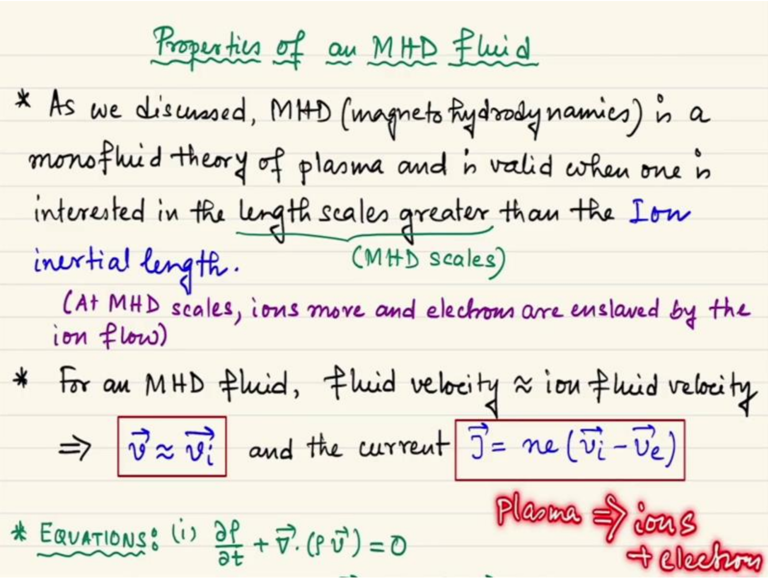

So, as we discussed last time that MHD is a mono fluid theory of plasma and it is valid only when we are interested beyond a certain length scale an ion inertial length. Of course, in terms of time scale you can also like estimate sometime corresponding to that.

So, that to be very honest I mean roughly speaking if we are interested in the time scales much greater than the inverse of the ion or electronic gyro frequency or the ion or electronic how to say that the gyro frequency or the reciprocal of the ion cyclotronic plasma frequency then where most of the cases it is almost guaranteed that we are in the MHD region.

So, roughly what happens in the MHD region is that we are in such a long length scale or time scale that even for our time scales or length scales the ions are moving considerably, and electrons are enslaved by the ion flow. As we described last time that if we simply with an enormous amount of patience if we study the movements of the clock’s hands, then it is true that it is possible to even see that the hours hand is also moving.

But, in most of the cases what happens that we do not have enough patience. So, we can only see the seconds hand and sometimes minutes hand, but hours hand to move we really have to wait for long time, and, if we have this level of patience here analogically then ions are also found to move and when ions are moved electrons just make a background and it simply follows the ions.

So, for an MHD fluid as we discussed that the inertia of the fluid is mainly governed by the ions, and that is why for the MHD fluid the fluid velocity is almost equal to the ion fluid velocity that was also evident from the formula. So, the mono fluid velocity which is nothing but, has velocity of the center of mass of the two fluids – electronic fluids and ionic fluids.

So, here when we are just talking this mono fluid model for the simplicity, we simply consider that our plasma is almost perfectly ionized and which is the case when this mono fluid model actually comes to be appropriate and then you have ions and electrons. So, the number of neutral atoms is negligibly smalls.

So, the $\boldsymbol{v}$ which is the MHD fluid velocity will be simply very close to the ionic fluid velocity, but the current on the other hand which is given by $\boldsymbol{J}$ is nothing, but is equal to $ne(\vec{v}_i - \vec{v}_e)$, n is the number density of both electrons and ions, e is the electronic charge.

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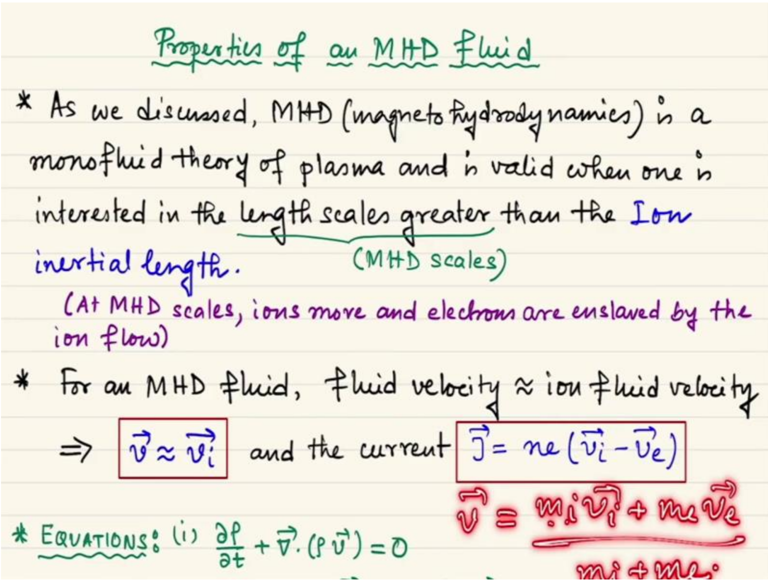

So, whereas, for this case if you remember that simply $\boldsymbol{v}$ was $\frac{m_i\vec{v}_i + m_e\vec{v}_e}{m_i + m_e}$.

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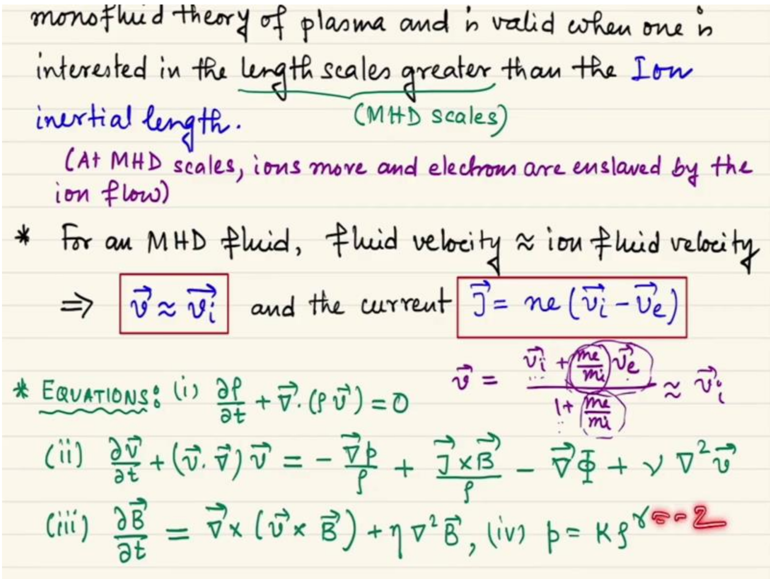

Now, since $\frac{m_e}{m_i}$ is negligibly small and we considered that both $\boldsymbol{v_i}$ and $\boldsymbol{v_e}$ are of roughly of the same order, then we can neglect the whole thing with respect to $\boldsymbol{v_i}$ and also, we can neglect this factor in front of 1 and that is why we finally, have $\boldsymbol{v}$ almost equal to $\boldsymbol{v_i}$. Now, for the current this is no longer true this type of the electronic contribution does not work here and we have to take the contribution of both these velocities.

Now, if you remember the four governing equations of magneto hydrodynamics as we said they are simply the continuity equation, 2nd is the momentum evolution equation on here, and the induction equation.

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \qquad \text{(i)}$$

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v}.\vec{\nabla}\right)\vec{v} = \frac{\vec{\nabla} p}{\rho} + \frac{\vec{J}\times\vec{B}}{\rho} + \vartheta\nabla^2\vec{v} - \vec{\nabla}\varphi \qquad \text{(ii)}$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{v} \times \vec{B}\right) + \eta\nabla^2\vec{B} \qquad \text{(iii)}$$

So, this term $\vec{v} \times \vec{B}$ comes from the if you remember the generalized Ohm's law and this $\eta\nabla^2\vec{B}$ is something analogous to the dissipation terms for the magnetic fields, and it is called the magnetic diffusivity.

Finally, we have to close the equation and since we are just again for the sake of simplicity, we are not considering the energy equation, we simply close the equation at this level just by saying that our fluid is poly tropic, even simpler than saying it to be barotropic. So, this is a simple version of barotropic where p has this specific form $k\rho^{\gamma}$.

Of course, once again γ can be anything like I mean real quantity, γ can be positive, negative, but just writing this form does not guarantee that the system is adiabatic. This is a general polytropic system, this γ can be equal to minus 2, for example, that is also possible.

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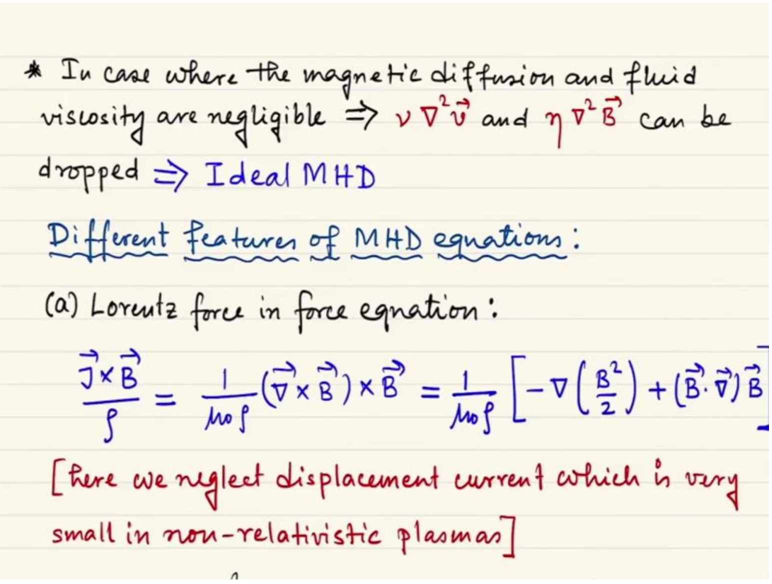

So, with these equations we can start studying the features of different properties of MHD equations. Now, just before that let me just point out one thing that where this magnetic diffusivity term $\eta\nabla^2\vec{B}$ with η and the fluid viscosity term is negligible, then I mean simply this $\eta\nabla^2\vec{B}$ and they can be ignored and we then talk of the regime which is known as ideal magneto hydrodynamics.

The good news is that in most of the cases of astrophysics, they had a strong exception, but in most cases of astrophysics and space physics I mean large number of phenomena can be explained just by using ideal MHD. Why? That I will be coming during some next discussion.

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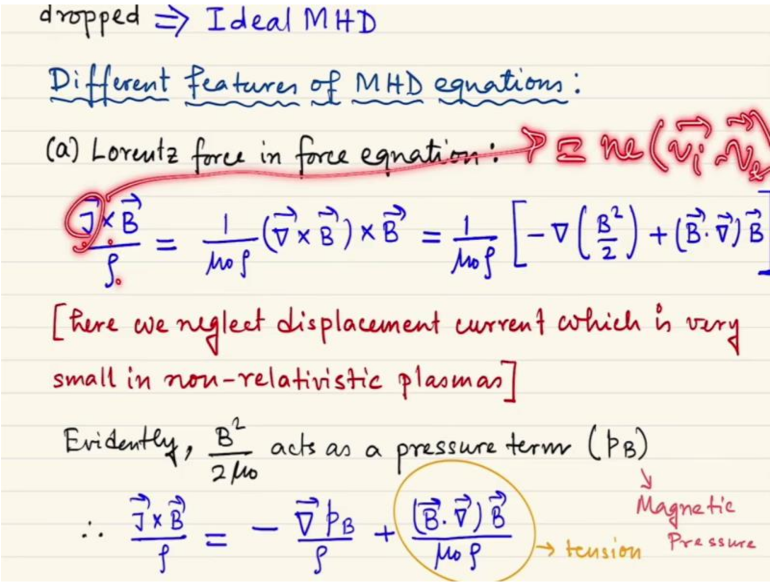

So, one specific feature of MHD equation is the type of Lorentz force in the force equation. So, that equation is absent in general for neutral fluids. So, neutral fluids are not charged. So, they do not have this Lorentz force in the force equation. So, Lorentz force is nothing, but $\frac{\vec{J}\times\vec{B}}{\rho}$. So, $\vec{J}$ is simply equal to $ne(\vec{v}_i - \vec{v}_e)$, but again if you remember Maxwell amperes law, what is $\boldsymbol{J}$?

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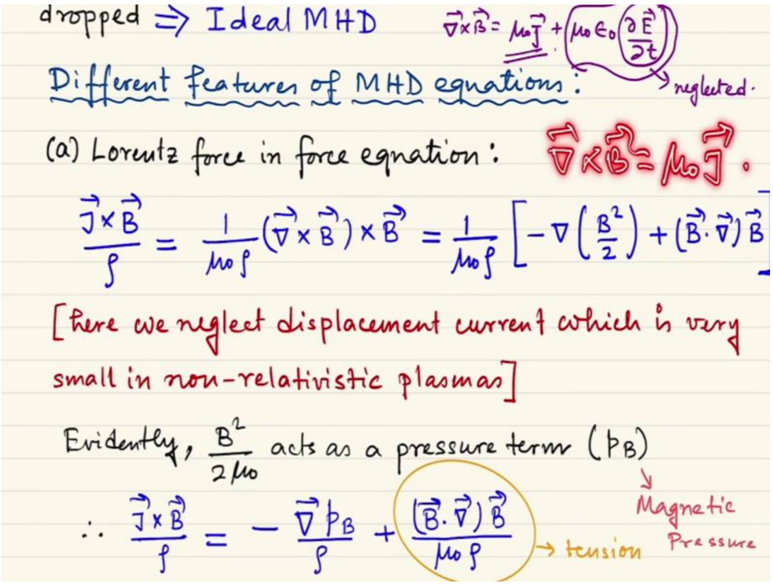

So, $\boldsymbol{J}$ can also be written as $\vec{\nabla}\times\vec{B}$ is equal to $\mu_0\vec{J}+\varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$. Now, this $\varepsilon_0\frac{\partial\vec{E}}{\partial t}$ is called the displacement term and this term $\varepsilon_0\frac{\partial\vec{E}}{\partial t}$ actually can be neglected with respect to this term $\mu_0\vec{J}$. This is neglected you can show by order analysis if the plasma is non-relativistic in nature, and we are considering here non relativistic plasmas. So, for our case $\vec{\nabla}\times\vec{B}$ is simply equal to $\mu_0\vec{J}$.

So, then we can write simply $\boldsymbol{J}$ as $\frac{1}{\mu_0}\vec{\nabla}\times\vec{B}$. So, this total thing comes then $\frac{1}{\mu_0\rho}\left(\vec{\nabla}\times\vec{B}\right)\times\vec{B}$ and if you know this special vector identity this is nothing, but $\frac{1}{\mu_0\rho}\left[-\vec{\nabla}\left(\frac{B^2}{2}\right)+\left(\vec{B}\cdot\vec{\nabla}\right)\vec{\mathrm{B}}\right]$. You see sometimes we will see in a short while that will be very important to understand two different aspects of Lorentz force.

Now, sometimes we say that why this is really useful to remember the vector identities or have a good grip over the vector identities both in vector algebra and vector calculus, the reason is that if you write you are turn in this expression you do not understand what is the physical significance, but once you ride in this expression you can actually see the effects are prominent then in this writing next.

So, that is why it is always good to have a good grip over the different type of vector identities and it is good to make a habit to write different expressions using vector identities like we are doing here.

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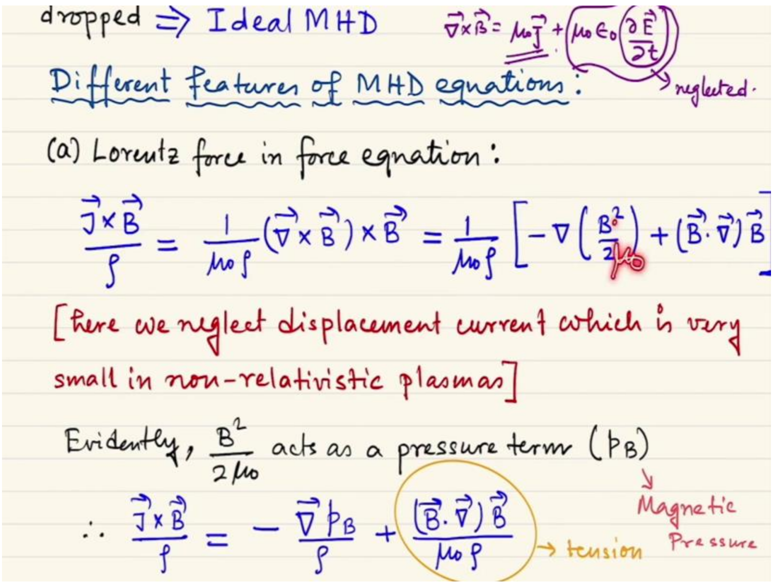

So, once I write in this form the whole thing you now see that I have $\frac{1}{\mu_0\rho}\left[-\vec{\nabla}\left(\frac{B^2}{2}\right)+(\vec{B}\cdot\vec{\nabla})\vec{B}\right]$. Now, μ_0 can be taken inside. So, it will be $-\vec{\nabla}\left(\frac{B^2}{2\mu_0\rho}\right)$ and what does that look like?

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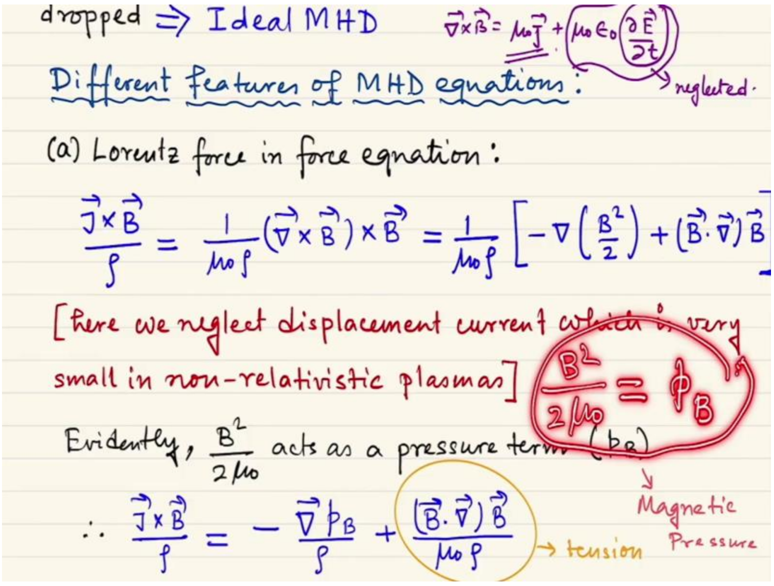

It simply looks that this $\frac{B^2}{2\mu_0\rho}$ is behaving like a pressure type of quantity which we call sometimes the magnetic pressure.

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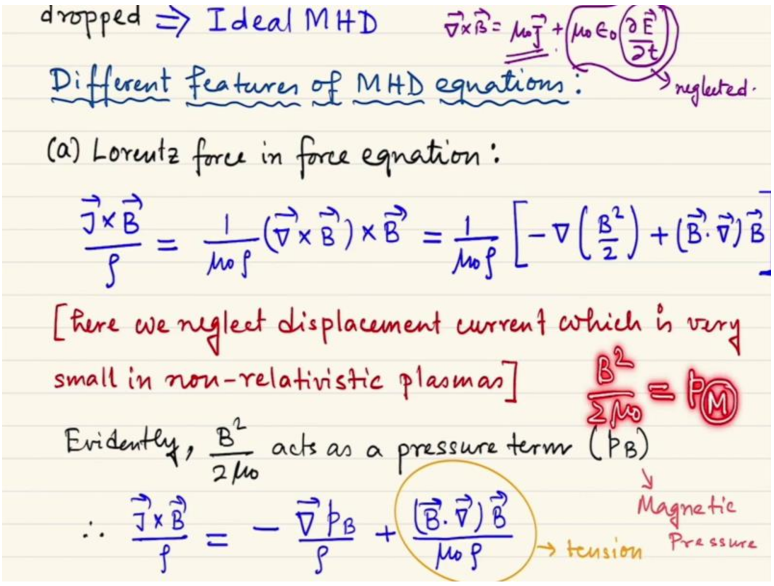

In several literature you can see $\frac{B^2}{2\mu_0\rho}$ this is nothing, but p_M, they use this symbol.

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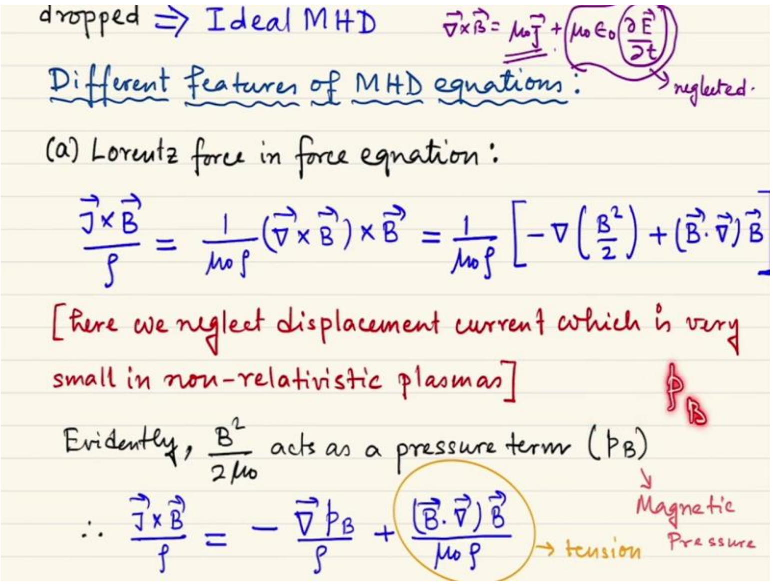

I am using here just p_B. So, this is a pressure term. So, in the total equation you will then have two terms – one is like this $\frac{\vec{\nabla}p_B}{\rho}$ and another will be like this $\frac{(\vec{B}\cdot\vec{\nabla})\vec{B}}{\mu_0\rho}$. So, the total pressure will be $p + p_B$, it is an effective pressure.

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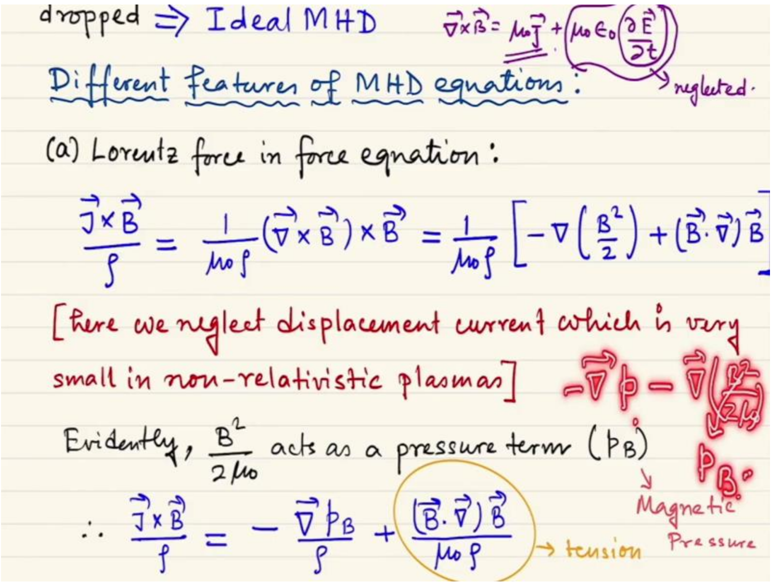

So, you see that this writing style gives us several significances of this whole expressions of this one $\frac{\vec{\nabla} p_B}{\rho}$ is like a pressure and we write this $\frac{(\vec{B}\cdot\vec{\nabla})\vec{B}}{\mu_0 \rho}$ and this one what is this? This one $\frac{(\vec{B}\cdot\vec{\nabla})\vec{B}}{\mu_0 \rho}$ is known as tension and why this is known as magnetic tension that we will be coming in a few minutes.

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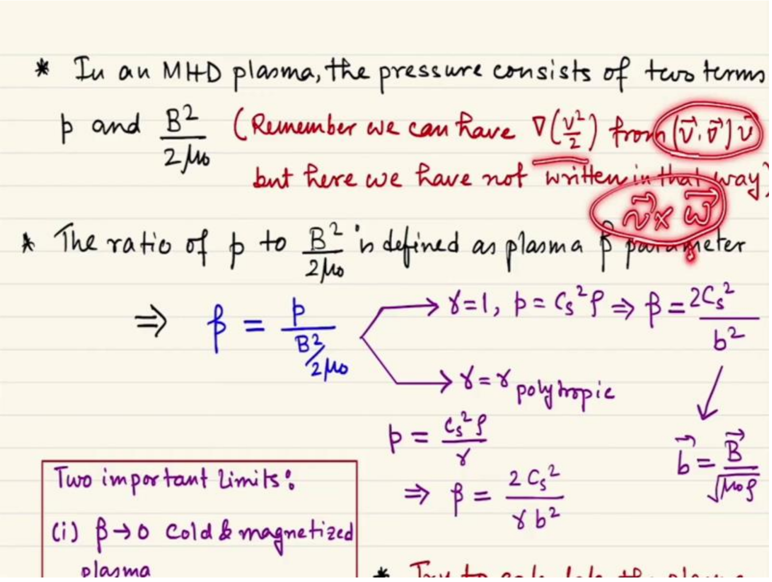

But, before that now, let us try to understand that in MHD equation we have two contributions from pressure type of thing. Of course, there one contribution can be coming which is I mean can come if we write this advective term $\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v}$ in terms of $\vec{\nabla}\left(\frac{v^2}{2}\right)$ and $\vec{v}\times\vec{\omega}$ which we wrote in the case where we were studying the effect of rotation.

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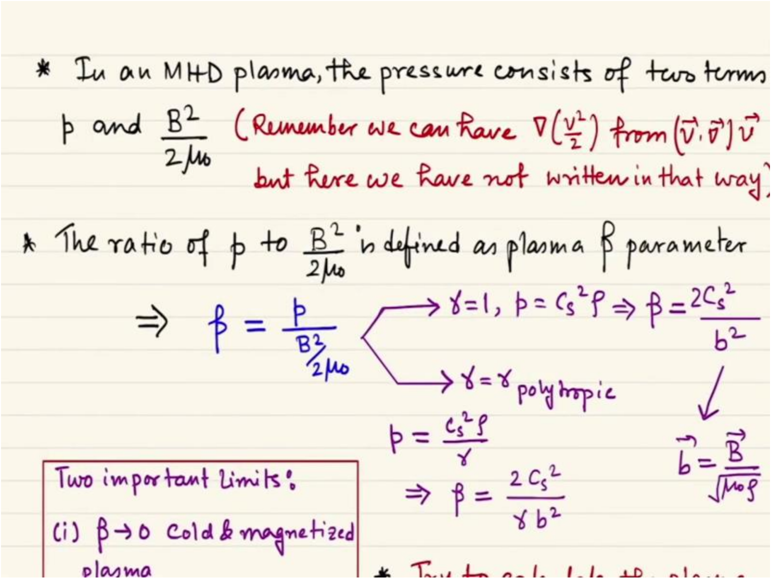

Here we will not write that simply because we know what is the meaning of this $(\vec{v}\cdot\vec{\nabla})\vec{v}$ and to be very honest, we have reduced this term $(\vec{\nabla}\times\vec{B})\times\vec{B}$ because in the equation we did not have any other terms like this, but $(\vec{B}\cdot\vec{\nabla})\vec{B}$ has a term which is identical to this and that is $(\vec{B}\cdot\vec{\nabla})\vec{B}$. So, we do not have to rewrite $(\vec{v}\cdot\vec{\nabla})\vec{v}$ in terms of $\vec{\nabla}\left(\frac{v^2}{2}\right)$ and $\vec{v}\times\vec{\omega}$, that we do not need.

So, in an MHD plasma the pressure consists mainly of two type of terms – one is p the normal pressure another is $\frac{B^2}{2\mu_0}$. One simple point to make here this $\frac{B^2}{2\mu_0}$ is also equal to the magnetic energy density, in an electromagnetic field if you remember. So, these two things now can act together on the MHD fluid and one can be greater than the other, one can be less than the other they can have different type of interplay.

According to the relative importance of one with respect to the other and how to measure that? A quantitative measure of that is given by the so-called β parameter of a plasma. So, β parameter is nothing, but the ratio of p to $\frac{B^2}{2\mu_0}$. So, p is in the numerator and $\frac{B^2}{2\mu_0}$ is at the denominator. So, finally, β is nothing, but p by $\frac{B^2}{2\mu_0}$. So, it will be simply $\frac{2\mu_0 p}{B^2}$.

Now, we will see two specific cases one is the isothermal case where γ is equal to 1 and so, p is simply proportional to the density that case we have already encountered previously for a normal fluid. So, this conclusion remains unchanged. So, an isothermal fluid is nothing, but p is equal to $C_s^2\rho$ and β is equal to $\frac{2C_s^2}{b^2}$, where this small $\boldsymbol{b}$ is introduced deliberately and this one actually much important for the physicists who work in space physics mostly.

So, this $\boldsymbol{b}$ is a magnetic field normalized to a velocity. So, just check that if $\boldsymbol{B}$ is divided by $\sqrt{\mu_0\rho}$, then it is just getting a dimension of a velocity. So, this one we just call as small $\boldsymbol{b}$. So, our β parameter will then simply equal to $\frac{2C_s^2}{b^2}$. Now, you can easily understand that if the temperature of the plasma increases then what happens? C_s^2 increases.

So, it's β value will increase. If on the other hand the magnetic field is very strong then β value decreases. Now, we consider another simple case which is the case of polytropic case. So, γ is equal to $\gamma_{\text{polytropic}}$, for example. So, then the pressure is no longer just proportional to ρ, but pressure can be written as $\frac{\rho C_s^2}{\gamma}$, but this C_s^2 itself is a variable.

So, just replacing the expression of p in this expression we can write β is equal to $\frac{2C_s^2}{\gamma b^2}$, and then you can easily see that both from these things we have the same conclusion that whenever you C_s is very important that means, the system has a very high temperature, β is high and if $\boldsymbol{B}$ is weak then also β is high. But on the other hand, if $\boldsymbol{B}$ is strong then β is low.

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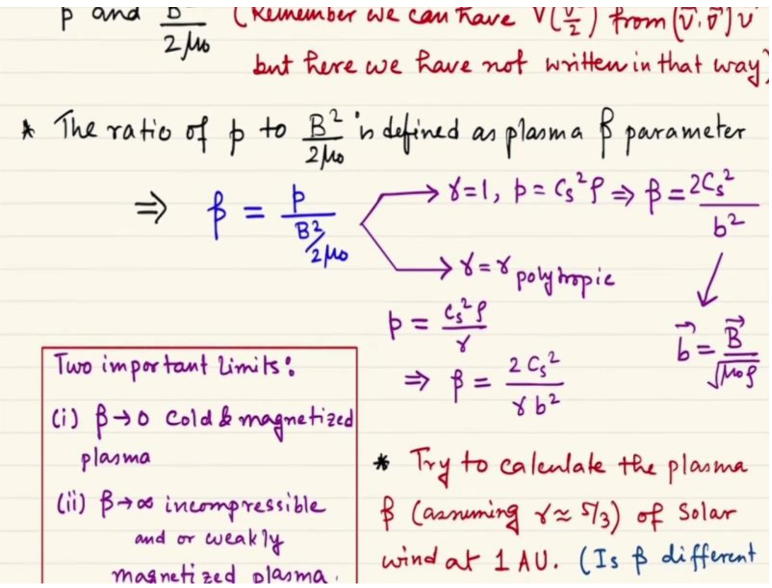

So, from this we now make two limits. So, β tends to 0, what does it imply? So, β tends to 0, can imply two situations, either of the two or both together that is first of all C_s is tends to 0 that means, the temperature is very, very low for the plasma. So, the plasma should be very cold and also there is another possibility is that $\boldsymbol{B}$ should be infinitely large.

So, it simply says that the plasma is subjected to a very strong magnetic field which is a combination of an external field plus the internal field. So, the total resultant magnetic field is very strong and this type of plasma is known as cold and magnetized plasma. So, it is true that if the plasma is very, very cold and even if the magnetic field is not very strong simply then also it is good enough to have a β value very close to 0.

But, if we have just two conditions together which we called cold and magnetized plasma then we must have β tends to 0. Now, on the other hand the diametrically opposite cases the β tending to infinity and that is the case of where you can easily see from here when β can tends to infinity when the temperature is very, very strong.

So, β tends to infinity for an isothermal plasma simply says that the plasma is very, very hot. But still the plasma can be written using this isothermal closure because if you try to go into the detail, you can actually see that if a plasma is very, very hot, then in general there can be much more sophisticated closures.

Now, for polytropic case if β is tending to infinity, it simply says that this C_s is a point to make actually. So, in normal literatures you can see that β tends to infinity gives as the incompressible plasma, but just try to think.

So, the thing is that first of all from this equation $\frac{2C_s^2}{b^2}$ you can have this idea that maybe when β tends to infinity it is simply makes C_s^2 tending to infinity and that gives us something nearly the incompressible medium, but this is a bit typical because this is not possible this relation is only true for isothermal plasma, and from isothermal plasma it is not evident to go to the incompressible limit.

On the other hand, from polytropic plasma one can go to the incompressible limit for γ tends to infinity very large value, but the problem is that the β is then equal to $\frac{2C_s^2}{\gamma b^2}$. Now, it is true that C_s^2 just tending to infinity and γ is also tending to infinity and actually C_s^2 by γ which is p by ρ is something which is finite for an incompressible flow.

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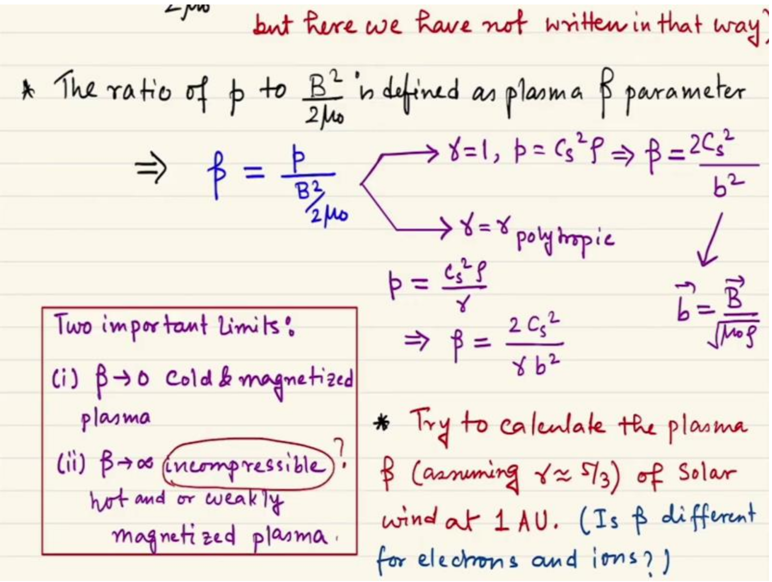

So, to be very honest in my opinion it is a bit. So, according to the literature although I wrote here this incompressible thing maybe it is more appropriate just to say that it is hot and weakly magnetize plasma.

But one thing is clear that when β is very, very large then C_s should be large and it should correspond to irrespective of γ high temperature and the same thing when the magnetic field is weak that means it will also correspond very high β.

Now, try to coming to a very practical question. So, we know now β value is important to know the nature of the plasma. So, now, try to calculate the plasma β assuming γ almost equal to $\frac{5}{3}$ for the solar wind at 1 astronomical unit. For that I suggest you to search over internet the typical values of different type of physical quantities so that you can calculate C_s^2 you have some idea about the magnetic field of solar wind at 1 astronomical unit.

Then you calculate both for ions and electrons and check which one is larger and finally, the question is that for a plasma what should be the combined or the resultant β, how to do that? So, think all these questions and if you are blocked you can write me over forum and we can discuss. Thank you.

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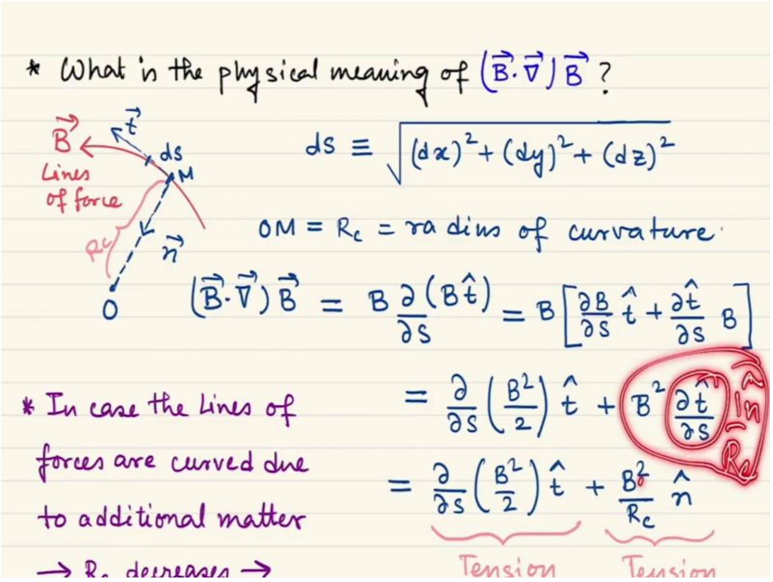

Now, coming to this question, so, we have already discussed about the magnetic pressure part, but I introduce this part $\left(\vec{B}\cdot\vec{\nabla}\right)\vec{B}$ which looks like an advective term, but in terms of the magnetic field only and this is known as the magnetic tension, why?

So, just consider this red curve I mean it represents one line of force of the magnetic field. So, it is the magnetic line of force. Now, I am simply interested in this small segment and within the small segment, the tangential direction is along $\boldsymbol{t}$ and this segment I call as ds and the radius of curvature is R_c and O is the centre of curvature.

Now, ds is nothing, but we all know $\sqrt{(dx)^2+(dy)^2+(dz)^2}$ there is no surprise and OM is the radius of curvature. $\boldsymbol{n}$ is the unit vector which is acting radially inward from that line of force to the center of curvature. Now, I write $\left(\vec{B}\cdot\vec{\nabla}\right)\vec{B}$ and try to analyze this, what is this? So, this is nothing, but the directional derivative of $\boldsymbol{B}$ in the direction of $\boldsymbol{B}$ itself and this is $B\frac{\partial}{\partial s}$ because $\frac{\partial}{\partial s}$ is the partial differentiation along this line of force.

So, ds is simply the segment along this line of force. So, $B\frac{\partial}{\partial s}$ which will be now acting on $\boldsymbol{B}$ and what is that $\boldsymbol{B}$ for this ds, this is the $\boldsymbol{B}$ magnitude times $\hat{t}$, this $\hat{t}$ is the unit vector. So, the magnetic field is at every point of a magnetic line of force the magnetic field is just tangentially directed that is the definition. So, we write this in this $B\frac{\partial}{\partial s}(B\hat{t})$ form and then we expand as $B\left[\frac{\partial B}{\partial s}\hat{t}+\frac{\partial \hat{t}}{\partial s}B\right]$.

Of course, here the unit vector changes its direction with time. So, that is why this term $\frac{\partial \hat{t}}{\partial s}$ is known nonzero and this term $B\frac{\partial B}{\partial s}$ is easy to analyze. So, $\frac{\partial}{\partial s}\left(\frac{B^2}{2}\right)\hat{t}$ because. You can simply write this $B\frac{\partial B}{\partial s}$ in this form $\frac{\partial}{\partial s}\left(\frac{B^2}{2}\right)$ and what about this term $\frac{\partial \hat{t}}{\partial s}B$? This term just using simple vector algebra you can simply find that this term $\frac{\partial \hat{t}}{\partial s}$ is given by $\frac{1}{R_c}\hat{n}$.

So, if this is the case then the whole thing will be $\frac{B^2}{R_c}\hat{n}$. So, this one $\frac{\partial}{\partial s}\left(\frac{B^2}{2}\right)\hat{t}$ is the tension along $\boldsymbol{B}$ because this is along the tangent direction, and this one $\frac{B^2}{R_c}\hat{n}$ we are calling tension perpendicular to $\boldsymbol{B}$. So, if for example, the magnetic field intensity increases, then this tension will be increasing and thereby making a force so that the magnetic lines of force are well tied, just like a cord under attention and this one is much more interesting.

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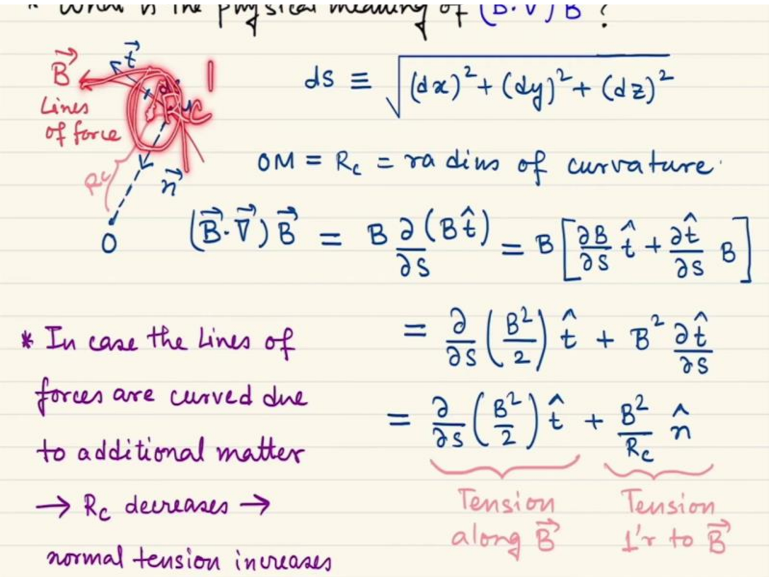

In the case, if you can see that you have the magnetic lines of force which are getting much more curved due to some additional matter or some increase in the magnetic field intensity and when it is getting curved then it's radius of curvature decreases. So, radius of curvature now decreases this is now R_c'.

When radius of curvature decreases this term $\frac{B^2}{R_c}$ increases and then it simply says that when it has this type of geometry then you have something net force which is acting in this direction. So, that means, whenever this tension, what does it do? Whenever this magnetic field gets deformed from its original position, it tries to bring it back to the original position. That is why it behaves like a tension.

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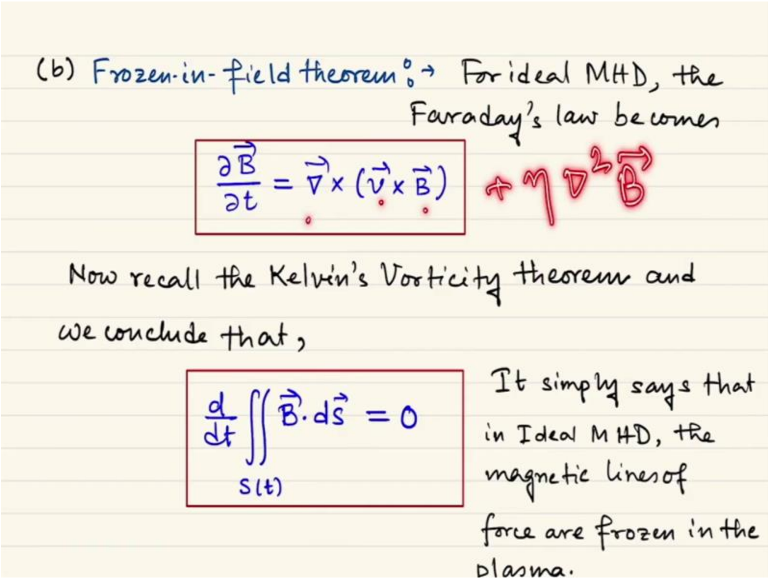

So, that was all about the tension part. Now, coming to another interesting thing which is the frozen in field theorem for MHD. So, for ideal MHD we all can now remember from the last discussion that the Faraday's law actually becomes $\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B})$ because the $\eta \nabla^2 \vec{B}$ is dropped. If we recall from our previous lectures when we were talking about the different properties of ideal fluids, then we talked about the Kelvin vorticity theorem.

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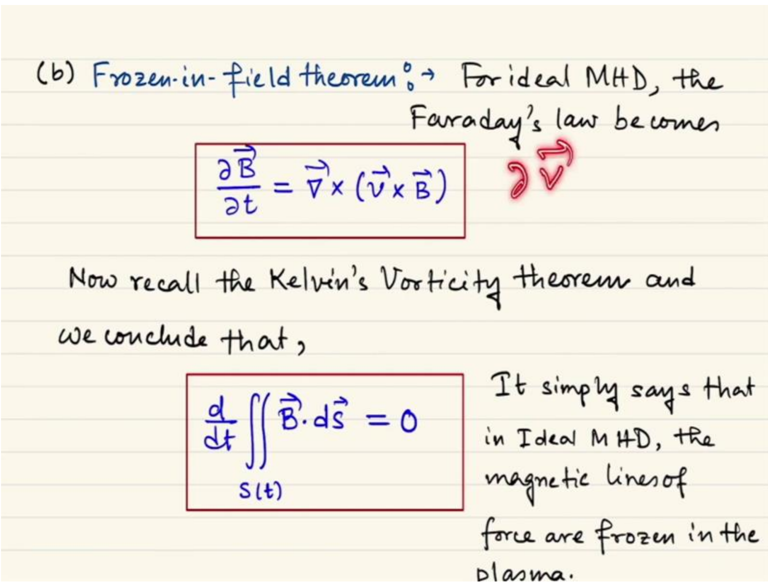

And, we said that any vector field when it just satisfies something like $\frac{\partial \vec{A}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{A})$.

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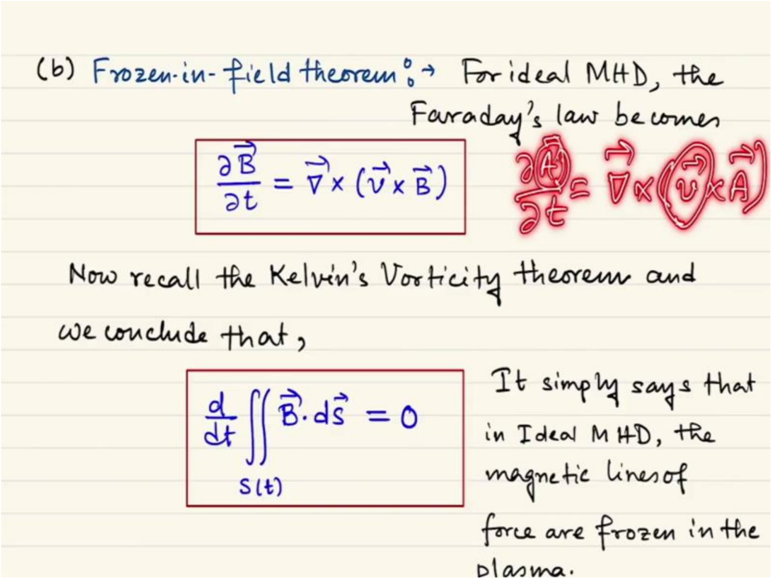

So, any vector field $\boldsymbol{A}$ if it satisfies evolution equation like this $\frac{\partial \vec{A}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{A})$, where $\boldsymbol{v}$ is the fluid velocity.

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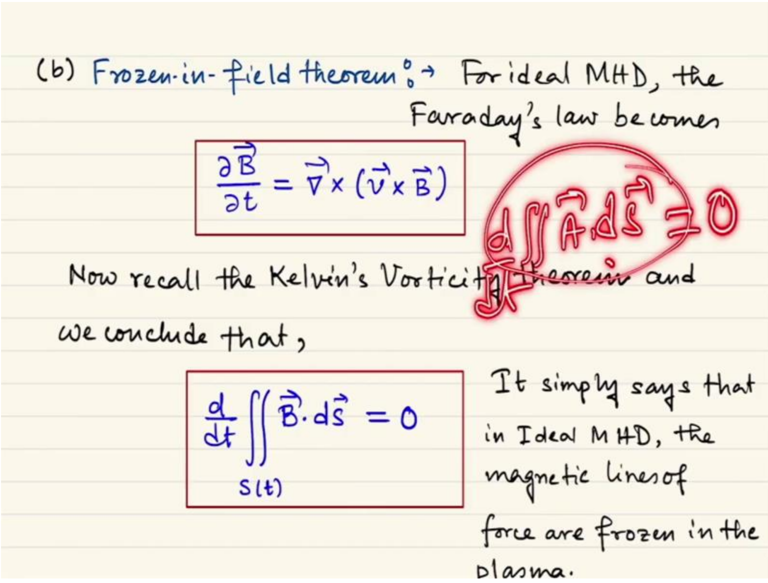

Then this quantity will have an associated flux which will simply be then $\vec{A} \cdot d\vec{s}$ which will be conserved along any surface inside the fluid and another way of writing this is

$$\frac{d}{dt}\iint \vec{A}\cdot d\vec{s} = 0.$$

So, that is exactly what we have written over here that the field which is satisfying this magic equation is $\boldsymbol{B}$, the magnetic field.

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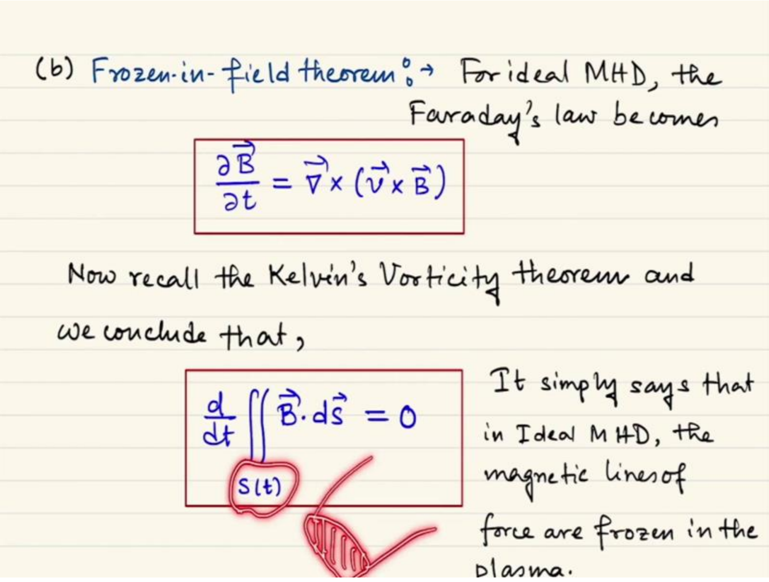

That is why $\frac{d}{dt}\iint \vec{A}\cdot d\vec{s} = 0$, where we are just talking about the any surface which is made by the fluid elements and moves with the fluid. So, of course, if it is made by the fluid element so, it is a real surface which is containing the fluid elements.

So, what is the meaning of this? That of course, this surface also changes its area with times. That is why I have written this $\boldsymbol{S}$ as an explicit time dependence. Now, it simply says this is equal to 0 means that in ideal MHD the magnetic lines of force they are frozen in the plasma or the matter.

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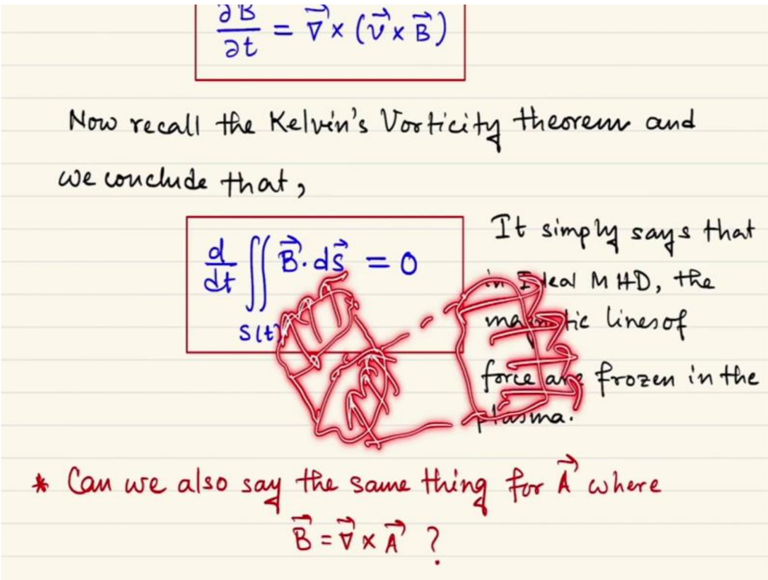

So, that is why whenever you take any arbitrary area in a fluid and then you just trace this area from here let us say now after sometime it is coming here or sometime let say it goes there. In both cases if you just calculate $\vec{B} \cdot d\vec{s}$ here or here you will always have the same value.

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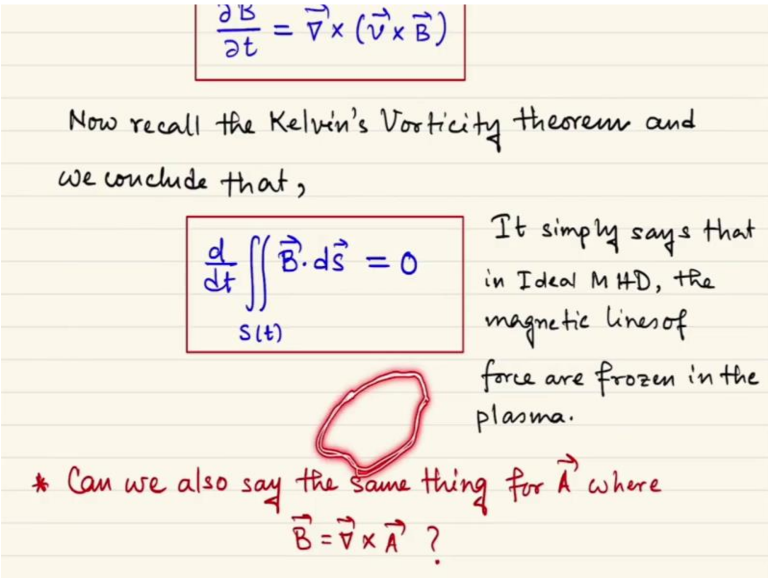

That simply says that because we are just following one surface made by the fluid elements, we are actually following a slice of matter. So, if we are keeping our target intact, then the flux is also intact. So, our target was just a slice of matter. So, it simply says that the magnetic field is frozen with the matter or with the plasma.

I have a small question for you can we also say that the same thing for $\boldsymbol{A}$, where $\vec{B} = \vec{\nabla} \times \vec{A}$? So, just then you have to derive the evolution equation for $\boldsymbol{A}$ and check whether this satisfies this $\frac{\partial \vec{A}}{\partial t} = \vec{\nabla} \times \left(\vec{v} \times \vec{A}\right)$ magic equation or not. This is typically very easy and you will see that $\boldsymbol{A}$ does not obey this or satisfy this type of solution equation.

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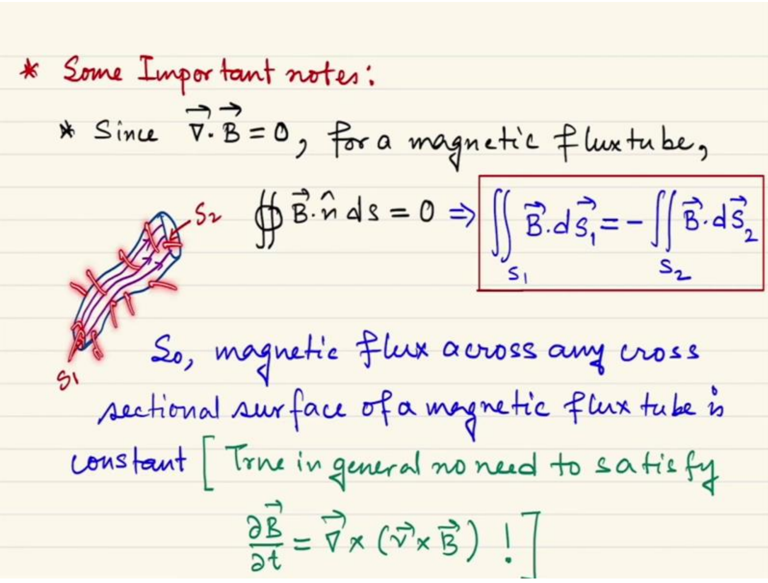

So, before ending this discussion, let me just tell you some important notes. So, $\vec{\nabla} \times \vec{B}$ is always 0. So, if you just take a magnetic flux tube in an MHD fluid, then if you consider a volume of this, then the total magnetic flux through the whole surface which encloses that volume of the magnetic flux of tube also is 0, that is simply because of the Gauss's divergence theorem.

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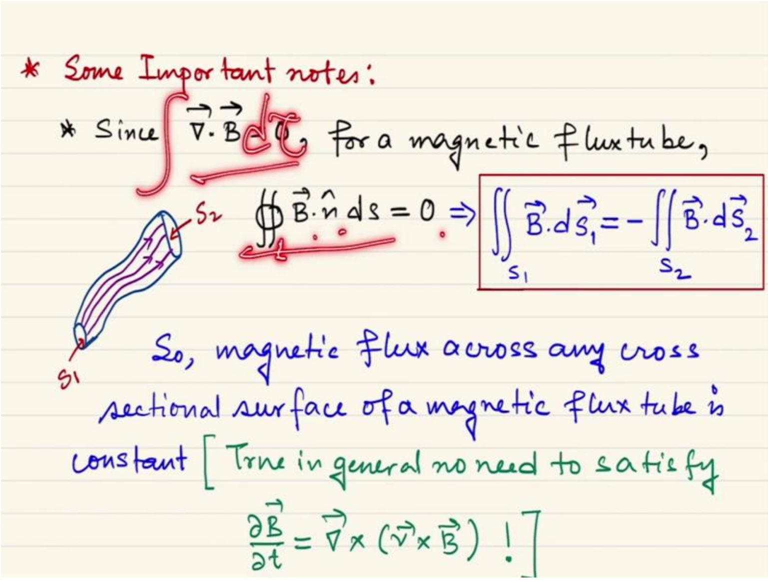

So, because you just have to do this $d\tau$ and that will be simply equal to close surface integral of $\vec{B}\cdot\hat{n}ds$ is equal to 0.

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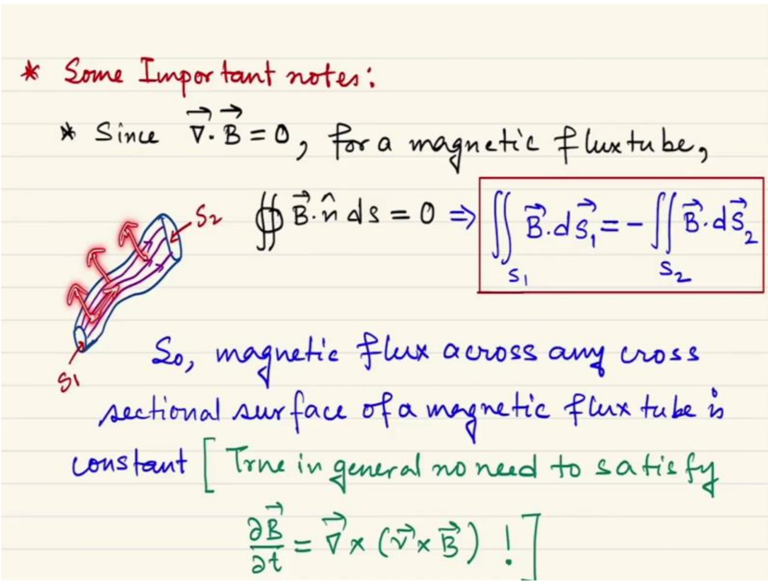

Now, the direction of the area of this lateral surface is always perpendicular whereas the direction of the magnetic field at every point is the tangential. That is why the total flux vanishes at every point of the lateral surface. So, this is only nonzero at this surface and this surface.

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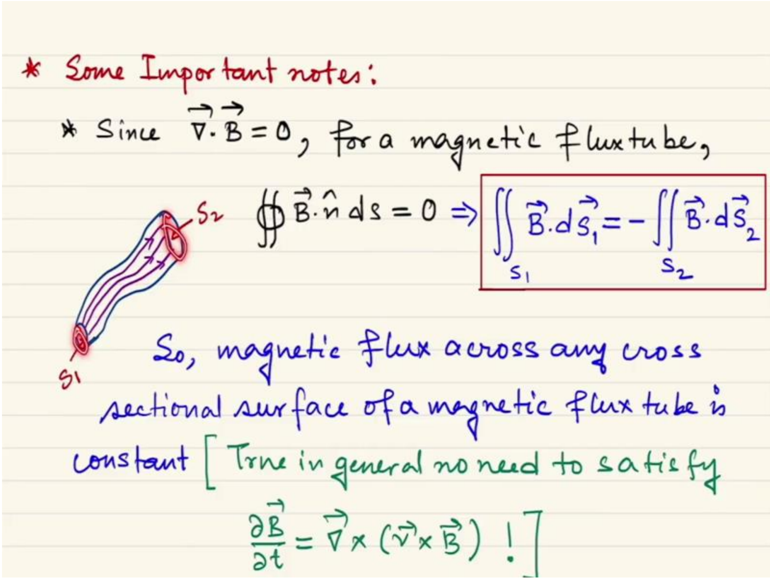

So, that is why the magnetic flux entering through the cross-sectional surface of a magnetic flux tube exactly is equal to the flux which is coming out of the opposite cross-sectional surface of that same tube.

So, magnetic flux across any cross-sectional surface of a magnetic flux tube is always constant. We do not need Kelvin vorticity theorem to be true that means, we do not need the magnetic field to satisfy the magic equation for that. It simply says that even if this is not true for some system, for example, if your system has some viscosity, then this is not true. There will be a term with viscosity $\eta\nabla^2\vec{B}$.

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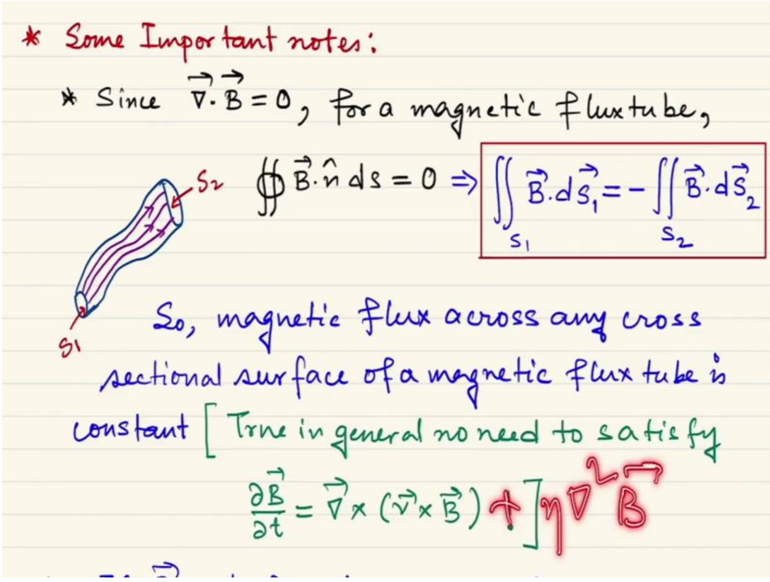

But, still then this fact is correct that the magnetic flux across any cross-sectional surface of a magnetic flux tube is constant.

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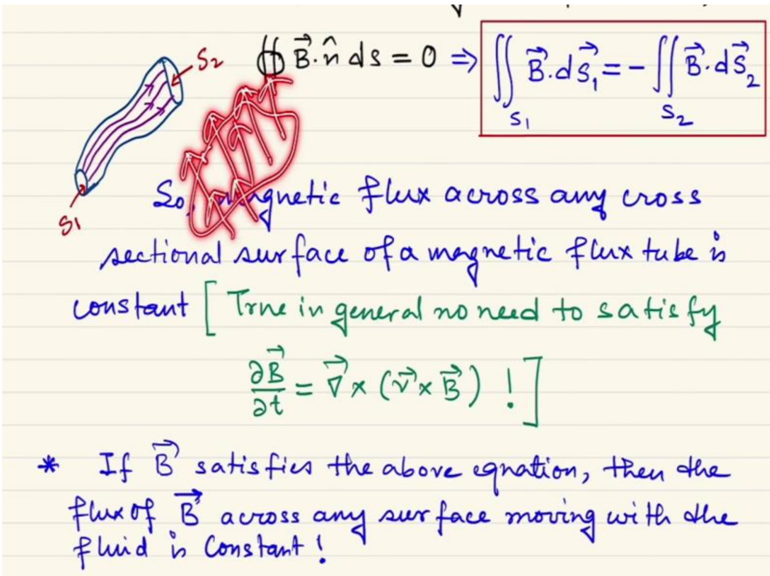

But which is not constant and correct if you take any arbitrary cross-section any arbitrary surface made by the fluid element inside the flow field, then if you calculate the total flux across the surface of the magnetic field.

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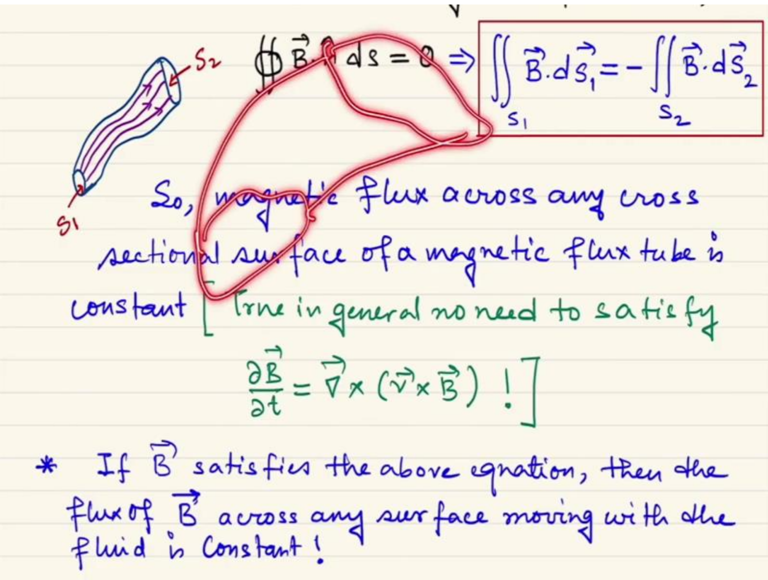

Then whether these will be the same to that surface which is evolved in time and now is situated to a new position.

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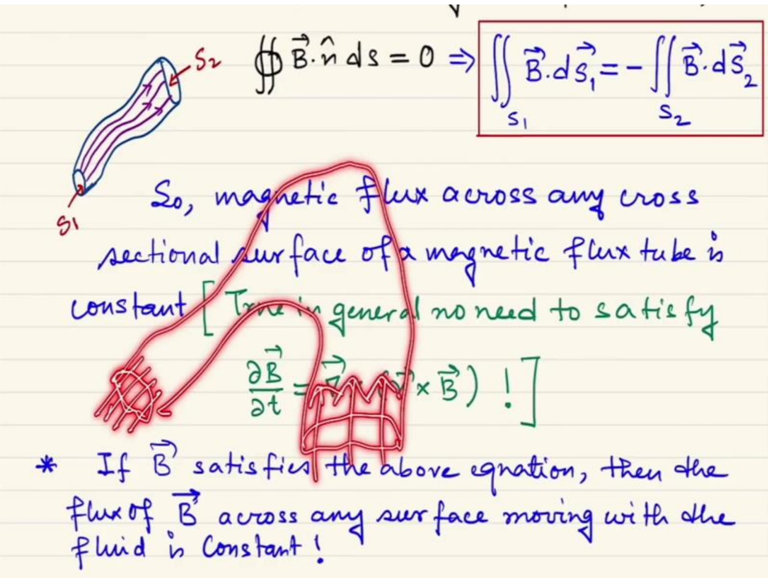

If you just start with a surface over here you calculate $\vec{B}\cdot d\vec{s}$ and then you take the integral over this surface then this surface with the fluid flow, for example, now comes here. Now, you calculate again integration $\vec{B}\cdot d\vec{s}$. The question is whether this will be equal. Yes, for that to be equal ***B*** has to satisfy magic equation.

So, this is all about the frozen-in theorem for the magnetic field and in ideal MHD.

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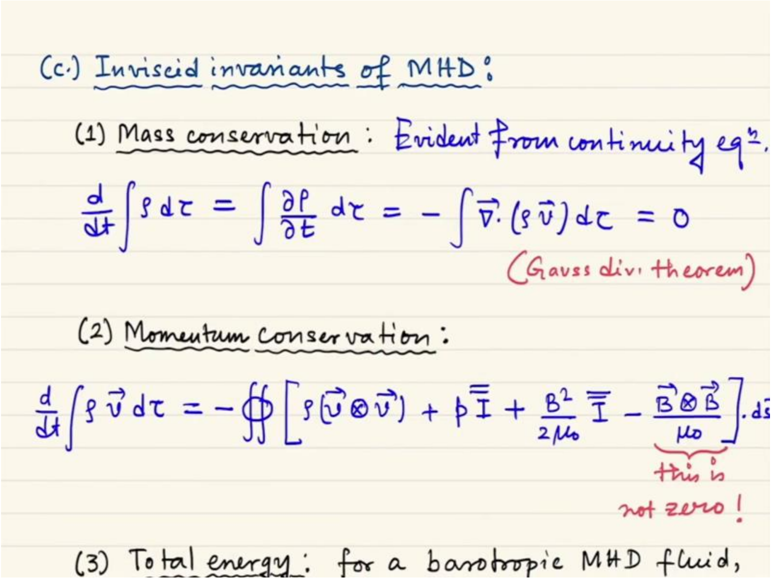

So, in the next discussion I will start discussing about the inviscid invariants of MHD fluids.

Thank you very much.