Introduction to Astrophysical Fluids
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Lecture – 28
de Laval nozzle and extragalactic jets

Hello, and welcome to another session of Introduction to Astrophysical Fluids. In this session, we will discuss a very interesting topic of Astrophysics that is the topic of de Laval nozzle and the corresponding physics of extragalactic jets.

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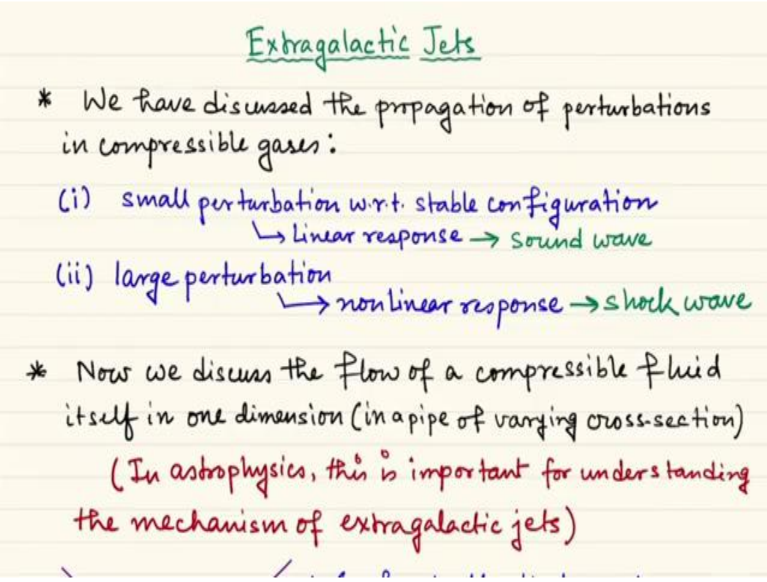

So, already in the previous discussions, we have discussed the propagation of perturbations in compressible gases, and we have seen that if we are considering small perturbations with respect to some initial stable configuration of the fluid then the fluid I mean if we try to perturb or damage any quantity of the fluid for example, pressure, velocity, density then the fluid responds to the perturbation in the terms of the linear modes.

That is for a normal compressible fluid, this mode is known as the sound mode or the acoustic mode. However, if the perturbation is large enough, then non-linear linearity is no longer valid and non-linearity comes into play, and we have seen that there are development of discontinuities in the flow and then we discuss about several aspects of what we call the shock wave.

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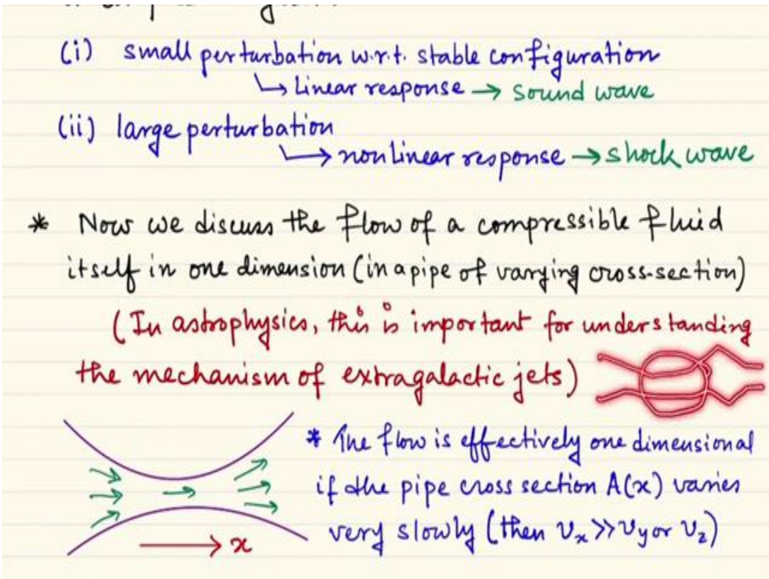

Now, in this discussion, we will discuss not the flow of any perturbation, but the propagation of any perturbation, and the flow of a compressible fluid itself in one dimension. What is the meaning of one dimension? That does not say that the total universe is one dimension that means, the fluid is flowing in 3D world or 3D universe, but the flow direction is effectively along one direction.

For example, when a fluid is flowing in a pipe with varying cross section so, that is a very simple and very fundamental problem of 1D fluid flow. Then how should we determine the density, the pressure, the velocity of the fluid at different parts of the pipe is a very well-known problem of fluid dynamics.

Now, in astrophysics, this type of problem is very important because of the understanding of the mechanism of extragalactic jets. I am coming to that later, but before that let me first address this problem from a fluid dynamics point of view and then we will go to relate this to the astrophysical observation.

So, let us just consider a flow which is effectively one dimensional. That means, the pipe is looks like this, let us say here I have already done some very specific structure of the pipe, but let us say the pipe has this type of thing whatever I mean this. So, in some part this is narrower, in some part this is wider and the fluid is flowing throughout the pipe and mostly along x direction.

So, there are motions which are oblique to x direction, but in our case, we are just saying that the pipes cross section A which is the cross section of the pipe perpendicular to the flow that varies very slowly with x, and that is why all these transverse components where whether we are talking about v_y or v_z, they are actually very, very small with respect to v_x and that is why the flow is effectively one-dimensional.

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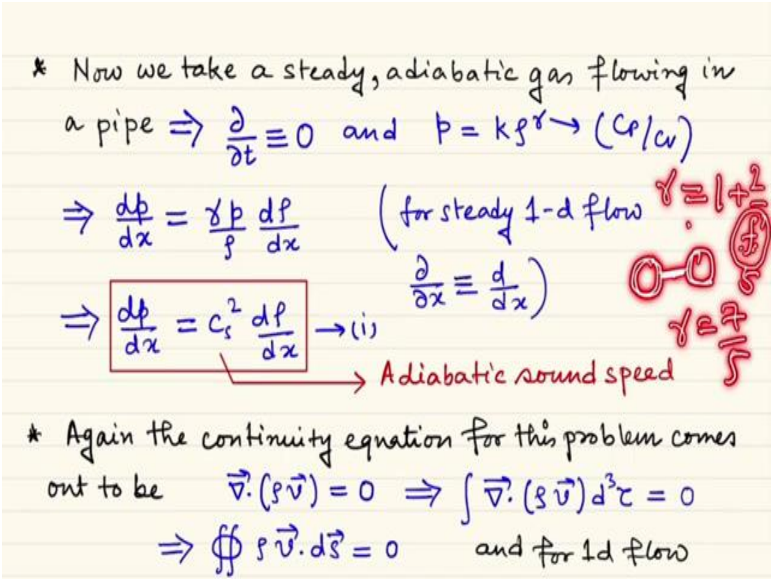

So, the flow is not strictly one dimensional, but this is the case of effective one-dimensional play flow. Now, we take a steady adiabatic gas flowing in a pipe. Now, when we are talking about steady flow that means, any explicit dependence on time will be vanishing, $\frac{\partial}{\partial t}$ will be equal to 0 and p will be equal to $k\rho^{\gamma}$, but this γ is not arbitrary polytropic index.

But this γ is equal to $\frac{C_p}{C_v}$ that is equal to, if you know this formula that for adiabatic index γ should be written as $1 + \frac{2}{f}$, where f is the number of degrees of freedom. So, if we are considering the monoatomic gas then f is 3. So, γ will be simply equal to $\frac{5}{3}$.

If we are talking about diatomic gas which is of the shape of a dumbbell two atoms are like connected with each other by some rigid rod type of thing then the number of degrees of freedoms are 5 and γ will be simply $\frac{7}{5}$. So, these things I hope you all know from your knowledge of thermodynamics.

So, if we have such an adiabatic system and that is completely reasonable because we will see that we will try to relate this to a process which is very, very quick. So, once again adiabatic closure is a very good closure.

Quick means, when the phenomena is very quick then the total system does not have enough time to establish thermal equilibrium with the surroundings by the sufficient exchange of heat. So, if the system is very quick then we can actually assume that the total heat content is already confined inside the system. So, that gives the adiabatic nature of the process.

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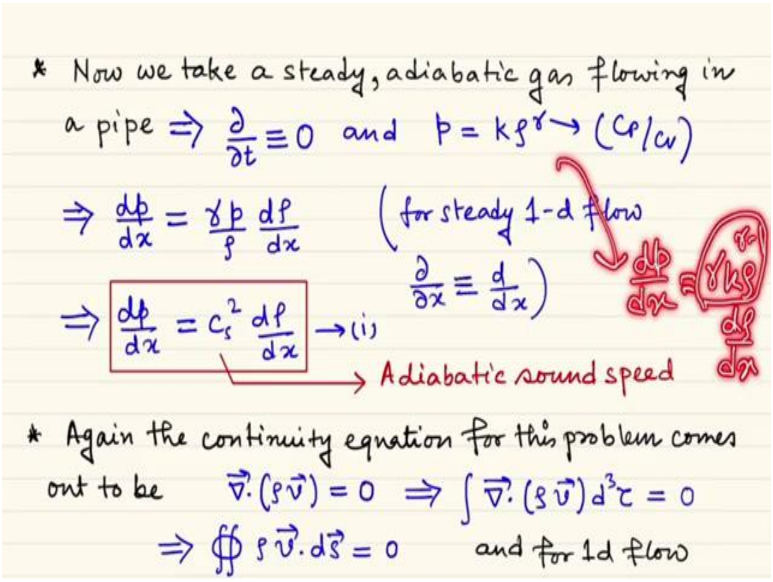

So, if we have this then $\frac{dp}{dx}$ is equal to $\frac{\gamma p}{\rho}\frac{d\rho}{dx}$ that is very easy to see. You can just see that $\frac{dp}{dx}$ is equal to $\gamma\rho^{\gamma-1}k\frac{d\rho}{dx}$ and this $\gamma\rho^{\gamma-1}k$ is nothing but $\frac{\gamma p}{\rho}$. You can simply replace the value of p over there, so, you can finally have

$$\frac{dp}{dx} = c_s^2 \frac{d\rho}{dx} \qquad \text{(i)}$$

Now, we know c_s^2 is nothing but the adiabatic sound speed and this is equal to $\frac{\gamma p}{\rho}$. Here, we are talking about steady flow. So, all the fluid variables, they are not explicitly depending on time and effectively 1D flow.

So, there is also no dependence on y or z directions. So, finally, $\frac{\partial}{\partial x}$ and $\frac{d}{dx}$ will be the same thing. So, x is the only variable which is the only independent variable in the problem. All the fluid variables are expected to depend only on x for this problem.

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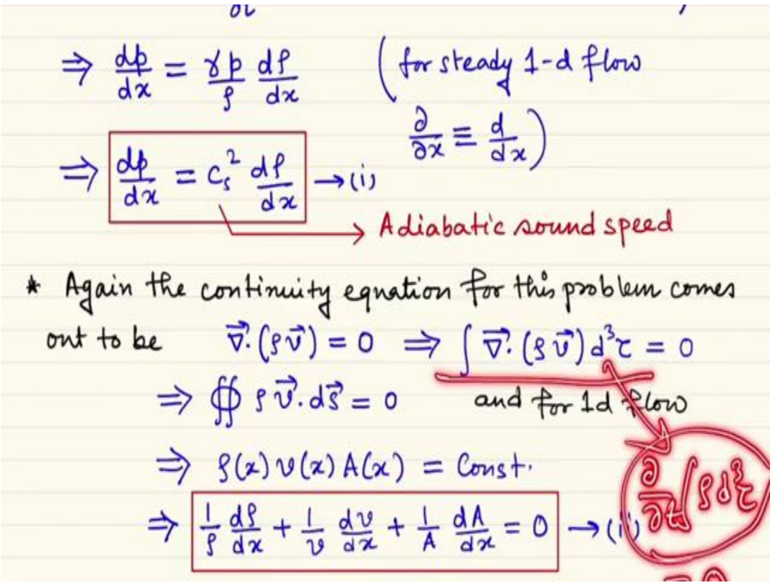

Now, we also know that is one equation for our interest. Now, we also know that continuity equation for this problem comes out to be $\vec{\nabla}.(\rho\vec{v})$ is equal to 0 and if you just consider that this thing to be integrated over the whole volume this is also 0. This is nothing but the mass continuity equation and the mass conservation equation and if you remember that this total thing is nothing but $\frac{\partial}{\partial t}\int \rho d^3\tau$.

So, this should be equal to 0, otherwise the mass is not conserved. Since we have assumed that there is no source or no sink of material inside the flow field, then we can easily apply this continuity equation and by Gauss's divergence theorem can be given by this the surface integral $\rho\vec{v}.d\vec{s}$ is equal to 0.

Now, we have a pipe and inside the pipe we have a fluid flow. Now, we take any imaginary volume element type of thing inside the fluid. So, if we take arbitrary volume, the surfaces will be also covering or surrounding the volume will also be arbitrary.

I mean here, in this case arbitrary means they should be always normal to the pipe flow or the fluid flow, but they should be arbitrarily placed. So, if I take, for example, this volume element then the surfaces will be this surface, this surface, this surface and this surface if we take another volume element like this then the surface will be this one, this one, this one and this one.

Now, if you remember that the surface area vectors are basically the vectors which are normal to the surface. So, these are the surface area vectors. So, because the flow is essentially along x direction, the dot product of the velocity with these two surfaces vectors will vanish, but only with these two surfaces will survive.

As they are 0, so, always actually you can say that for the one surface like this will be exactly equal and opposite to the other surface. So, that is simply saying that the mass of fluid which is entering through one surface must be going out through the opposite surface.

Finally, if you can take actually arbitrary volume element and that is true for all the cross-sectional surface which are normal to the fluid flow this type of thing. So, if we have some fluid flow like this, then this one or this one we have two normal surfaces, and you can do it for any arbitrary surface which is perpendicular to the flow that means, this type of surface.

Finally, you can show that we can write in a general manner that $\rho(x)v(x)A(x)$ is equal to constant for any x inside the fluid flow. Now, if we differentiate this equation with respect to x, because x is the only independent variable we can actually write as

$$\frac{1}{\rho}\frac{d\rho}{dx} + \frac{1}{v}\frac{dv}{dx} + \frac{1}{A}\frac{dA}{dx} = 0 \qquad \text{(ii)}$$

It is zero because this constant when gets differentiated with respect to x gives 0. So, we now have two equations, equations (i) and (ii).

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* We again neglect the body force and the viscosity and the momentum evolution equation (for steady flow) now becomes,

$$\boxed{v\frac{dv}{dx} = -\frac{1}{\rho}\frac{dp}{dx} = -\frac{c_s^2}{\rho}\frac{d\rho}{dx}} \rightarrow \text{(iii)}$$

* Finally using (ii) & (iii), we get

$$-\frac{v}{c_s^2}\frac{dv}{dx} + \frac{1}{v}\frac{dv}{dx} = -\frac{1}{A}\frac{dA}{dx}$$

$$\Rightarrow \boxed{(1-\mu^2)\frac{1}{v}\frac{dv}{dx} = -\frac{1}{A}\frac{dA}{dx}} \quad \left[\mu = \frac{v}{c_s}\right]$$

$\rightarrow$ (iv)

Now, in this case, we neglect again the body force and the viscosity actually you will see in most of the cases of very high energetic astrophysical phenomena, we neglect body force and the viscosity, and then our momentum evolution equation is no longer Navier Stokes, but it is an equation called Euler's equation that means, only the inertial term and the pressure gradient term.

So, we can write that

$$v\frac{dv}{dx} = -\frac{1}{\rho}\frac{dp}{dx} = -\frac{c_s^2}{\rho}\frac{d\rho}{dx} \qquad \text{(iii)}$$

So, this is the pressure gradient term. So, finally, then we have another relation equation (iii) relating this one $\frac{dv}{dx}$ and $\frac{d\rho}{dx}$.

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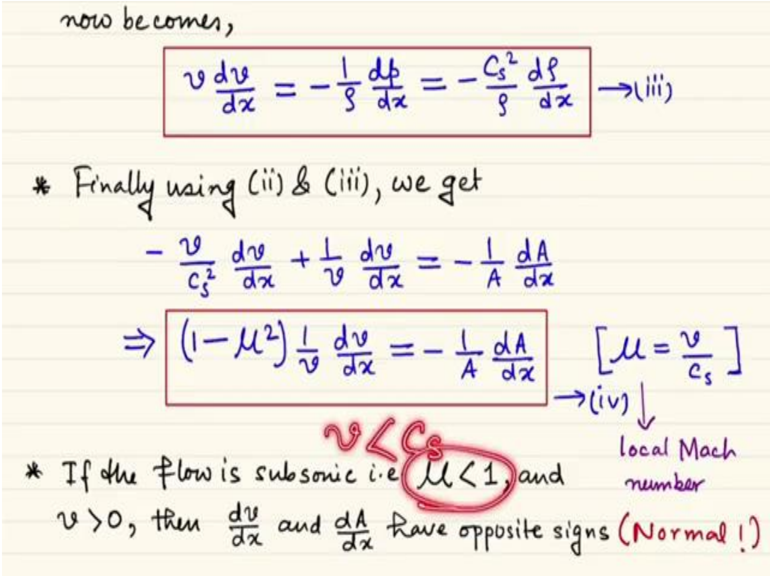

So, in this equation (ii), if you remember we can eliminate $\frac{1}{\rho}\frac{dp}{dx}$ by $v\frac{dv}{dx}$ and you will see finally, you can have this total expression to be written like this

$$-\frac{v}{C_s^2}\frac{dv}{dx} + \frac{1}{v}\frac{dv}{dx} = -\frac{1}{A}\frac{dA}{dx}.$$

Now, if you just take everything inside the bracket in the left-hand side this will give you

$$(1-\mu^2)\frac{1}{v}\frac{dv}{dx} = -\frac{1}{A}\frac{dA}{dx} \qquad \text{(iv)}$$

What is $\frac{v}{C_s^2}$? This is nothing but the local Mach number. Local Mach number means, the Mach number at every point because v is the velocity at every point, C_s is also the sound speed of every point at each point.

So, this is local velocity by local sound speed gives us local Mach number. So, this actually changes as x changes. So, this is the final and the most important relation for this problem. Now, remember that if the flow is subsonic, just the calling of the definition of subsonic and supersonic flows, we know that v should be less than C_s.

So, your Mach number will be less than 1, and in that case, if we are talking about the fluid which is mostly flowing along the x direction, so, v_x is greater than 0 and in that case, you can say that $\frac{dv}{dx}$. So, this term μ is less than 1, that means, this term $\frac{dv}{dx}$ is greater than 0, this $\frac{dA}{dx}$ is also greater than 0.

So, $\frac{dv}{dx}$ and $\frac{dA}{dx}$, they are related with opposite signs because something positive times $\frac{dv}{dx}$ is equal to minus something positive time $\frac{dA}{dx}$. So, $\frac{dv}{dx}$ and $\frac{dA}{dx}$ they have opposite signs and we claim that this is very normal.

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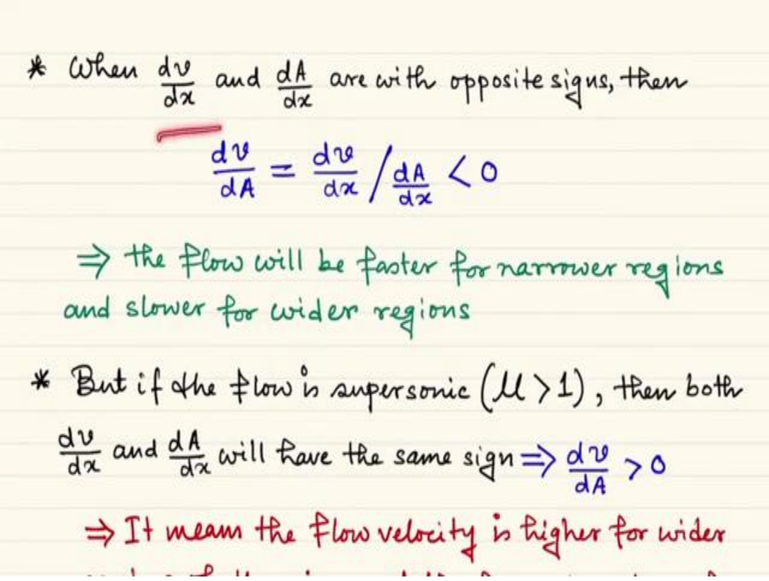

Why? Because if $\frac{dv}{dx}$ and $\frac{dA}{dx}$ they are with opposite signs then $\frac{dv}{dA}$ which is nothing but $\frac{dv}{dx}$ by $\frac{dA}{dx}$ is less than 0 because they have opposite signs. So, $\frac{dv}{dA}$ is negative. That means velocity goes down when cross sectional area increases.

It simply says that the flow will be faster for narrower regions and slower for wider regions that we actually see in practice that in a pipe when the cross-sectional area is less than the fluid velocity is very larger and here the cross-sectional velocity is smaller, right. So, that is the normal case, but now, we see that this is true only when the flow is subsonic.

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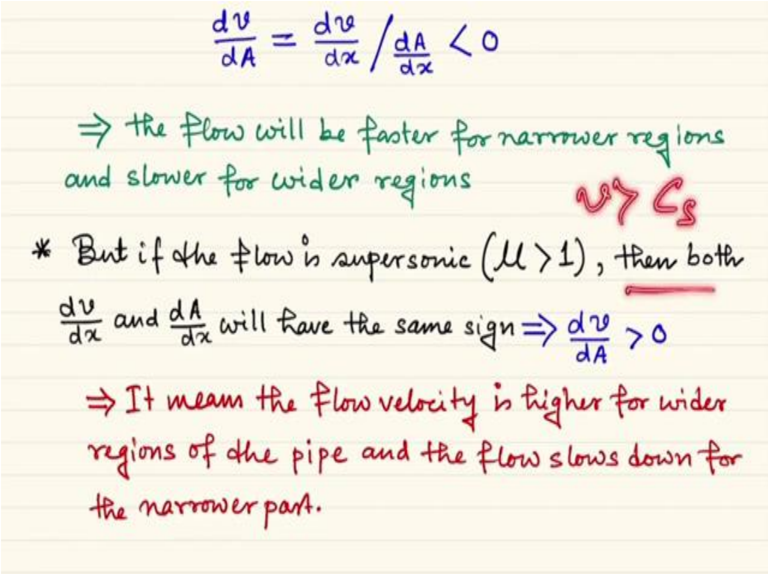

But if the flow is supersonic that is, Mach number is greater than 1 or the velocity is greater than sound speed then both $\frac{dv}{dx}$ and $\frac{dA}{dx}$ will have the same sign, why is that? Because then this will have negative sign, this is always positive. So, this negative and this negative will cancel each other and then $\frac{dv}{dA}$ will also be equal to $\frac{dv}{dx}$ by $\frac{dA}{dx}$ which will now be greater than 0 because they will have the same sign.

It simply means that the flow velocity is higher for wider regions of the pipe and the flow slows down for the narrower part. That is something a bit counter intuitive for us and that is the whole essence of this current theory.

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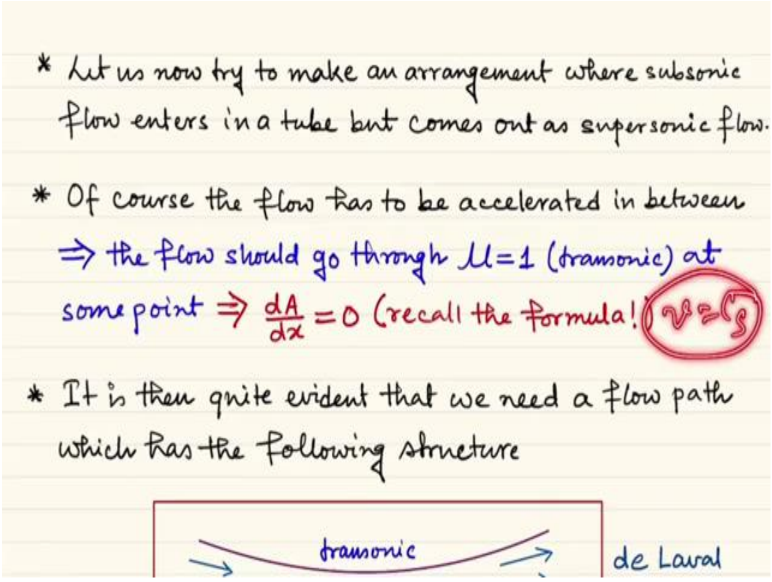

So, that is because, till now we have always discussed in the framework of fluid dynamics with the subsonic flows. For supersonic flow, this is the normal case that means, the wider cross section the fluid will have acceleration, and greater velocity the narrower region the fluid will have lower velocity. So, let us now try to make an arrangement where subsonic flow enters in a tube or a pipe, but comes out as a supersonic flow.

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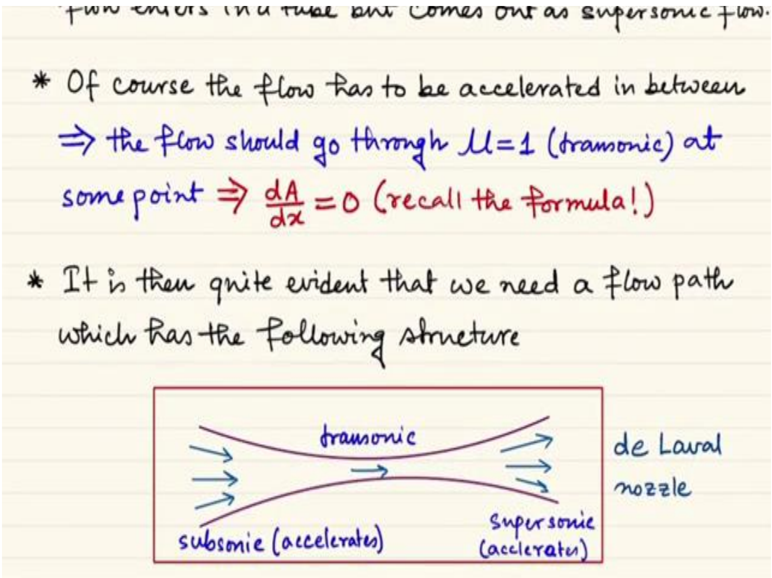

So, of course, such a flow has to be accelerated in between otherwise it is not possible. If it enters like a subsonic flow and comes out as a supersonic flow then in between, the flow should be or must be accelerated otherwise how can it be, and that means, that the flow should be in between at some point should be transonic.

That means, a Mach number is actually transiting from something less than 1 to greater than 1 and that is why it should pass through the value μ exactly equal to 1 which is the transonic case. That means, the velocity is exactly equal to the sound speed, and for that point what happens?

If the fluid velocity is equal to sound speed, μ is 1 and this is 0. So, even $\frac{dv}{dx}$ is non 0, $\frac{dA}{dx}$ is 0. So that means, that the system will have an area where the cross-sectional system in the pipe or the tube should have a part for which the cross-sectional area is effectively the same over a range of x.

So, $\frac{dA}{dx}$ simply says that the area has some extremum value over x, and it is then quite evident that we need to use a flow path which has the following structure. So, the fluid first enters into this wider region and then the area basically comes down to its minimum value giving birth to this narrowest region.

After that this fluid will be again going to a part where the cross-sectional area is greater, and then you will see that what happens? That first the fluid was subsonic while it was entering and then it was accelerating only because that from wider to narrower region, so, simply it got accelerated because it was by going to some narrow regions.

After passing through this region with constant cross section finally, it is again going to another part. Now, so, in this part this is now becoming supersonic, and now, actually when it comes to another part where the cross-sectional area is again increasing then it again accelerates itself right and because now this is supersonic flow.

So, finally, the flow as a result accelerates throughout the whole nozzle and this specific construction is known as de Laval nozzle, and de Laval nozzle is the most promising model for the aspects I mean extragalactic jets till now. Of course, there are subtleties, but being simplistic this is the most promising model.

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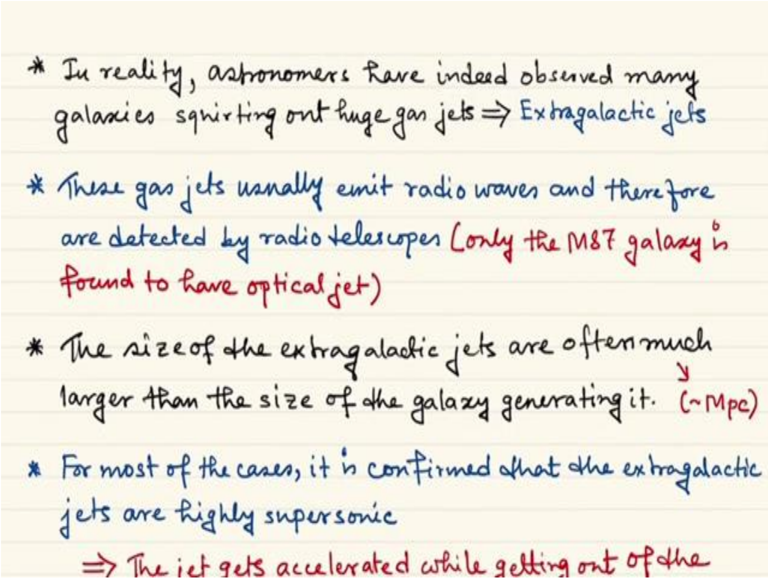

Now, in reality, astronomers now, coming to some astrophysical connection that astronomers have indeed observed many galaxies which squares out huge gas jets and these are known as the extragalactic jets. Now, these gas jets usually emit radio waves and therefore, they are detected by radio telescopes.

So, only there is one exception that within our knowledge that only M87 galaxy is found to have optical jet otherwise mostly they have radio jets. The size of the extragalactic jets is often much larger than the size of the galaxy generating it, and sometimes this is so large that the size is of the order of mega per sec and which is even much larger than the parent galaxy and they are in general the largest coherent fluid flows which are visible in the whole universe.

So, for most of the cases this is confirmed that the extragalactic jets are highly supersonic in nature. The jet gets accelerated while getting out of the galaxy to the ambient medium of less pressure and greater available cross-sectional area. So, as you can easily understand that I mean how the jet is highly supersonic.

So, we can easily use our theory which we have just seen previously that this type of supersonic flow will be accelerated whenever it will go to surrounding or a medium with less pressure and greater available cross section, and that is why when this type of extragalactic jets they are spitted out of the galaxy to a very dilute ambient medium, I mean that is the intergalactic medium then this flow is also accelerated. So, this is something which we already know.

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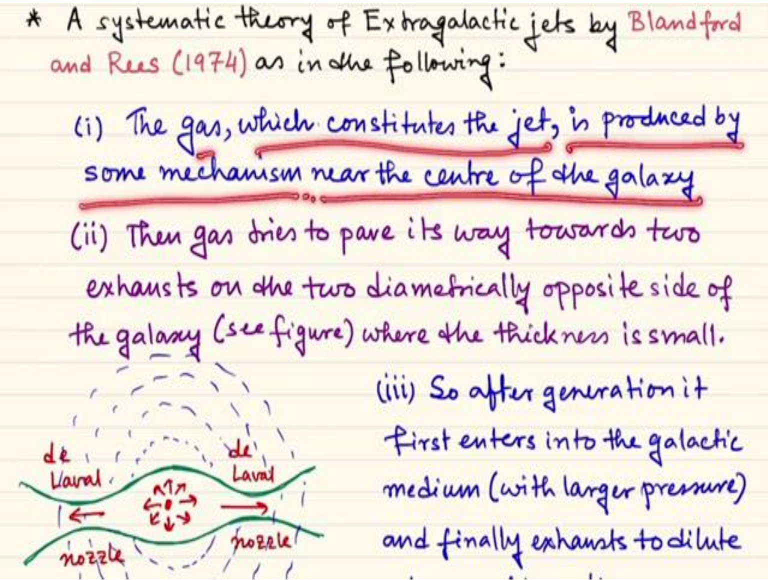

Now, what basically happens inside and how this acceleration happens? It is not really 100 percent known, but a systematic plausible theory of this extragalactic jets were for the first time proposed by Bland ford and Rees in the year 1974, and they simply say that the gas which constitutes the jet is first produced by some mechanism that we do not know much clearly near the center of the galaxy.

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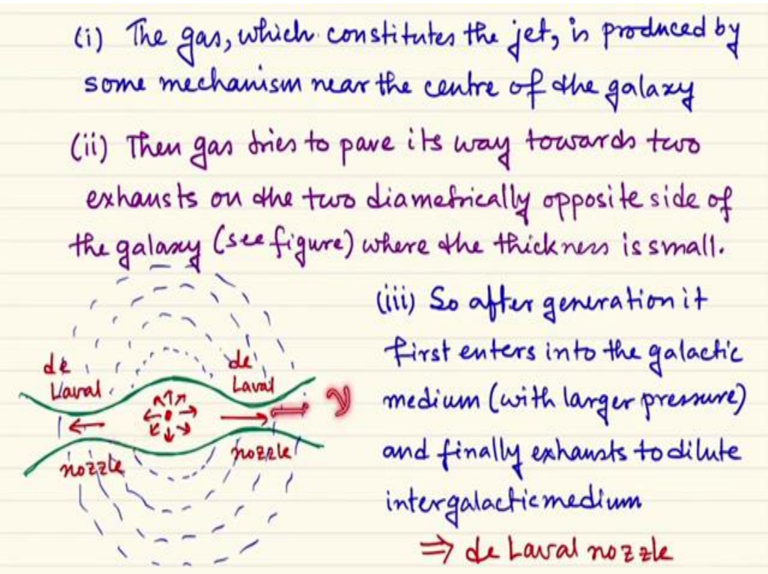

Now, you see this figure. So, the jet is produced here or generated here near the center of the galaxy and then the gas tries to pave its way towards two exhausts on the two diametrically opposite side of the galaxy where the thickness is small. So, exactly this type of gas highly energetic and light gas they will try to pave their way out to get out of this galactic medium to the outer medium and they will search for the paths where there is least resistance.

So that means, they will search for the path where the thickness is small. So, first what happens? After just generation it is subsonic and then when it enters into the galactic medium which is quite dense and the pressure is high then first it gets accelerated because it is almost like entering from a wider region to a narrower region.

When it finally, becomes supersonic by acceleration then it finds its path of exhaust, it again accelerates because of the de Laval nozzle principle. So, finally, when it exhausts to the dilute intergalactic medium this becomes highly supersonic and accelerating.

However, we know very well nowadays that Bland ford and Rees model, they are not the best one because of various things. One of the reasons is that the size of the de Laval nozzle which can be estimated for several type of observed extragalactic jets are actually not realistic for the corresponding galaxy sizes.

Then the people actually thought that the efficiency for the efficient acceleration of the extragalactic jets, actually there is an indispensable role of the magnetic field. So, that part is another domain, but at least for being inside the domain of the neutral fluids a very simplistic model of one-dimensional compressible flow.

We could show how the system, how a subsonic flow first gets accelerated while entering to a region or zone of higher pressure, and then it first becomes supersonic and then it again gets accelerated whenever to go to another region with less density or less pressure. So, that is the moral about this theory.

So, in the next lecture we will discuss something about the instabilities of the astrophysical medium.

Thank you very much.