Introduction to Astrophysical Fluids

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Lecture - 44
Kinetic to fluid picture of plasmas

Hello, and welcome to another lecture session of Introduction to Astrophysical Fluids. Now, we have already started discussing the plasmas, and I have already introduced some very basic concepts of plasma, how to define a plasma, how to characterize a plasma, how to describe the plasma, and we saw that as plasma is an ensemble or I mean is a system consisting of charge species then we need actually 2 sets of equations.

So, one is for the describe in the matter part another is to describe the electromagnetic fields, right, and for electromagnetic fields we know that of course, the Maxwell's equations. There is no ambiguity about that. But how to describe the matter, then we have several options.

Depending on the interest from case to case sometimes, we will describe plasma in terms of particles, where the particle behaviors are important to explain certain phenomena. Sometimes try to describe the plasma as a continuum then we define a plasma fluid, and sometimes we actually need the 2, then we talk about hybrid models as I said. So, this really creates a very interesting structure of plasma theory.

That is why this is much vaster with respect to the normal hydrodynamic theory, and also as unlike the normal kinetic ensembles which is made up of normal neutral gas molecules, a plasma actually consists of particles which always have long range interaction through coulombic interaction which is totally absent in case of neutral gas molecules. Then, the analytical treatment of such a system is also non-nontrivial and sometimes becomes very much complex.

Sometimes even we do not know whether we can treat sufficiently those the sets of equation, the systems of equation just by analytics then we have to simulate. So, in this lecture I will give you a very brief overview, on the different stages of description of a plasma. So, there are a lot of mathematical details which I will skip, and I will just say very qualitatively I mean maybe I will take help of some equation which are indispensable.

Otherwise, it will be mostly a very superficial guided tour from kinetic aspect to a fluid aspect, and finally, our interest is to talk about the fluid models, and the corresponding utilities or I mean how to say that corresponding importance or applications in case of space and astrophysical context.

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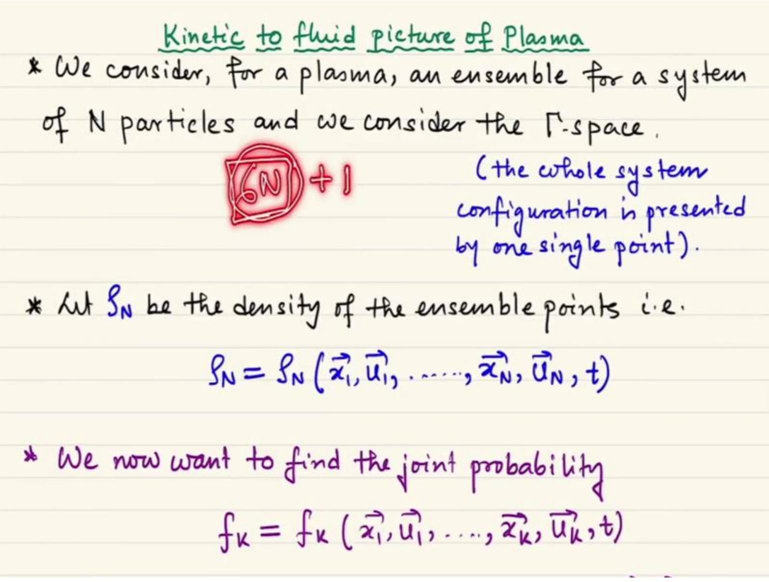

So, if you remember that we will just proceed as we did for normal hydrodynamic case that we consider even for a plasma that it is an ensemble of N particles, but this N particles can be charged, that is the thing, and we consider the Γ space. Now, if you remember from the lectures of the first week of this course.

So, Γ space is the whole system configuration space where every single point. So, Γ space for example, for a system with N particles and each particle have, for example, 3 degrees of freedom in space and 3 velocity components. So, a Γ space will be in general of $6N$ dimensions, right, and if you want to include the time as 1 dimension then this will be $6N$ plus 1 otherwise you will just say my Γ space is 6N dimension and time acts as a parameter.

So, in Γ space, the total system configuration at one instant is presented by one single point in the space, and we can also just as you know that we can define ρ_N which is the density of this ensemble points. Then we know that we can also create ensembles.

So, an ensemble is nothing but collection of points was describing the same type of system only differing from each other by the choice of initial conditions. So, then ρ_N we can define to be the density of such ensemble points and this ρ_N is of course, a function of x_1, u_1, up to x_N, u_N, and time t as well.

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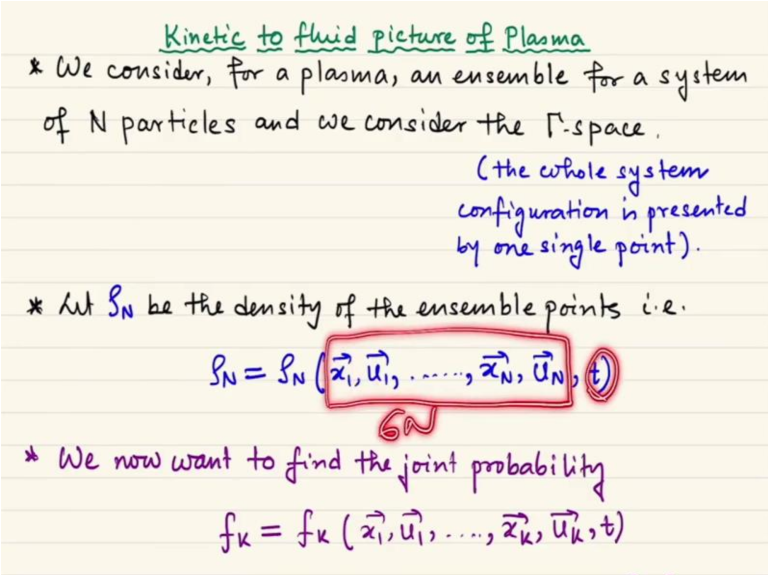

So, every variable has 3 components because these are vectors. So, we have $2N$ such variables, so it will have $6N$ and then t.

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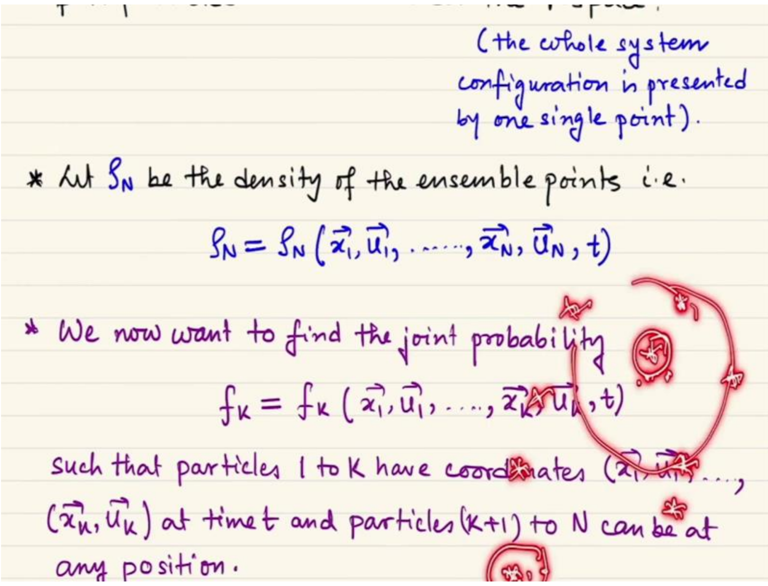

Now, since unlike a normal system of neutral gas molecules here always one species is acting with the other by coulombic interaction. Then, we actually will be interested to find the joint probability because the one particles velocity and position is always governed by another particle.

So, for example, if this one is getting closer to this one, irrespective of whether these are I mean of the other irrespective of the positions and the velocities of the other particle there will be some effect as well. Of course, we know from our previous discussion that if these 2 particles are very far away, so that this particle is actually screened by its neighboring particles then the effect of this particle to this particle is negligible.

But, in general, we cannot neglect the effect of any arbitrary particle with the other, and then, we will be interested to find the joint probability to say that which is f_k. So, f_k is a joint probability distribution, so which is a function of x_1, u_1, x_2, u_2 and also x_k, u_k and t.

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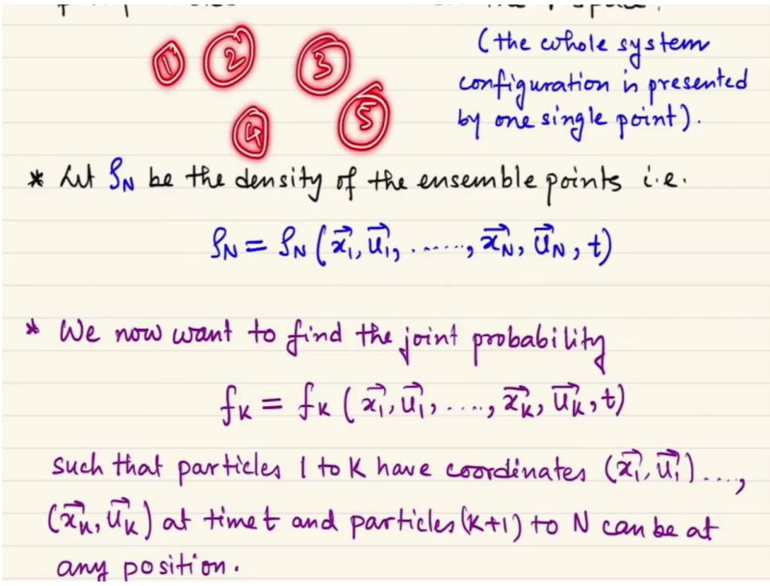

So, this is simply the probability that the particle 1 to k have coordinates x_1, u_1, I mean for example, if we just label the particles 1, 2, 3, 4, like this, then this joint probability distribution just says that this is the probability distribution. So, that the particle 1 is at x_1, u_1, at a given time t_0. We are talking about something which is simultaneously occurring, and particle 2 will be at x_2, u_2.

Particle 3 will be at x_3, u_3 and particle k will be at x_k, u_k. If we specify all these things then the question is that what is the probability that this positions and velocity are attained simultaneously.

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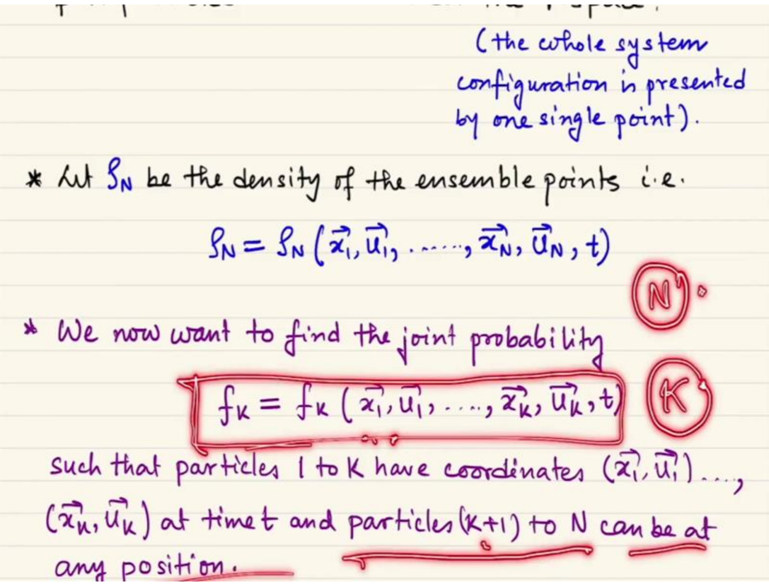

So, let us say we have N particles, and from N particles we just say that we are just interested in k particles out of N whose velocity and position we have designated already, and we want to know the joint probability distribution, so that they have this design I mean when p designated positions and velocities at a given time. But all the other particles for example, particles $k+1$ to N, all the $N-k$ particles, they can be at any position and velocity.

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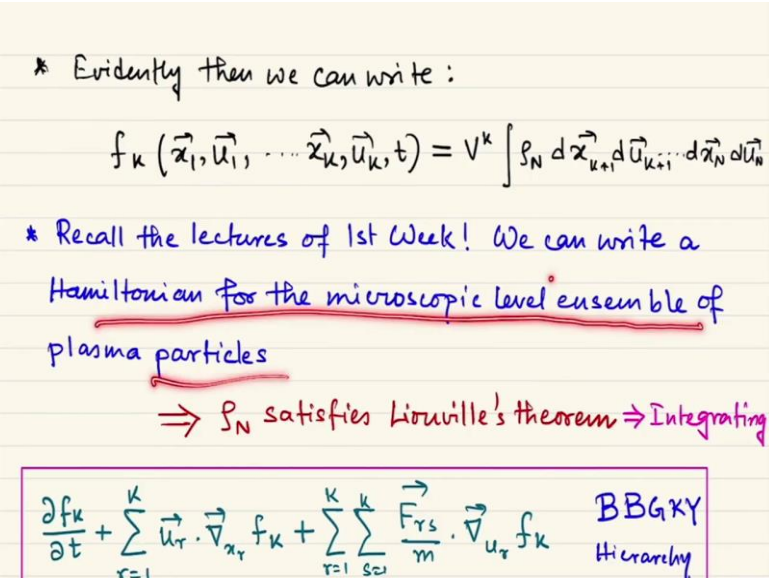

So, if it is the case then of course, we know this one is given by is equal to V^k integration over, so this V is the volume, $\rho_N dx_{k+1} du_{k+1}$ up to $dx_N du_N$, where all these things can vary from minus infinity to plus infinity, right. Because they can have any position and momentum that is the story for the rest $N-K$ particles, is it clear?

Then, the thing is that if we recall the lectures of the first week, then we can write a Hamiltonian for the macroscopic level ensemble of plasma particles because at microscopic level there is no dissipative force. All the fundamental forces are there and they are actually conserving energy, so we can write a Hamiltonian for the system, and we know that ρ_N which is the density of ensemble points these will or rather should satisfy Liouville's theorem.

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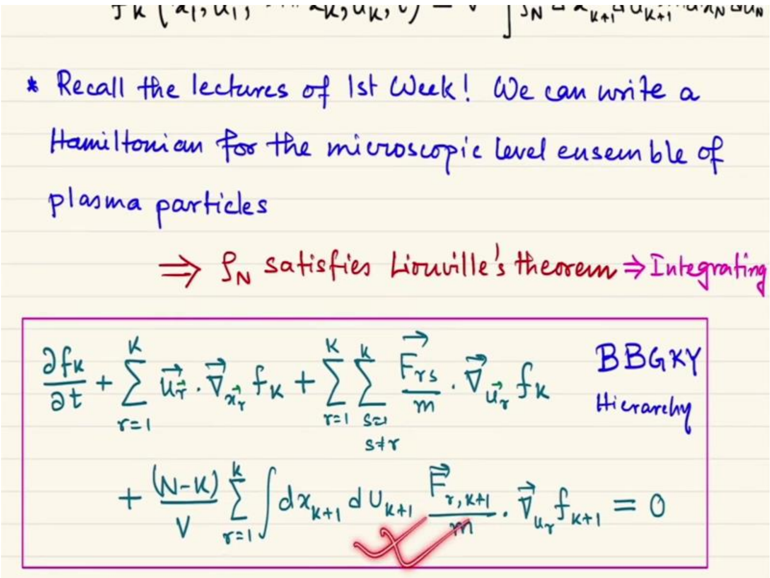

So, if you now just use this one this Liouville's theorem, let me just tell you something like this. So, this is the density of ensemble points and this is the joint probability distribution and if you just follow that ρ_N satisfy Liouville's theorem then after integration basically if you simply see that every ρ can be actually attend from some probability distribution.

Then you can finally, get the evolution equation for the joint probability distribution I mean the joint probability distribution of k^{th} particle. Just a minute. So, once again, so this V_k is simply introduced just for the normalization purpose. V_k is nothing, but the volume occupied by K particles.

Those k particles for which we are like now interested with a designated position and velocity, and then from this one, you have to actually just follow the same rule as we did for normal hydrodynamic case, and then you just can get this type of equation. Just write the Liouville's theorem with this type of expressions, and then you can simply write this the equation for this f_k. Now, f_k does not say that this is the probability distribution function for k^{th} particle.

This is a bit confusing here. So, f_k means this is the joint probability distribution of k particles, and for example, f_1 that will be the distribution function of one single particle. If it is f_2 then it will be the joint distribution function of 2 particles. If it is f_3 then this will be the joint distribution function of 3 particles, this type of thing. So, this is quite complicated actually. I do not want to enter into the details of this.

So, if you are interested you can just search in any books, any standard book of plasma physics. So, you see that statistical mechanics that is also a good choice. But there is that book should contain the statistical mechanics of plasma.

Now, you see finally, we have an evolution equation for f_k and where you can see that this is nothing, but the equation where you have this $\vec{u}_r.\vec{\nabla}_{\vec{x}_r} f_k$ of x_k, this is the space variation, of course, and this is the convective type of term, and this is the term which comes from its gradient in the velocity space, and here you can actually, all of them should be vectors not very much correct in writing like this. So, all these things are vector.

Then you will see, this $\frac{\vec{F}_{rs}}{m}$ is the force type of term, and this $\vec{\nabla}_{\vec{u}_r}$ is the velocity gradient of f_k, and this third term, basically comes as an extra and this one gives you the information about the f_{k+1}. So, it simply says that if you want to study the evolution equation for f_k that means, let us say if you are interested to study the evolution equation of f_1, that means, how does the single particle distribution function evolve in time.

Then, you have to know the information about f_2, for example, that means, at least the functional form of joint probability distribution. The same thing if you want to know $\frac{\partial f_2}{\partial t}$ you have to know about f_3 and so on. So, this gives us a hierarchy of equations and this is known as BBGKY hierarchy. There are 5 scientists. So, you just search their names that is you to do that.

Then you will see that this basically creates a problem in closing the system of equation because at every point if the evolution equation of k^{th} of the distribution function, k particles include the distribution the knowledge or needs the knowledge of joint distribution function of $k+1$ particle.

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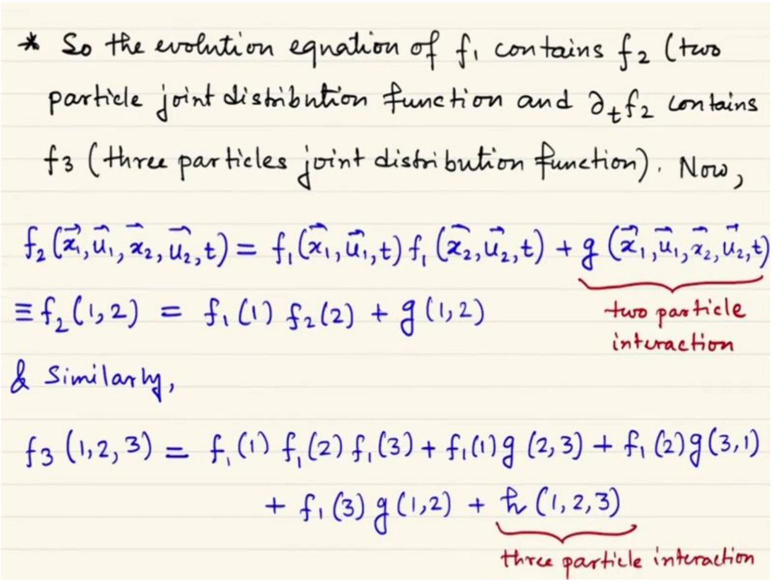

So, this is true. So, that is what I said here. The evolution equation of f_1 contains f_2 and also f_2 contains f_3 and so on. So, sometimes if you now just can write or rather you can express f_2 in terms of f_1 and the 2-particles interaction.

So, you can write always the joint probability distribution of 2 particles which are interrupting with each other as a sum of the 2 terms, this one $f_1(\vec{x}_1,\vec{u}_1,t)f_2(\vec{x}_2,\vec{u}_2,t)$ is the product of the 2 probability distributions of single particles. That means only this one $f_1(\vec{x}_1,\vec{u}_1,t)f_2(\vec{x}_2,\vec{u}_2,t)$ survives if the 2 particles are independent. Now, this one $g(\vec{x}_1,\vec{u}_1,\vec{x}_2,\vec{u}_2,t)$ is nonzero, only two particles are interacting which with each other, right.

So, for example, if we just say that if we are interested in first particle and second particle, this is the level number of the particles, then that will be equal to, so, $f_2(1,2) = f_1(1)f_2(2) + g(1,2)$.

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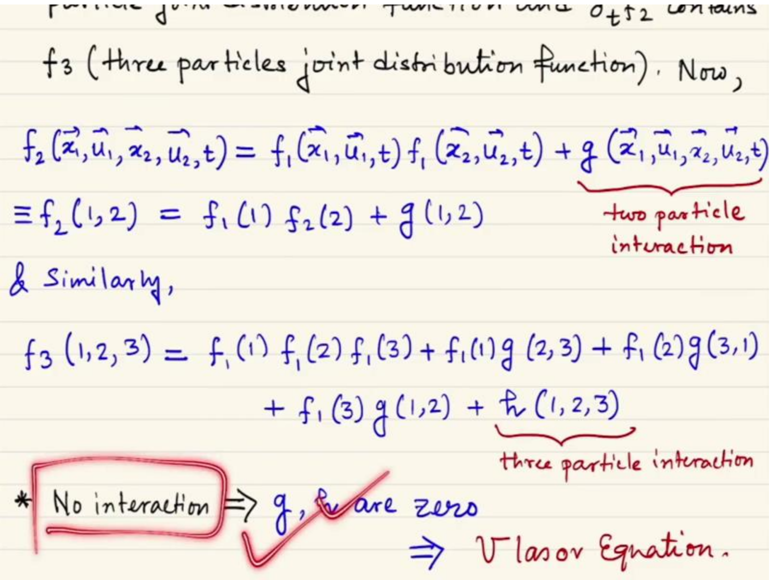

Similarly, you can think of the joint distribution function of 3 particles then you have the first term which designates the product of the 3. That means, if the 3 particles are non-interacting, then their joint distribution function would be simply this $f_1(1)f_2(2)f_3(3)$. Then $f_1(1)$ should be multiplied with this one $g(2,3)$ that means, that if 2, 3 are interacting.

But they are independent of 1. This is the case. If 3, 1 are interacting, but they are independent of 2, and if 1, 2 are interacting, but they are independent of 3, all these things are sum because they are the net effect and they are mutually exclusive. So, that total possibility will be the sum of the 3, and finally, there is a possibility that all the 3 particles are mutually interacting then you have $h(1, 2, 3)$. So, of course, if there is no interaction, we have g and h equal to 0.

So, only this term $f_1(1)f_2(2)f_3(3)$ will survive, and in such case, you know that the kinetic equation which we should obtain is nothing where something similar to Vlasov equation.

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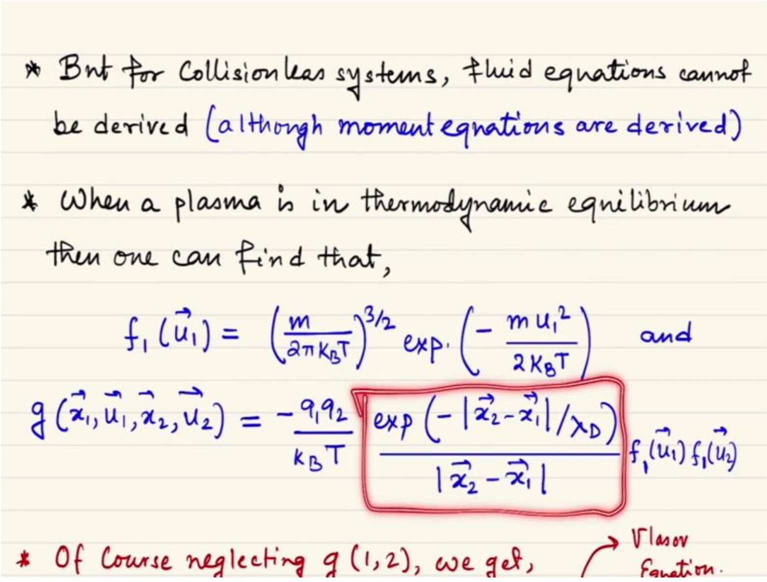

But there is a physics problem that is Vlasov equation is only valid for collisionless systems, but for collisionless systems we know that in principle fluid equations cannot be derived because collisionless systems in principle should not relax towards a Maxwellian distribution, right. Although, you can always derive the moment equations, but you cannot close the equations that is the problem. That is why finally, a dynamical theory for the continuum is impossible to write in theory.

But if a plasma is in thermodynamic equilibrium, then actually one can find this f_1 that is a single particle distribution is like this $\left(\frac{m}{2\pi k_B T}\right)^{3/2}$ exp $\left(\frac{-m u_1^2}{2k_B T}\right)$ and g at least the 2-particle introduction function is given by this $\frac{-q_1 q_2}{k_B T} \frac{\exp\left(\frac{-|\vec{x}_2-\vec{x}_1|}{\lambda_D}\right)}{|\vec{x}_2-\vec{x}_1|} f_1(\vec{u}_1) f_1(\vec{u}_2)$. So, it is a simplified thing and which says that this is not very much different from their individual products, but it is just modulated or multiplied by this factor of this screening effect. So, this is nothing, but a screening effect factor due to coulomb’s interaction.

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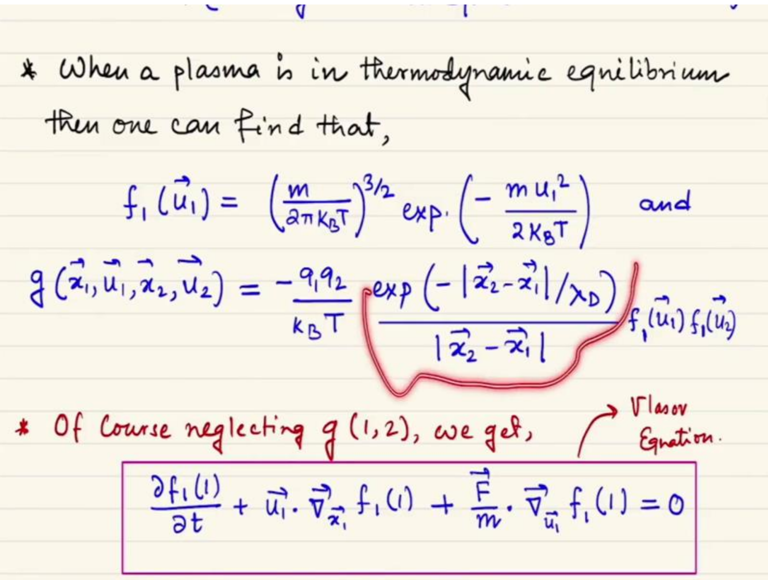

So, from here you can actually see that if $\vec{x}_2 - \vec{x}_1$ is very, very large than finally, this $\frac{\exp\left(\frac{-|\vec{x}_2 - \vec{x}_1|}{\lambda_D}\right)}{|\vec{x}_2 - \vec{x}_1|}$ term is actually negligibly small. So, in practice for the case where we are talking about collisionless systems we can simply neglecting $g(1,2)$.

We can write the kinetic equation which is, so, f_1 once again this simply designates that this is the distribution function of 1 particle,

$$\frac{\partial f_1(1)}{\partial t} + \vec{u}_1 . \vec{\nabla}_{\vec{x}_1} f_1(1) + \frac{\vec{F}}{m} . \vec{\nabla}_{\vec{u}_1} f_1(1) = 0.$$

So, this is the Vlasov equation.

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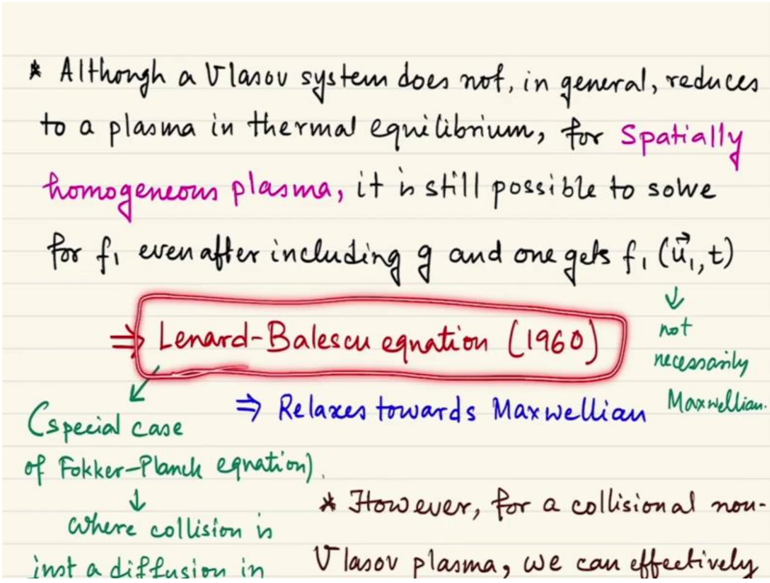

Now, we also know that a Vlasov equation does not reduces to a plasma in thermionic equilibrium that means, it never leads to a Maxwellian, in principle. But we have the good news if we consider a spatially homogenous plasma then it is actually still possible to solve for f_1 that means, the single particle distribution function even after including g, and one then gets f_1 as a function of u_1 and t, although this f_1 is not necessarily Maxwellian. I am just writing this. Not necessarily Maxwellian.

But, in large times this will tends towards a Maxwellian distribution and this equation while we are talking about an especially homogenous plasma the corresponding evolution equation for the single particle distribution which is known as Lenard-Balescu equation. It was derived by Lenard and Balescu both in the year of 1960.

This one basically relaxes toward Maxwellian. So, that was the thing that even for our system, which is not really collisionless, which is collisional, we can actually still do something very easy like we do in case of a Vlasov equation. That means, we can directly start from the very beginning with an only velocity dependent single particle distribution function, and it actually tends towards Maxwellian, but then we have to use Lenard-Balescu equation.

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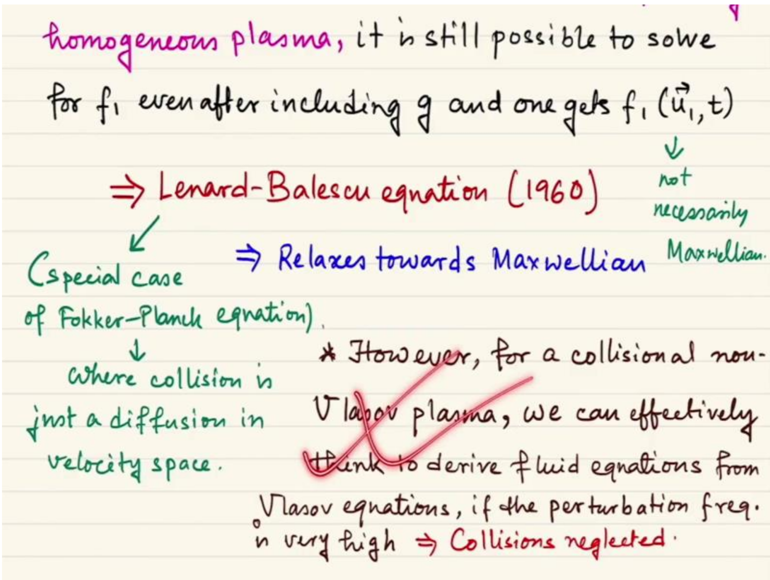

This is a spatial case of Fokker-Planck equation. A Fokker-Planck equation is a class of equations where collision is not neglected at all, but it is modeled as a diffusion in velocity space. Once again, Fokker-Planck equation is very important for plasma physics, in general.

But for the current scope of this course well it is just I am giving as an information if you are interested you can search further. Now, we say that analytically how to simplify a plasma just by assuming this to be spatially homogeneous. Now, for a collisional non-Vlasov of plasma, that means, a collisional plasma which actually should relax to a Maxwellian distribution, we can effectively think to derive fluid equations from Vlasov equation as well.

We do not have to go through Lenard-Balascu, but directly Vlasov equation under a certain condition where we are considering perturbations of very high frequency, and that means, within this perturbation time period almost no collisions are there. So, the system is very much collisional, but finally we can still consider our basic kinetic equations to be Vlasov equation just by thinking that the frequency of the perturbation is a very, very high value.

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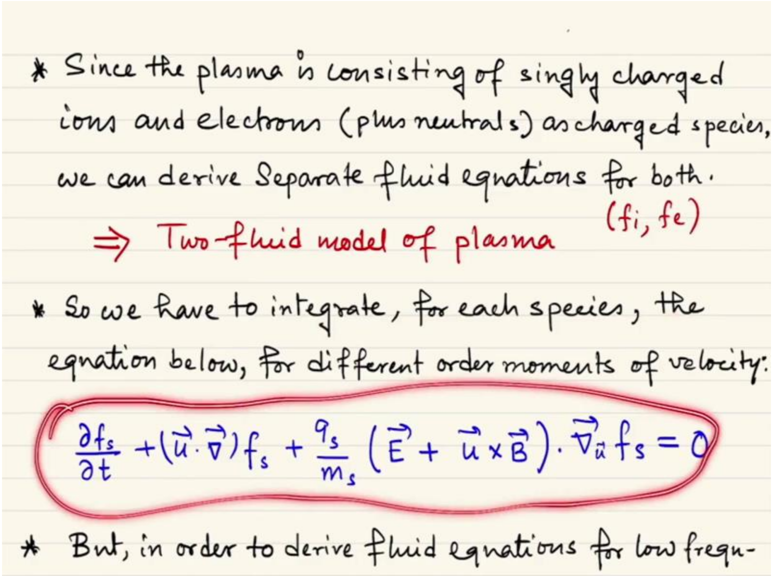

So, the period is very, very small. Now, since the plasma is consisting of singly charged ions and electrons, of course, in principle plus neutrals, but they are these two charged species, we can actually derive separate fluid equations for both. Actually, for plasma I mean for ion, you have single fluid evolution equation as f_i, for electron you have f_e. So, I am just writing in case you forget f_i, f_e like this (f_i, f_e) and then that leads us to the abstention of 2 fluid model of plasma.

Now, we have finally, integrate to get the macroscopic equations, for each species these equation of course corresponding to several indifferent order of movements of velocity, and then you know this is the traditional perception for obtaining macroscopic equations.

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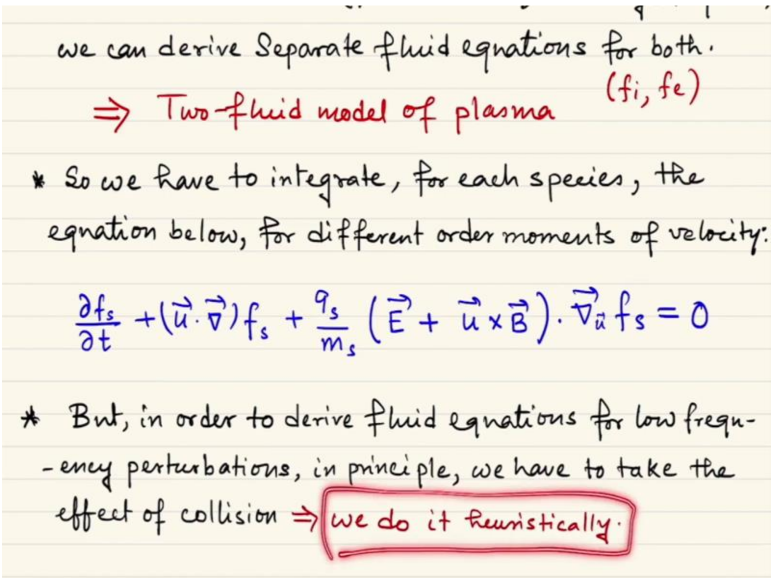

But the question is that practically we need equations where we can part of the system with any arbitrary frequency, the frequency can be low actually. Then how to do that? Then we cannot use Vlasov equation. So, in order to derive fluid equations for low frequency perturbations in principle, we have to take the effect of collisions, but once again analytically I mean handling the collisions is not a matter of joke.

So, we do it for the time being heuristically and we simply say that we are no longer considering particle to particle collision effects.

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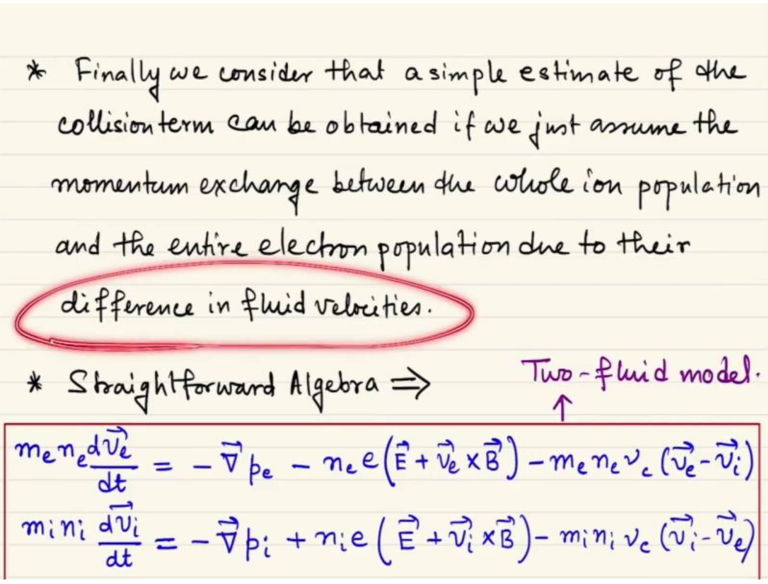

So, what we do? Actually, we think that the particle-to-particle collisions are no longer considered here. What you consider is a very simplistic picture of the collision, and we say that we are more interested in the global effect of the collision. Then, we just say that the collision effect, at least the level of collision effect of the collision where we are interested is just the momentum exchange between the whole ion population.

The whole in electron population due to their difference in the bulk velocities or the fluid velocities. So, for every charged species you can actually define a derive macroscopic velocity or fluid velocity, and if you can do that simply there will be a discrepancy in the fluid velocities of the electron population and the ion population, and since the electrons are lighter in nature when they will just encounter. So, once again this is something a very simplistic picture.

You can actually think like that. So, it is not really particle to particle collisions, but the global collision effect is that the total electrons are making a fluid, the ions are making a fluid, and ions are mostly heavy, they have almost the same number density as that of electrons, at least nearly same, not necessary exactly the same but ions are heavier, so electrons are just colliding with the ions and they are losing momentum.

So, there they are losing momentum, that is exactly the same thing. So, from a revolver you have a bullet. So, if you hit the bullet to let us say a wooden board or something and the board is very massive, so what happens, then the bullet simply loses some amount of the momentum to this massive, and I mean all massive wooden board which is addressed.

Here the ions are not addressed in general, but they are much bulkier. So, electrons are much losing their momentum, to this ion population, and we just say that this is only the case of I mean the grow of effect of the collision, this momentum exchange between the ion fluids and the electron fluids. Although, the real story of the collision is much more complicated.

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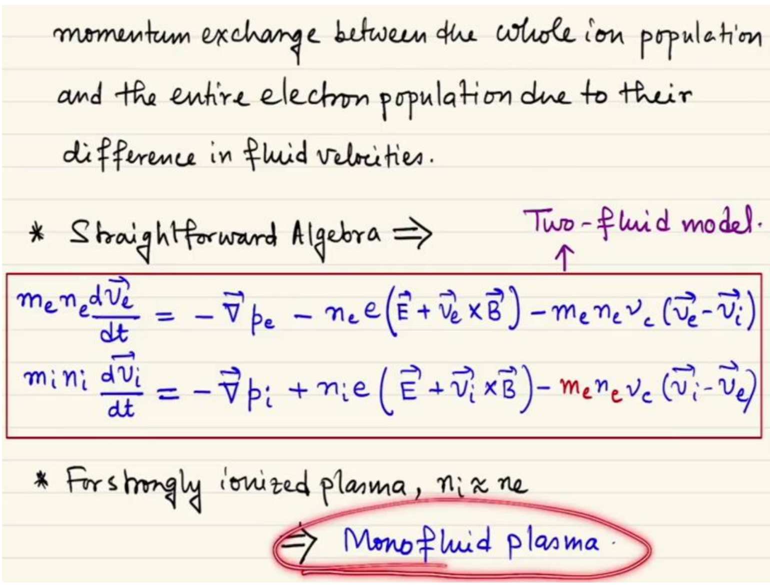

So, in that case what we can do? We can simply say that. Just a minute before just going to that. It should be m_e as well. So, here you see that if you just consider this fact then you will see that after a few steps of straightforward algebra you can finally, like derive the fluid equations for both the populations and this effect of collision which is of our primary interest here is given by simply this $m_e n_e \vartheta_c (\vec{v}_e - \vec{v}_i)$.

So, this $m_e n_e \vartheta_c (\vec{v}_e - \vec{v}_i)$ is just the exchange term of the electron and ion fluids once again. So, now, this $m_e n_e \vartheta_c \frac{d\vec{v}_e}{dt}$ thing is nothing, but the inertia term. So, there are 2 terms $\frac{d}{dt} + (\vec{v}_e.\vec{\nabla})\vec{v}_e$ and of course, there is $(\vec{v}_i.\vec{\nabla})\vec{v}_i$ type of terms. So, that you can easily understand. This $-\vec{\nabla}p_e$ is the electronic pressure, this $-\vec{\nabla}p_i$ is the ionic pressure. This $n_e e(\vec{E} + \vec{v}_e \times \vec{B})$ is the Lorentz force on electrons and this $n_i e(\vec{E} + \vec{v}_i \times \vec{B})$ is Lorentz force on the ions, and these are the momentum exchange terms $m_e n_e \vartheta_c (\vec{v}_e - \vec{v}_i)$, and $m_e n_e \vartheta_c (\vec{v}_i - \vec{v}_e)$.

This is the same momentum, so one is $(\vec{v}_e - \vec{v}_i)$, one is $(\vec{v}_i - \vec{v}_e)$. So, simply as you can see that here this will be minus and this will be plus, we have a different sign. So, this one $m_e n_e \vartheta_c (\vec{v}_e - \vec{v}_i)$ is losing momentum this one is $m_e n_e \vartheta_c (\vec{v}_i - \vec{v}_e)$ gaining momentum in fact. The one is gaining that means, that electronic velocities are much more I mean important than the ionic velocity, so, this total part $m_e n_e \vartheta_c (\vec{v}_i - \vec{v}_e)$ is a gaining momentum for the ionic fluid.

Now, up to that this was the 2 fluid models. So, we have 2 separate fluids, one for the electrons one for the ions. Then, for strongly ionized plasma, in general what happens that even locally n_i and n_e, they are very much close and almost equal.

Then you can actually add and subtract those two equations to find a representative fluid which is not a real fluid, but it is a representative fluid which globally represents the thing and this is called the monofluid model of the plasma.

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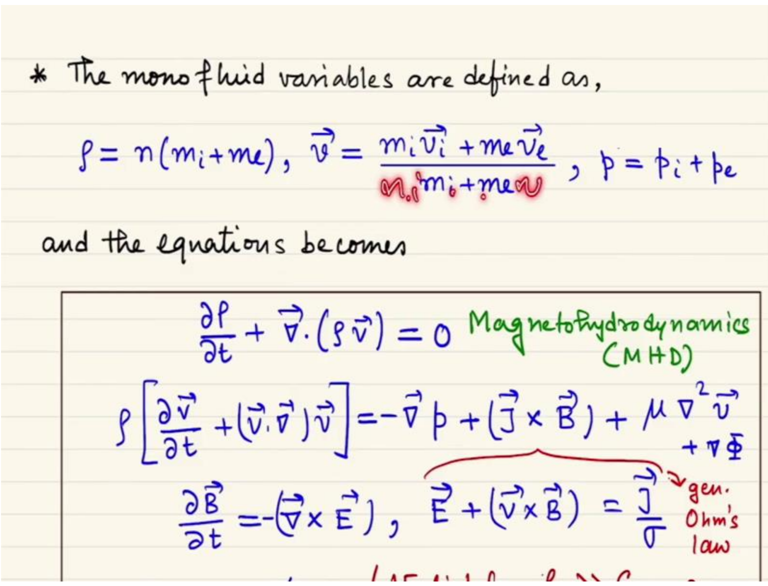

Then, you do not have any longer existence of the ion electronic fluid or ionic fluid, but it is a representative global fluid whose variables they are not defined by this type of things. So, the ρ will simply be the common density times that total mass.

Total mass I mean mass of one ion plus mass of one electron. The velocity will simply be given by $\frac{m_i \vec{v}_i n_i + m_e \vec{v}_e n_e}{m_i n_i + m_e n_e}$, but since n_i and n_e they are the same, then actually 1 n is cancelled from numerator and denominator and you can simply see this is the center of mass velocity.

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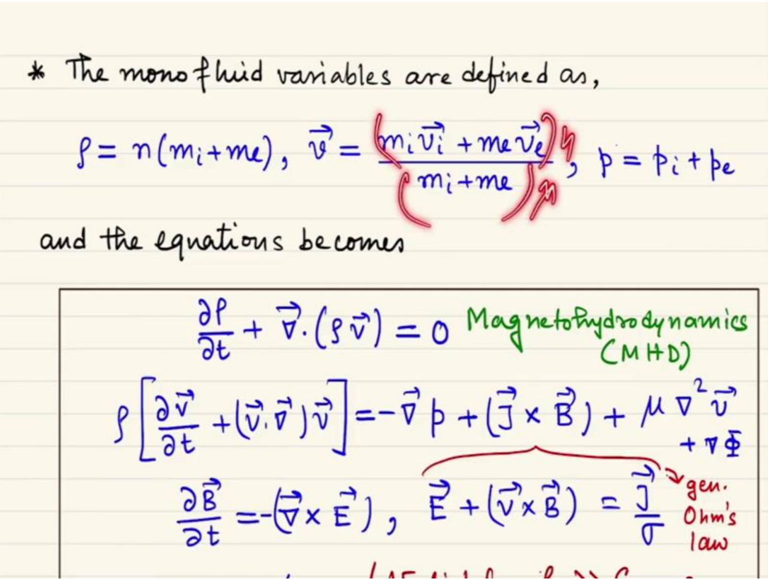

So, pressure is just algebraic sum of the ionic pressure and the electronic pressure $p = p_i + p_e$.

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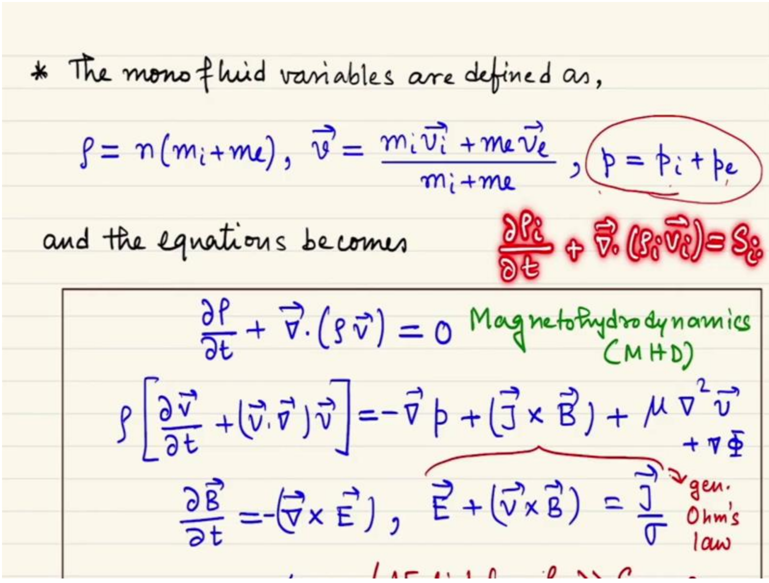

So, of course, although here you do not have to forget that here, we are just talking in terms of the momentum evolution equation. But for every single species you have actually a continuity equation. But this continuity equation should not look like very simple as that.

Actually, just because it is too detailing, but I will just be saying that every continuity equation should look like this $\frac{\partial \rho_i}{\partial t}$ plus $\vec{\nabla}.\rho_i\vec{v}_i$, so, for example, this is for the ions is equal to S_i, where S_i is the source of ions. So, the source of ions is nonzero in a plasma. But of course, if you are thinking that there is no longer production of ions and electron then this is also 0, and the same thing for the electrons.

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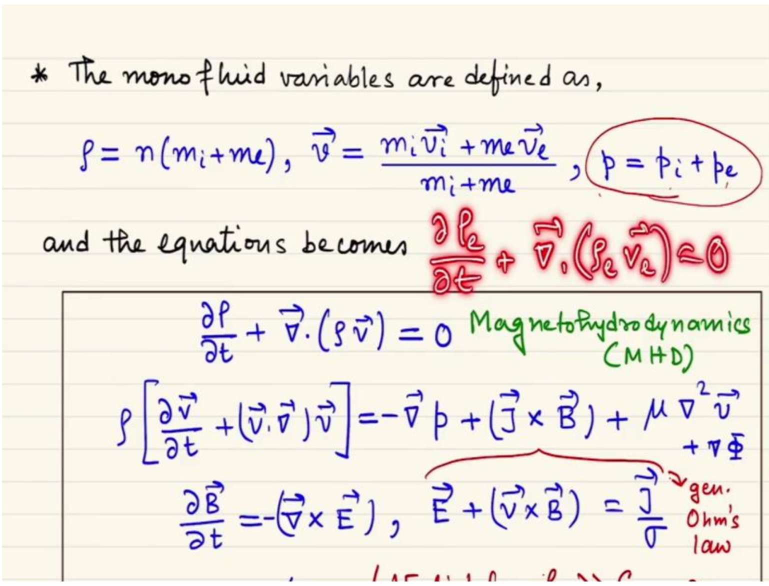

You have an equation like this $\frac{\partial \rho_e}{\partial t} + \vec{\nabla}.\rho_e\vec{v}_e = S_e$ and finally, you add them up and if you do that algebra very carefully, we will see that it will give you a combined equation for the global monofluid variable. So, you have a single fluid continuity equation. In the same way, if you add these two equations you will get the momentum evolution equation for the resultant fluid.

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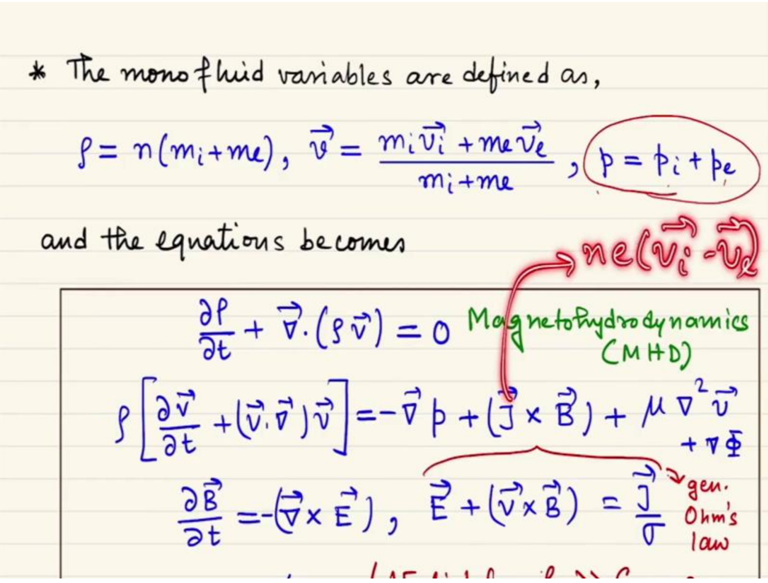

Once again v is the monofluid velocity, is equal to $-\vec{\nabla}p + (\vec{J} \times \vec{B})$. Now, J is the current density which is nothing, but $ne(\vec{v}_i - \vec{v}_e)$, plus $\mu\nabla^2\vec{v} + \rho\nabla\varphi$. This $\nabla\varphi$ is the body force.

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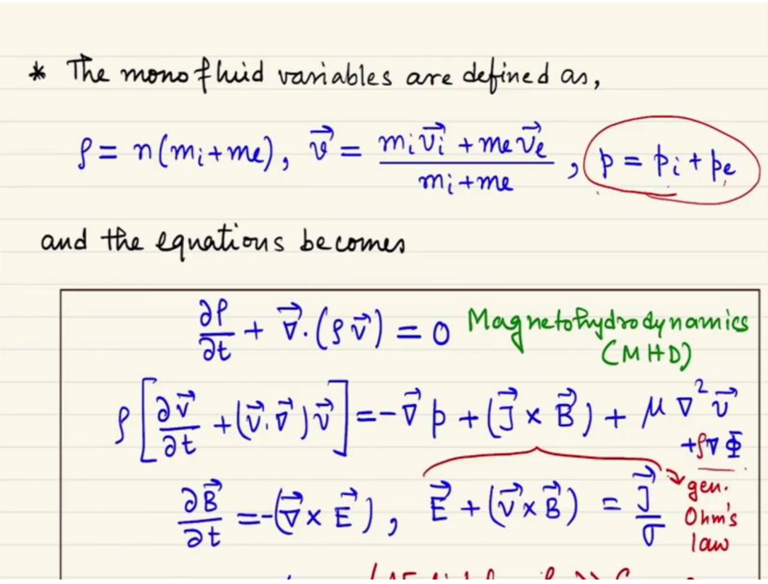

So, body force terms actually I did not include here, but it can be included any time. I have just written because most of the cases in astrophysics we write this, and then if you add these 2 equations you will get the evolution for this bulk velocity, if you subtract these 2 equations as you can easily understand you will get an evolution equation for the current.

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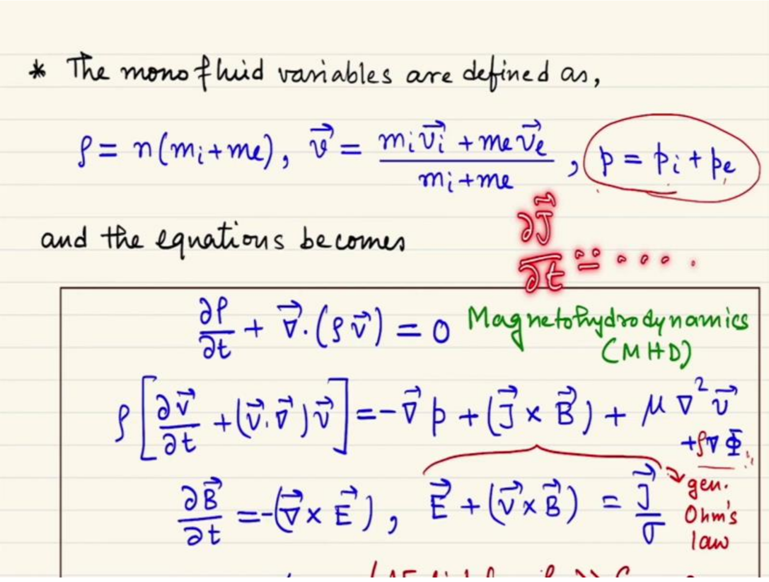

Actually, this is known as generalized Ohm's law.

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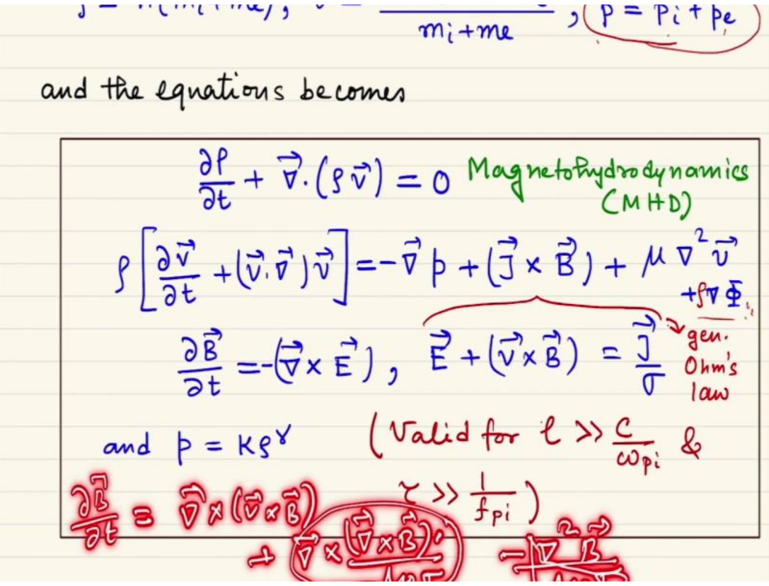

After doing some water analysis which is not very, very simple, but they are sufficiently useful, you can simplify considerably the total evolution equation where finally, the $\frac{\partial J}{\partial t}$ term is no longer existing and it simply gives you a constraint between the E, B and J field.

This is known as the very practical form of generalized ohms law $(\vec{E} + \vec{v} \times \vec{B})$ is equal to $\frac{J}{\sigma}$, and you have your Faraday's law from Maxwell's equation. So, if you just like replace your electric field by this equation you will simply get something like this $\frac{\partial \vec{B}}{\partial t}$ is equal to $\vec{\nabla} \times (\vec{v} \times \vec{B})$ because $\vec{\nabla} \times \vec{B}$ is equal to $\mu_0 J$, since the displacement current is neglected here.

If you again do expand this you will simply have something like $-\frac{\nabla^2 \vec{B}}{\mu_0 \sigma}$ and then this $\frac{1}{\mu_0 \sigma}$, you can call η. That we will discuss in the next lecture that will be the magnetic diffusivity. Now, for the closure, we can again go to the energy equation as we did it for normal fluids. But we can also for the simplicity at least for our astrophysical context, close the system at this position just by saying that this is poly tropic in nature.

Now, you see that this total sets of equation they are call the monofluid equations of a plasma or the magneto hydrodynamics equation. Why magneto hydrodynamics? Because they are, I mean the charge densities they are very, very equal, so locally the electric force part is absent.

So, you see that in the Lorentz force component there is no electrostatic part, so only magnetic part is there, that is why we are actually saying, and actually when in this case you can easily see that the total E is also removed by $\vec{v} \times \vec{B}$ and some $\nabla^2 \vec{B} \eta$. So, we do not need the information about electric field when we are talking about this magneto hydrodynamics. That is why it is called magneto hydrodynamics and not electro hydrodynamics.

So, last thing is that it is useful to understand that magneto hydrodynamic model is actually valid. So, let us say if our system is such that the ions are not actually moving then what will happen, that the electrons are primarily moving and then the fluid is no longer monofluid model.

Then this is not possible to maintain the charge quasi neutrality at every point in space. So, what do we do? So, this is also I mean not charged neutrality actually this is also the equality in the number density at every point in the flow or in the plasma.

So, what is needed? It is needed that. So, if you just try to understand the physical part that the charge neutrality or the number density quality is maintained very properly at every point in space the requirement is that the ions, we should be interested in such link scales.

In such time scales, in which the bulky ions are also moving and when ions are moving the electrons are actually following them. If this is true then only, we can talk about the charge neutrality at every point or that equality in number density, and this is only possible when we are interested in the length scales beyond so called ion inertial length scale. So, ion inertial length scale is the length scale beyond which you can see the ion particles to move.

Let us say, let me just give you an example, for the time scale it is the same, you have to be interested in a time scale which is greater than the 1 by the plasma frequency times scale, the electronic plasma of frequency time scale. So, just I am giving you one example let us say you and 4 of your friends are going to just to walk in a garden and one of your friends has a problem in leg, maybe he has been injured.

So, he cannot walk today for some reason and he cannot put the rhythm with the others. Now, then if you just say, no problem. I mean, there can be 2 approaches one is that you just say we do not care about this person, and we just maintain our original rhythm of walking then he will be lagged behind. Now if you say, hey we are all friends and we have to take care of this guy. So, we will go together today. So, everyone actually says that, we are not in hurry, so no problem.

So, we should go together then everyone is actually waiting till your friend is moving. So, when your waiting period is long. So, your movement is also getting slow down, right. So, that is the essence of magneto hydrodynamics.

Now, you can say that, we have the intelligence of smartness, we have to slow down. But how do electrons do? So, actually what happens? Those electrons are not slowing down their movements, but their movements are much more. So, they have some drift motion, but their movements are much more thermally dominated because they are always the temperature is high and their mobility is very high, so they are mostly randomly moving.

So, what happens? If you are actually waiting for very long time the ions are moving in a specific direction due to its inertia in specific direction, in the meantime they are moving here and there. But if we have waiting for long basically the random motions with respect to the ions is becoming negligibly small. It is almost about when you are just checking the motion of the seconds, I mean the motion of the hour's hand.

Then what happens? Then you see that, when the 1 hour is completed then again, the second's motion is again getting up to its original position. It does not say that the second was slow down or it did not move at all. That simply means that in the whole period or whole interval of time its movement is averaged out.

Then what happens? That the total picture gives us that the bulky ions they are just moving slowly and electrons are, because their random motions, are averaged out, they are simply following the ions as a background, and that is exactly the picture of magneto hydrodynamics, and if you can imagine that picture then you are all set for studying magneto hydrodynamics. So, this $\frac{c}{\omega_{pi}}$ is an estimate of the ion inertial length scale.

This is the time scale which is $\frac{1}{\omega_{pi}}$. So, what happens? So, this one ion inertial length scale is given by the light speed by the ionic plasma frequency.

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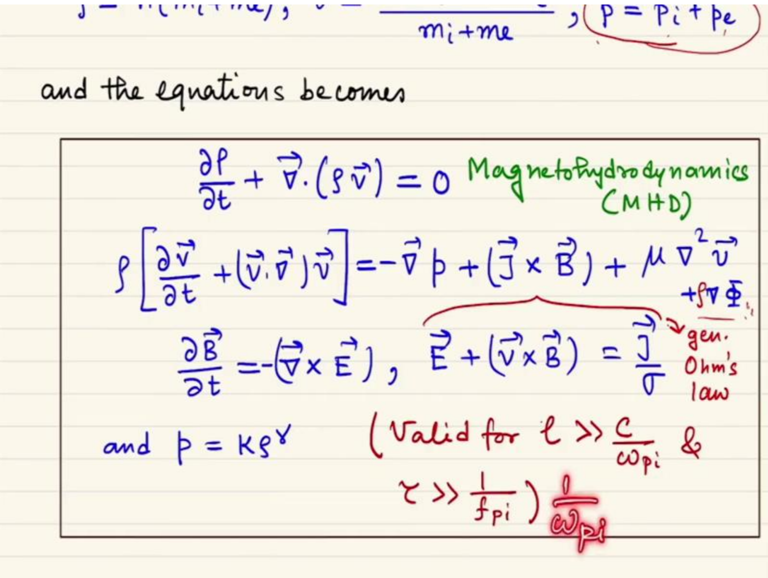

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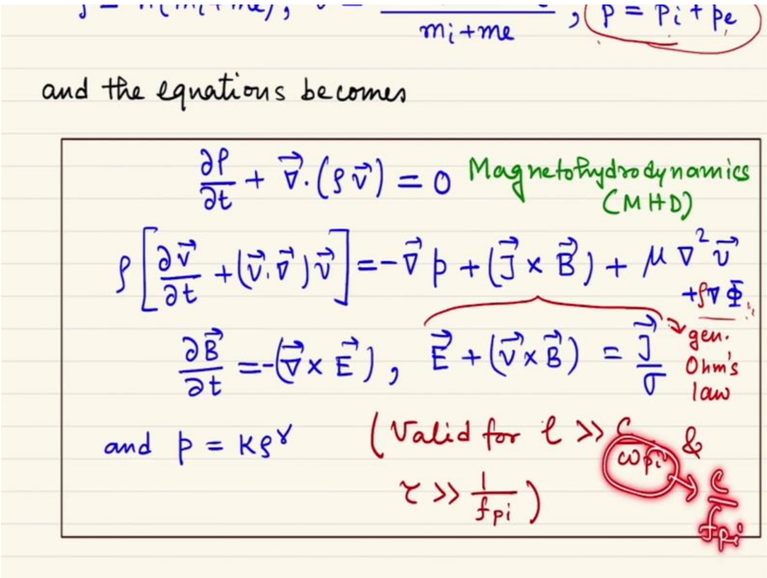

$m_i + m_e$, $(p = p_i + p_e)$

and the equations becomes

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho \vec{v}) = 0$$

Magnetohydrodynamics (MHD)

$$\rho\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = -\vec{\nabla} p + (\vec{J}\times\vec{B}) + \mu \nabla^2 \vec{v} + \rho\nabla\Phi$$

$$\frac{\partial \vec{B}}{\partial t} = -(\vec{\nabla}\times\vec{E}), \quad \vec{E} + (\vec{v}\times\vec{B}) = \frac{\vec{J}}{\sigma}$$

gen. Ohm's law

and $p = K\rho^{\gamma}$ (Valid for $\ell \gg \frac{c}{\omega_{pi}}$ & $\tau \gg \frac{1}{f_{pi}}$) $\frac{c}{f_{pi}}$

The time is of course, $\frac{1}{\omega_{pi}}$. This is not much different you can also use here $\frac{C}{f_{pi}}$ that is just an order, they are at the same order. So, magneto hydrodynamics or the monofluid model is valid for a plasma under 2 situations one is that the charge neutrality or the equality in number densities they are followed or they are obeyed at every point, and for that what happens that we have to choose length scales and timescales which are very long, and as you will see that it is simply saying that or the perturbations, so in magneto hydrodynamics is the regime where we are only interested in the low frequency of the perturbations, or large scales. The same thing actually happens when we will talk about the turbulence. Then of course, we will not talk above the length scale, but we will talk about the fluctuation length scales.

So, this is all about a very brief guiding tour from kinetic to the fluid theory, and actually monofluid theory of magneto hydrodynamics. In the next lecture, I will start discussing different properties of an MHD fluid and observations with modes, etc.

Thank you very much.