Introduction to Astrophysical Fluids

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Lecture - 35
Oscillation of stars

Hello and welcome to another lecture session of introduction to Astrophysical Fluids. In the last week, we discussed about the several types of instabilities. We showed that there can be instabilities on the interface of two fluids and we talked about Rayleigh-Taylor instability and Kelvin-Helmholtz instability. We also showed that there can be also a wave mode, which corresponds to the stability criterion and that was the surface gravity wave.

We also showed that, due to the interplay of self gravitating force and the hydrostatic pressure there is also another possible instability, which is very much possible in the star forming clouds that is the famous Jeans instability. But we started with another instability which which we said that it is very very useful for our domestic life as well as for astrophysical context, that was the Rayleigh-Benard convection.

So, we actually started the discussion with convective instabilities and then we talked about Jeans instability and then we talked about finally the two fluid interface instabilities. Now, if you remember for analytical convenience our models were very much simplified. So, in case of real fluids and real astrophysical contexts, the subject is very very difficult and sometimes this is not even evident to solve them analytically. Then we have to take the numerical support, so we have to do simulations. So, that is something very interesting and useful piece of information for you, because very soon some of you may start your research.

Now, in today’s discussion, we will discuss another aspect which is also related to the stability or instability part, that is called the oscillation of the stars. So, to be very honest the emergence of the subject astrophysics is much more recent, but the subject astronomy was very very old and very very ancient actually. People used to see stars and they got enthralled and that is why they tried to make the observation process schematized and systematic.

And that made the field of astronomy much more interesting and much more systematic as well as organized. And then of course, came the intervention of geometry, trigonometry, calculus, all these mathematical things and then of course with the increase of complexity they started to use topological techniques sometimes for very complicated things, this is also geometry of course. Other complicated geometric considerations relativistic considerations and also then the physical aspect came and astrophysics was born. But the physics of this stellar oscillation is a very very interesting subject, that is beyond the scope of the current course, see if you are interested you can go through the book of Kippenhahn and Weigert. That is a classical book on this and there is another book which I will tell you maybe in the live session, if someone asks me that, but also during the lecture I will mention about that.

So, let us start the discussion by saying that, rather than the physical aspect even astronomically the oscillating stars were very much interesting. That is because of the simple fact that, people used to observe in the sky that the luminosity of certain stars were actually changing with time. And not changing in an arbitrary way, but periodically and then they thought about the whole story of this oscillation.

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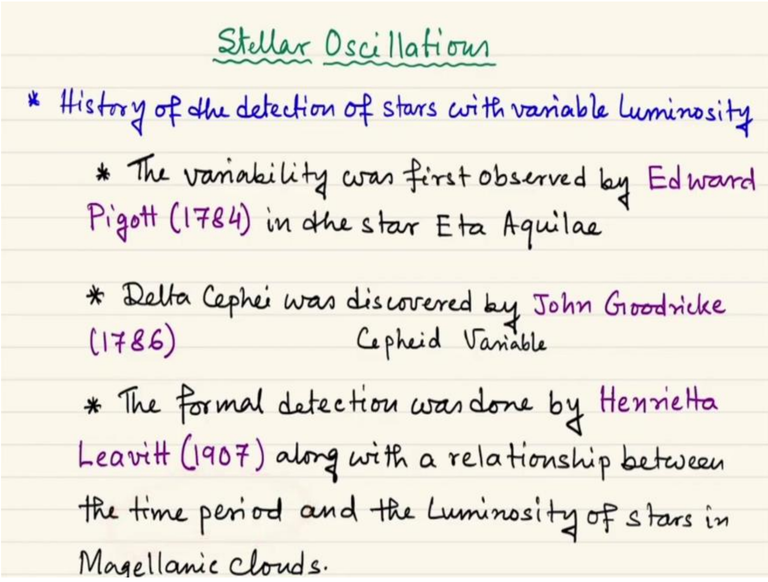

So, now before going to the equations and then some analytical part or something we are treating mathematically, let us first have a look at the history of detection and the formulation of the stars with variable luminosity. So, we will cover some of the elements of the history of the detection of stars.

Now, for the first time the variability of such a star was observed by Edward Pigott in the year 1784 and that was in the star Eta and in the multiple star system Aquilae. So, the stars name is Eta so it is like Greek letter η. So, the member stars of the Aquilae multiple star system are named by Greek letters like α, β, δ and most of the multiple systems are actually named like that and Pigott observed a variability in the star which was the η of that family.

Then in 1786, the famous star Delta Cephei that was discovered by John Goodricke, where he also found the variability in the luminosity of the star. Now, this Cephei belongs to the famous class of the Cepheid stars. So, whenever we talk about the stellar oscillation, we talk about the Cepheid variables. So, Cepheid variables are a class of stars whose luminosity changes periodically with time.

So, that was the story for old part of the detection, but the formal detection was done and she is called the pioneer of detecting this Cepheid variables, Henrietta Leavitt in the year 1907, and when she detected these variables she actually came up with a relation between the time period and the luminosity of the stars for the Magellanic clouds in the year 1907.

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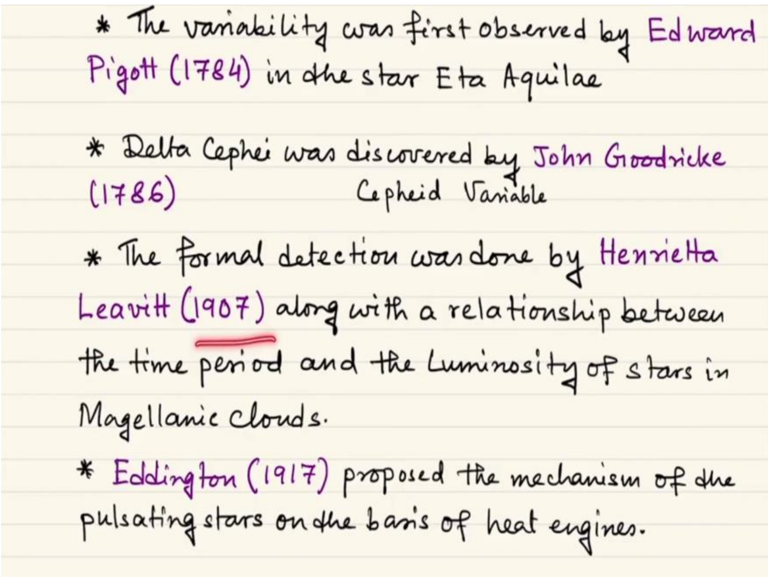

Now, finally maybe all of you have heard of the famous astronomer Eddington who was an engineer actually and he is famous for verifying Einstein's theory, maybe you have heard of this famous story.

So, Eddington in 1917 actually proposed for the first time a plausible mechanism of the pulsating stars, that means the oscillating stars, on the basis of the heat engines. Now, till now I was saying the stars are pulsating or something but how should it look like really.

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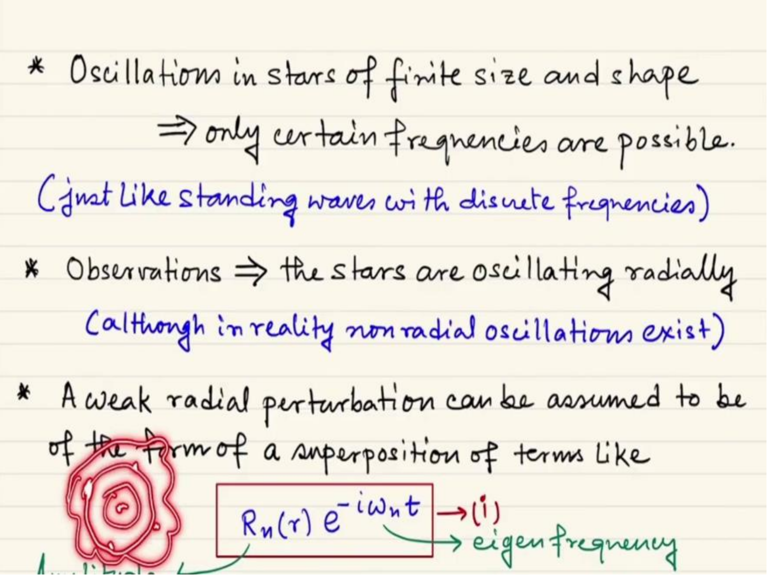

For that, I will come to a picture in a minute (see the rough picture above).

So, of course this was all about the history, now coming back to the theory part. So, Eddington gave something using the concept of heat engine. So, that was of course the first proposed mechanism for this oscillating stars. But nowadays, we know that there are several mechanisms possible for such pulsation, one is known as kappa mechanism, one is known as the convection mechanism, one is known as the tidal excitation mechanism all these things are there, you can search over all these things.

So, this is not really the interest of our current course, but here we will just see that in this situation when a star is oscillating can we have some indication of its stability, can we have what should be the fluid property of that star, so that if a weak perturbation is made to the stars equilibrium configuration or some initial steady configuration, then should it be unstable or should it be stable.

So, first let us start the theoretical part by considering that oscillations in stars of finite size and shape must be an oscillation with not arbitrary frequency, but only with certain discrete frequency. Because, here I have put a constraint on the size and the shape of the star and here you can easily understand that this will be actually equivalent to the case of a standing wave.

So, for example, if you have a string which is actually tied to two iron rods, which are rigidly fixed at floor at the at the ground and then you just put a small oscillation or create a small perturbation in the string, the string will not vibrate with any frequency possible, it will have certain characteristic frequencies. So, the string will actually oscillate with certain frequencies, which are known as the eigen frequencies of the whole problem.

So, that is the exactly the same thing over here. So, here nothing is actually bound or nothing is actually confined, but for example, let us say if you have a finite size spherical membrane and you put some very weak perturbation on this, maybe some of you have heard of the famous Indian instrument Tabla, which is a percussion instrument. So, if you just put some weak perturbation on this surface of Tabla, then how should it propagate? Actually, it also cannot propagate with any arbitrary frequency, it propagates with some certain frequencies determined by the problem and its boundary conditions, because the system is finite.

So, here this is the same type of case and here actually, you can see that it is very very reasonable to assume that just like standing waves what I just said, the oscillation of the stars is also expected to take place with certain discrete frequencies only. Now, astronomical observations actually led us believe that most of the stars, that means for example, the Cepheid variables they are oscillating radially, although in reality we know and actually we will discuss a bit about that, we know the stars also form non-radial oscillations as well, but most of the Cepheid variable stars are oscillating predominantly radially.

That means, that these stars even if it is pulsating, if you are at a fixed distance r, from the center, you will see the identical perturbation. So, if your r is fixed then the perturbation is fixed, of course for a given time. So, there is no θ and ϕ dependence. So, that is the conclusion or observational information.

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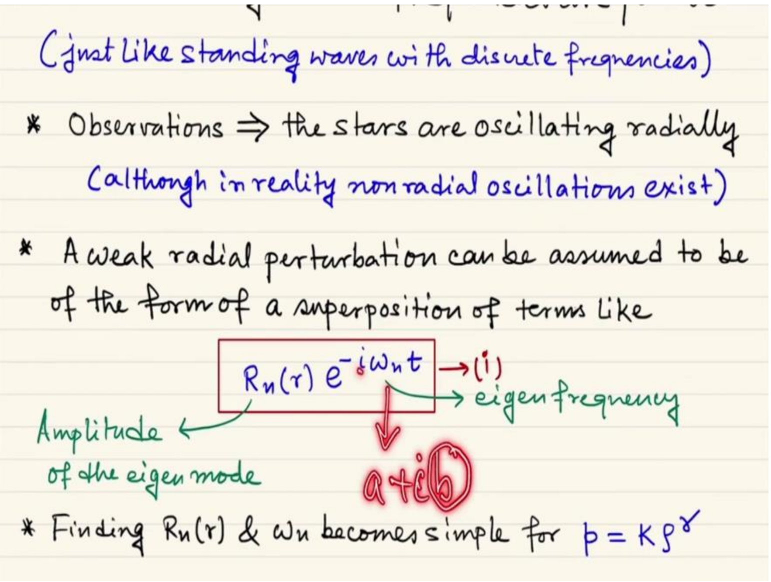

So, that is why just following the same piece of information, we can actually say that a weak radial perturbation for this type of stellar oscillation can therefore be modeled to be the composed of a superposition of terms like $R_n e^{-i\omega_n t}$. So, there will be $\sum_n R_n e^{-i\omega_n t}$, R_n is the amplitude of the perturbation which will be a function of r only, that means the radial distance from the center of the star.

So, we are again try giving as a trial solution the plane wave type of solution and you know that this is nothing but the decomposing the whole problem in terms of its eigen frequencies. So, once again you have to remember that, the system just oscillates with certain discrete frequencies and those frequencies are called the eigen frequencies. And the amplitude of the eigen mode is given by R_n, sometimes they are also called normal mode and normal frequency as well. So, that means any arbitrary perturbation which, can have some coupled oscillation as well, but if you somehow can write them in terms of normal frequency and normal modes, then you finally will be succeeding to represent it as a sum of a number of uncoupled oscillations, that is the thing. So, once again this is nothing but equivalent to saying that any periodic function can be written as a sum of infinite sum of sines and cosines. For example, Fourier series.

Now, this is true that for an arbitrary star finding R_n and ω_n, are not really evident, because there is no given concrete prescription for that.

But, somehow the life becomes simpler, if one assumes $p = K\rho^{\gamma}$, a polytrophic relationship. γ need not be equal to $\frac{c_p}{c_v}$, it is a polytrophic index.

Previously, we also saw that polytrophic relations are very very useful for having some very interesting and simple analytical relations between several quantities. And also this acts as a useful closure. So that you do not need, in your treatment, the energy equation explicitly, that is a very important advantage. So, if we do that then we just have to talk about the stability problem or the dynamics problem until the momentum equation.

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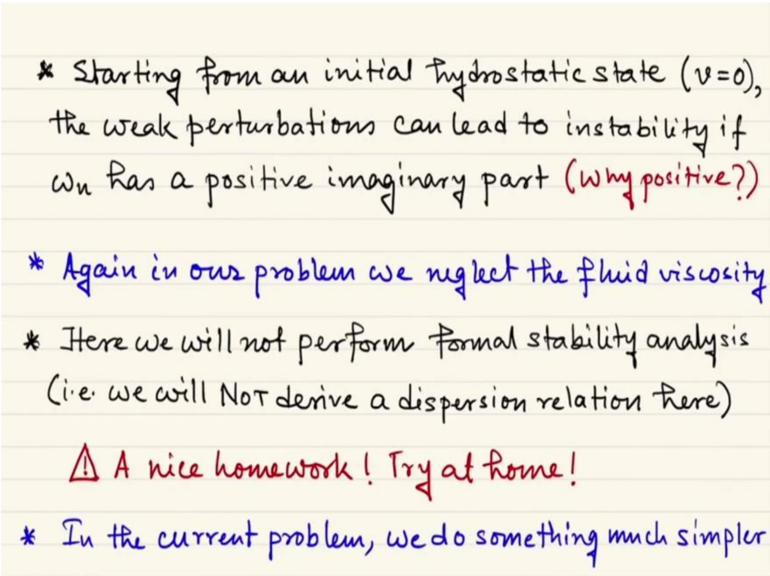

So, of course, you understand that if we start from an initial hydrostatic state, which is of course a steady state as well. So, here we start from some hydrostatic state where $\boldsymbol{v} = \boldsymbol{0}$, the weak perturbations actually can lead to instability. If ω_n has a positive imaginary part, why? that is easy to understand.

So, the time dependent part is given like $e^{-i\omega_n t}$ so, if ω_n now is like $a + ib$ and b is positive then, two i's will give you -1 and two -1's gives you +1. So, finally, it will give a growing nature of the perturbation in time. And that is exactly the reason for which we can say that for instability ω_n should have a positive imaginary part. Now, coming to one point that if we start from the initial hydrostatic state, where $\boldsymbol{v} = \boldsymbol{0}$, then we actually do not know whether the system will behave in terms of or will respond in terms of some linear wave mode or it will lead to some instability.

So, we also know that the routine process for that, is the linearization and then by deriving the dispersion relation, but here should we do it now? we will see. So, before that just let me just mention that in this problem again for simplicity and this is very much reasonable actually for astrophysical context that we neglect the fluid viscosity with respect to the gravitational force and the pressure gradient force.

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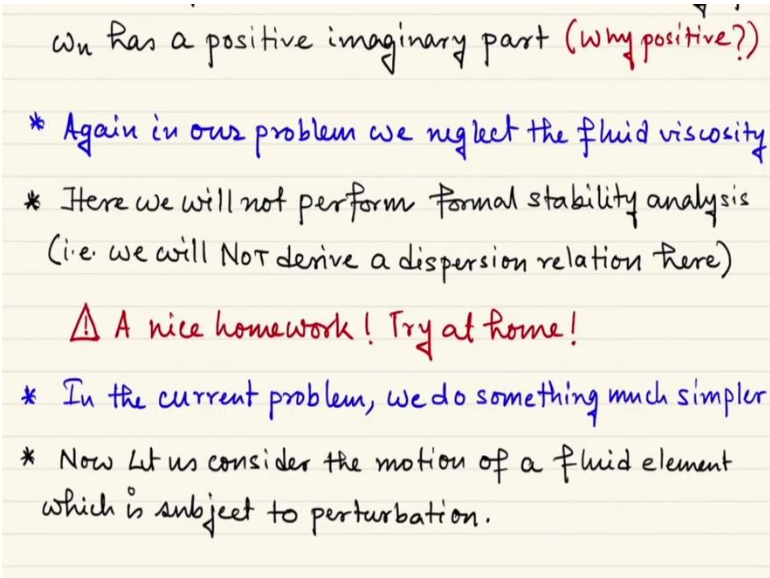

So, that is exactly what I was just saying one second ago, that here in our case we will not perform as formal stability analysis and we will not derive a dispersion relation here. Although, you can start from $\sum_n R_n e^{-i\omega_n t}$ and you can actually check how should the wave modes or the dispersion relation actually should look like.

So, this is the nice homework. So, I request you to try at home. But for our current course, what we will do is much more simpler and we will not even do the normal perturbation technique, we will now study something which was historically studied for the first time in order to understand the stability criterion for the oscillating stars and that is the study of the motion of a fluid element in the star.

So, of course, for that let us consider the motion of a fluid element which is subject to perturbation. So, now, we should not write the Euler equation or in terms of the Euler fields, but rather we have to think in terms of the Lagrangian derivative. So, we will not use, in the left hand side the Eulerian derivatives, but the Lagrangian derivatives.

Thank you very much.