Introduction to Astrophysical Fluids
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Lecture – 26
Supernova explosions and blast waves I

Hello, and welcome to another lecture of Introduction to Astrophysical Fluids. I hope that you have been enjoying the course thoroughly. So, as promised in this week's lecture, we will be discussing the application of shock waves in astrophysics and one application I already mentioned in the last lecture, that is the physics of the blast waves, which appear during the explosion of the Supernova.

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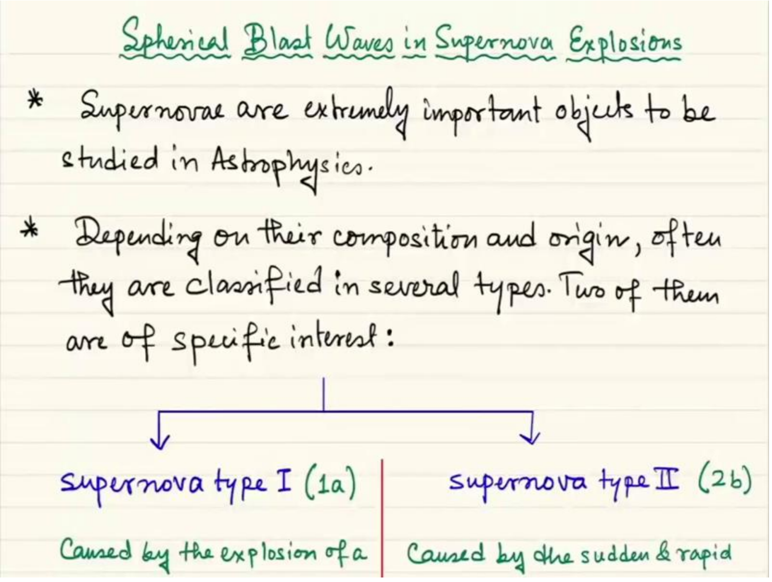

So, supernova are extremely important objects to be started in astrophysics. So, I am not sure how many of you know what supernovae are, here, I am trying to give you a very brief introduction about supernova. So, depending on the composition and origin, often the supernova they are classified in several types. But two of them are of very specific interest.

So, in general supernova means that, I mean very roughly, a very high energetic explosion type of thing, explosion of a star. So, that is the very rough meaning of supernova.

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studied in Astrophysics.

* Depending on their composition and origin, often they are classified in several types. Two of them are of specific interest:

supernova type I (1a)	supernova type II (2b)
Caused by the explosion of a white dwarf due to mass accretion from its neighbour in a binary system.	Caused by the sudden & rapid collapse and a violent explosion of a massive star of mass ranging from $8 M_\odot - 40 M_\odot$
(does not contain H-emission spectra)	(Contain H emission in spectra)

Now, whenever in the framework of normal astrophysics, we talk about supernovae. So, we talk about supernova of type I and most frequently we talk about supernova type $1a$. Now, there is another supernova, supernova type II and usually we talk about supernova type $2b$. Now, supernova I is basically defined as the supernova which does not contain any hydrogen emission line in the spectrum. So, that is the formal definition of supernova type I.

Now, there are $1a$, $1b$, and of which $1a$ is very interesting and this is caused by the explosion of a white dwarf, white dwarf is one of the very famous compact objects in astrophysics, already we have mentioned about it while discussing the physics of accretion disks.

So, this supernova is caused by the explosion of white dwarf due to mass accretion from its neighbor star in a binary system. Now, that neighbor star may be a less dense star as we discussed in the case of an accretion disk or a comparative white dwarf type of another compact object that is also possible.

So, then after accreting a sufficient amount of mass, it actually the super compact object or the white dwarf explodes and that causes supernova. Why that explodes that is another story and for that I want you to search actually through internet or in various books to go into the detail.

So, this current course is beyond the scope of discussing all those details. Another thing is supernova type II and this means that, this type of supernova contains hydrogen emission line in the spectra. Now, this is whenever we are talking about type II, usually we talk about type

$2b$ and this is caused by the sudden and rapid collapse and a violent explosion of a very massive star of mass ranging from 8 solar mass to 40 solar mass.

So, this needs a mass accretion from a binary neighbor I mean this does not need any other star, but it explodes due to its own collapse. So, that is the roughly difference between these two.

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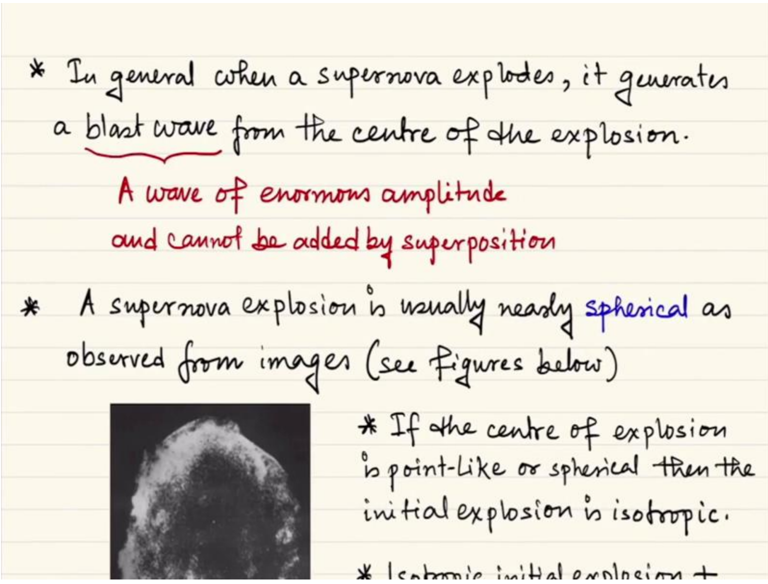

Now, whatever be the origin or the nature of the supernova, it is true that in general whenever a supernova explodes, it generates a blast wave type of thing from the center of the explosion.

You can simply think that a very small volume in space it emits suddenly an enormous amount of energy like this, for example, the explosion of a bomb, this type of thing. So, it creates a blast wave. So, the blast wave is nothing, but a wave of enormous amplitude and cannot be added by superposition. So, you cannot just add them linearly to get the resultant.

So, if you want to have the effect of two blast waves, the combined effect, the addition of the waves cannot give you simply the combined effect, because this is, then non-linearity should be taken into account. Now, so this blast waves are caused by the supernova explosion and in our discussion, we will mainly be addressing the nature and the physics of that blast wave.

Now, whenever a supernova explosion takes place from the images which are obtained in generally by astronomical means, they are usually found to be nearly spherical .

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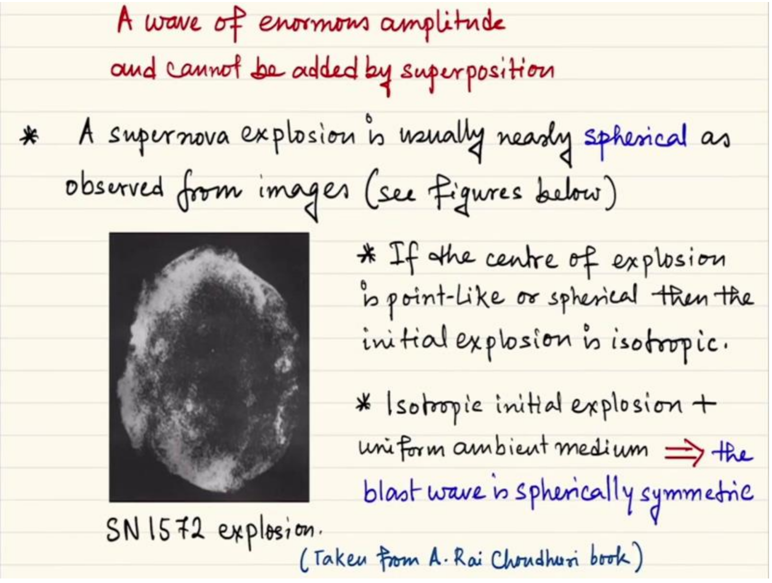

So, this is a typical image of the supernova SN 1572 explosion, this image is taken from A. Rai Choudhury book, one of the reference books which I gave you for this course. Now, this SN 1572 is a supernova which exploded in this manner and you can see this is almost spherical.

Now, it is true that, this is very common sense that, if the center of the explosion is nearly point like and let us say it is not like a dumbbell shaped or it is not like something like a conical shape type of thing but it is a point like thing like this or spherical, then the initial explosion can be expected to be isotropic, that means there is no directional preference for the explosion.

So, if we have an initial isotropic explosion plus if the ambience so, after the explosion the blast wave should be expanded in the ambient medium, this is the surrounding medium.

So, if the ambient medium does not have any stark, I mean non uniformity that means it is more or less homogeneous or uniform, then the blast wave is actually expanded also in a spherically symmetric manner, that means there is nothing to damage the directional unbiasedness of the whole process, right.

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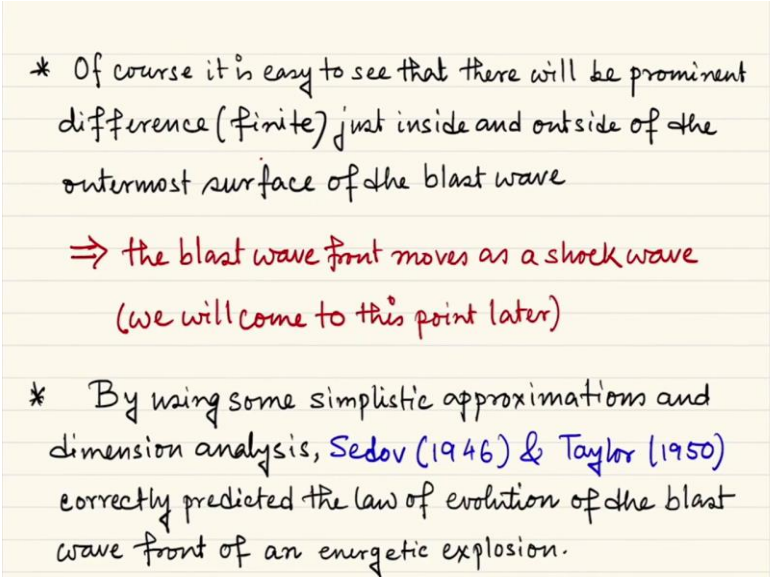

Of course, it is now easy to see that, there will be prominent difference which is finite just inside and outside of the outermost surface of the blast wave. So, if you just check of this for example, so, this is the blast wave front, right, and so, let us say the fluid just outside this and just inside this is very, very much different; because the inside this contains the exploded material right, it has a totally different density, energy, pressure, velocity.

Whereas the, medium outside is simply undisturbed medium, undisturbed surrounding ambient medium, right. So, it can be this front, this blast wave front can then be considered like a shock front correct, this one. So, this can be considered like a shock front because here this is the medium, which is totally influenced by the explosion and this medium is till now undisturbed or uninterrupted by the disturbance.

Now, you see that this shock wave then assumed to expand spherically outward in the outward direction. But we will come to this point later that, how we will use the different properties of a shock wave. As we did last time, that means the famous Rankine Hugoniot condition type of thing, and using that how can we determine or how can we like infer on the structure of the blast wave front and also the jumps of the density, velocity this type of thing, that we will do later but now we will do something very interesting and very simple.

By using some simplistic approximations and very crude dimensional analysis, actually Sedov in the year 1946 and Taylor in the year 1950, both of them correctly predicted something very interesting. They predicted that the law of evolution of the blast wave front of an energetic explosion correctly and that we will see.

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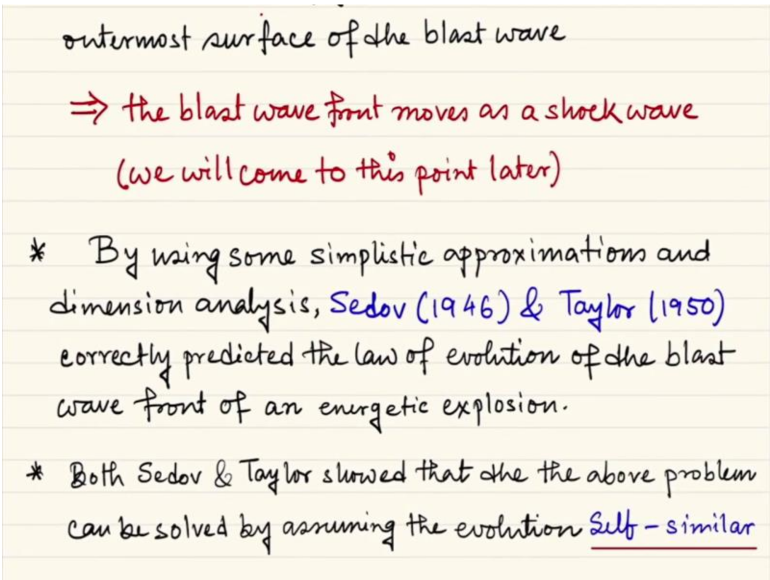

So, they did not use any formal analysis for that, they just used very rough dimensional analysis but they used another very simplistic approximation, which comes out to be very, very useful and very, very reasonable for this type of blast wave expansion.

So, both Sedov and Taylor they showed that the above problem of expansion can be solved very easily by assuming the evolution to be self-similar. So, they assumed that the expansion of the blast waves takes place in a self-similar way. Now, of course, the first question comes to our mind is, what a self-similar process means in this context?

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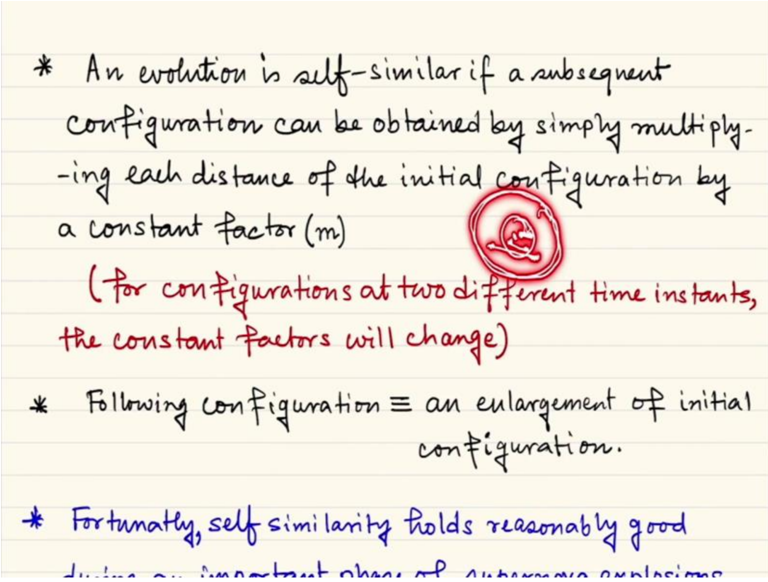

So, self-similarity is something which is tremendously used in various domains of physics, starting from large astrophysical things as you will see to microscopic systems. So, here we will try to understand what self-similarity from a very practical point of view. There are much more mathematically rigorous definition of self-similarity in several cases.

But here we just say very simply that, an evolution is self-similar, if a subsequent configuration, that means a configuration at a later instant, can simply be obtained by multiplying each distance of the initial configuration by a constant factor.

That means, if the initial configuration is given by this and the later configuration is given by this then if the distance between two points on the sphere is multiplied by 2 or when we trace these those two points on this sphere, then if we take any other points here and calculate the distance of these two points, between these two points on this sphere and this sphere, we will see that this distance is also multiplied by 2.

If this is true, then we will say that, the system is expanded or evolved in a self-similar manner. Another way of saying this is that if it is a self-similar expansion, then after that the configuration globally does not change.

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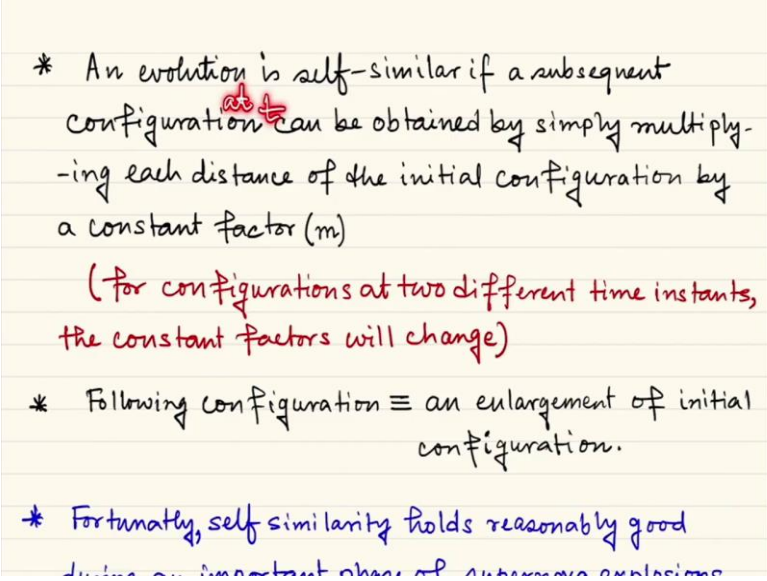

Now, of course, when I say that the subsequent configuration that is a subsequent configuration at a later instant let us say t can be obtained by simply multiplying each distance of the initial configuration by a constant factor m.

But this m is constant for any two points on the spheres, we are now concentrating on a given value of t. Let us say after 5 seconds from the initial point or starting point I mean 5 seconds after the explosion, for example, then this m has a constant value.

Of course, if we now try to go to another instant that means we try to study how it looks like at t is equal to 10, that means 10 seconds after the explosion instance, then there will be every distance will be again multiplied by the same constant factor, but that will be some m'. So, for configurations at two different time instants, the constant factors will change. So, for every time instant, there is a specific constant factor.

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-ing each distance of the initial configuration by a constant factor (m)

(for configurations at two different time instants, the constant factors will change)

* Following configuration $\equiv$ an enlargement of initial configuration.

* Fortunatly, self similarity holds reasonably good during an important phase of supernova explosions and in other problems involving blast waves

(Explosion of an atom bomb)

We will come into that in a much more schematic way that will be much clearer at that point. Now, you will see all these things specifically leads to the conclusion that, the following configuration or the subsequent configuration is nothing, but an enlargement of initial configuration, right.

So, fortunately we can see that, this is a very useful assumption but we do not know how reasonable or how relevant this assumption is for our case. But fortunately, self-similarity, actually holds reasonably good during an important phase of supernova explosions and in other problems involving blast waves.

One such problem is the explosion of an atom bomb, and for your kind information, for the first time, for example, Taylor was working on this blast wave thing, he was actually working to analyze the effect of the explosion of an atom bomb, and that is why only after he was working during the Second World War and only after the war ended, his research papers were declassified by the government and it got published and finally, people knew about it.

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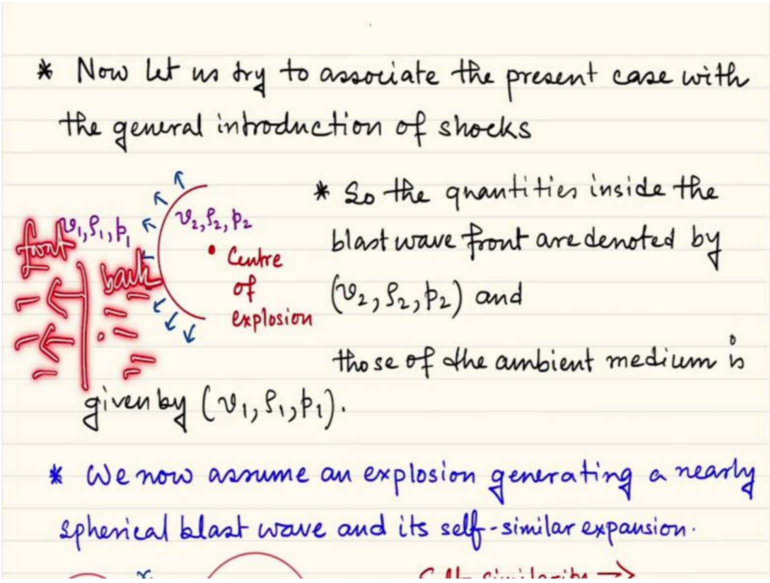

So, now let us try to associate the present case. Now, the present case with the general introduction of shocks. So, for example, now we are talking about the something is expanding, this surface of discontinuity is expanding in a spherically symmetric way, from the center to the outward direction.

But previously when we talked about the surface of the shock then we said that the shock is actually going like this way in a medium and disturbed medium like this. So, this is the front part of the medium and this is the back part of the medium that we said right, if you remember the picture.

So, here if we just think that, we have a shock, but the shock itself has a spherical shape then the shock is now no longer like this upright front type of thing, but it is a spherical front. Then we can actually think that the part which is in front of the shock is this part, that is the surroundings or the ambience.

Then the velocity, density and the pressure are given by v_1, ρ_1, p_1 for the medium which are behind the shock and contains all the information about the explosion, they are designated by v_2, ρ_2, p_2 and here this is the center of explosion.

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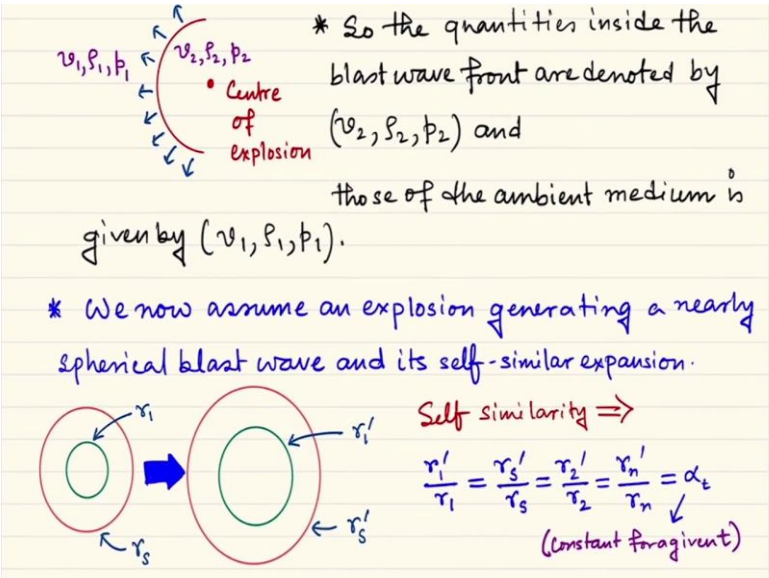

Now, in our case, we have now assumed that, the whole thing is taking place in a self-similar way. So, an explosion let us say is generated and due to the explosion, nearly spherical blast wave is generated and its expansion is nearly self-similar then self-similarity simply says that, if we just take the previous configuration and the subsequent configuration and we have two shells, one is an intermediate shell of radius r_1, another is the let us for example, any other radius, let us take the front, the outermost surface of the blast wave, which is of r_s radius , you can see here clearly.

Now, if we simply say that, this one is for a later instant, this one is now going into this and this one is going into this then this now becomes r_s' and this one becomes r_1'. So, the definition of the self-similarity simply says that, r_1' by r_1 will be equal to r_s' by r_s and it can be true for any other two radiuses, two radii actually, and the initial configuration.

So, it can be for any other let us say r_2' by r_2 or any other r_n' by r_n, and this constant factor is a constant for a time instant, for a given dimension. Let us say we are now checking what happens after, what happens t seconds after the point of explosion, and we call that proportionality constant or constant ratio factor to be α_t.

So, α_t is constant for all type of ratios but these changes when I change the t. So, one thing is true that r_1' will be $\alpha_t r_1$; r_s' will be $\alpha_t r_s$; and for any arbitrary r_s' will be $\alpha_t r_n$.

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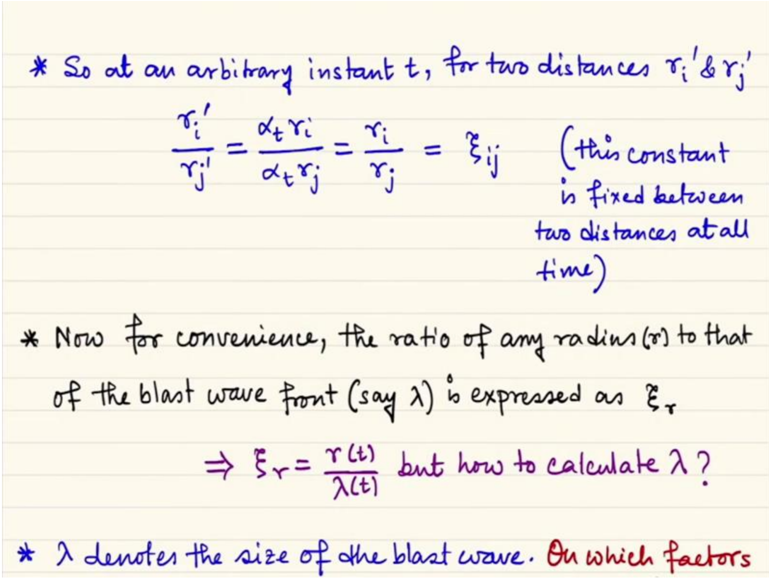

So, at an arbitrary instant t, for two distances r_i' and r_j'. So, now, let us say we are just concentrating on one sphere but everything in a later instance. But we are now concentrating, before we were concentrating on the green one in two configurations, the red one in two configurations, now we are concentrating on the two different radii in the same configuration and we will see that what it gives.

So, let us say two distances or two radii we have taken from the letter configuration, so, r_i' and r_j'. So, r_i' by r_j' will simply then be equal to $\alpha_t r_i$ and $\alpha_t r_j$ and you know, so, α_t will go out, then this ratio will simply be r_i by r_j.

So, this r_i by r_j will be actually equal to r_i' by r_j'. So, the ratio of two radii is independent and it is constant over time. So, this constant value does not depend on the time instant chosen but this constant value is depending on the choice of i and j.

So, let us say if you now take for example, red and green you have one value of the ξ and if you take, I mean red and this one intermediate one, you will have another value of ξ. If you take green and this intermediate, sorry this intermediate radius, you will have another value of ξ.

So, ξ's value is independent of j because this gives us the fixed ratio of two give consent I mean two given shells over time. So, this one, so this factor is independent of time.

So, we can say that a self-similar expansion is nothing but an expansion in which the ratio between two given shells, I mean in our context of course where we have a number of concentric shells, spherical shells, then the ratio of two given shells in the later configuration and in the primary configuration, they are independent of time.

Now, for convenience, the ratio of any radius r to that of the blast wave front, let us say λ is expressed just as ξ_r. So, we are just removing two indices like this and we just say that, let us say this r_j is nothing but the wave front itself, that is λ and r is any arbitrary radius. So, then ξ_r will be nothing, but $r(t)$ by $\lambda(t)$.

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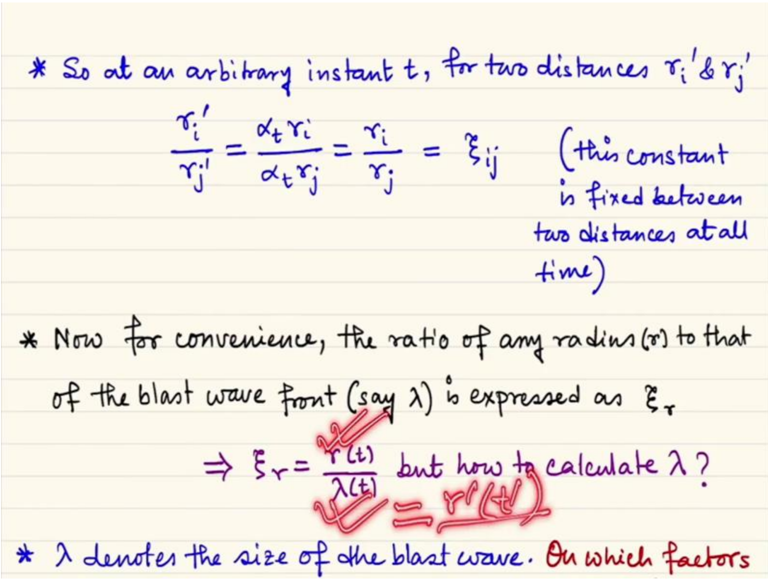

So, that is simply ξ_r and of course, you have to remember that, for a given r and if you just level your shells in one instant, actually I should have written like this is equal to $\frac{r(t')}{\lambda(t')}$, this is also equal to ξ_r.

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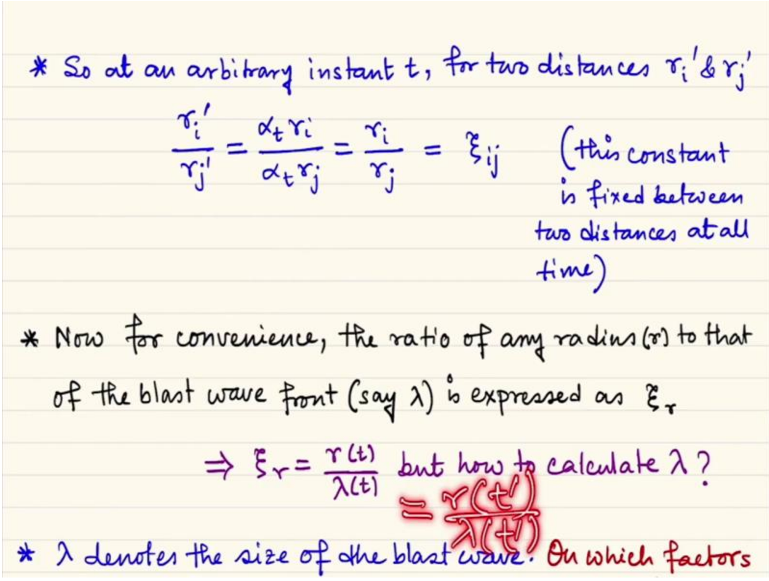

Now, the question is, how to calculate λ? If so, this is something fixed, for a given r if we know λ.

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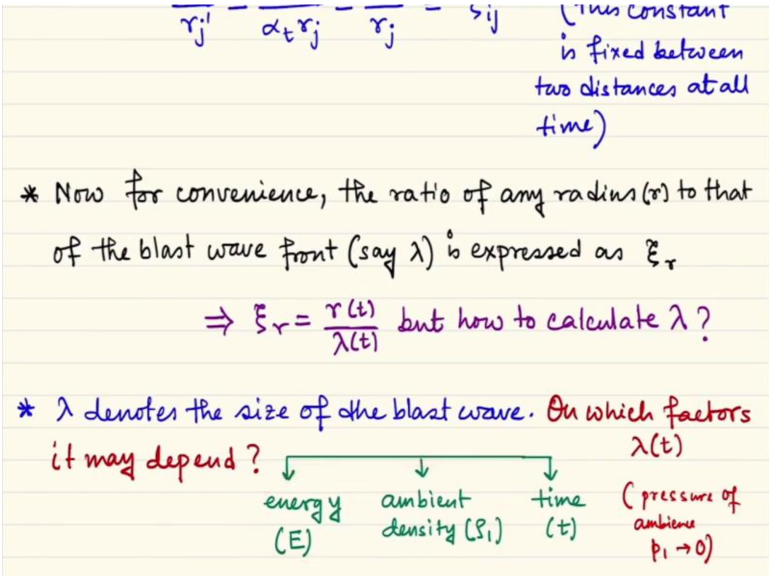

Now, λ denotes the size of the blast wave. Now, how to determine that? At this point we need to use the very powerful, but very simple dimensional analysis and for that we have to understand first or we have to determine on which factors $\lambda(t)$ depends.

Now, for a system, by using common sense one can simply say that, $\lambda(t)$ is an explicit function of time that is true, because this is an expansion. If nothing changes, but only time changes, λ will change, because of the size and it changes with time, because it expands. The system, the front is in motion.

Now, there is another thing, the energy of the explosion. let us say if the time is frozen, but we are just checking two explosions, one has an energy E_1, another is E_2, of course, from our intuition we can say that, the energy will mostly be converted in the kinetic energy of the blast wave envelope.

So, if E_1 is greater than E_2, then the blast wave velocity, front velocity will be greater than that of the blast wave front velocity for E_2. So, the size will be greater for greater E_1 and the ambient density. So, now, the whole thing should go.

So, if you now remember this picture, the whole thing may should go or should expand in the ambience. If the ambient density is very low, then the system will have facility or the system will facilitate the expansion process of this shock front. If this is high, then the system has to fight against the density of the surrounding right and then the size is not that large.

So, we can think that this can actually play an inverse role with the size of the blast wave. Now, how is that, we can do with a very simplistic way, and any other thing there can be pressure of the surrounding but for instance we just assume that the pressure of ambience this is p_1 is tends to 0, it is very, very low. So, we are just taking all these three energy, ambient density, and time.

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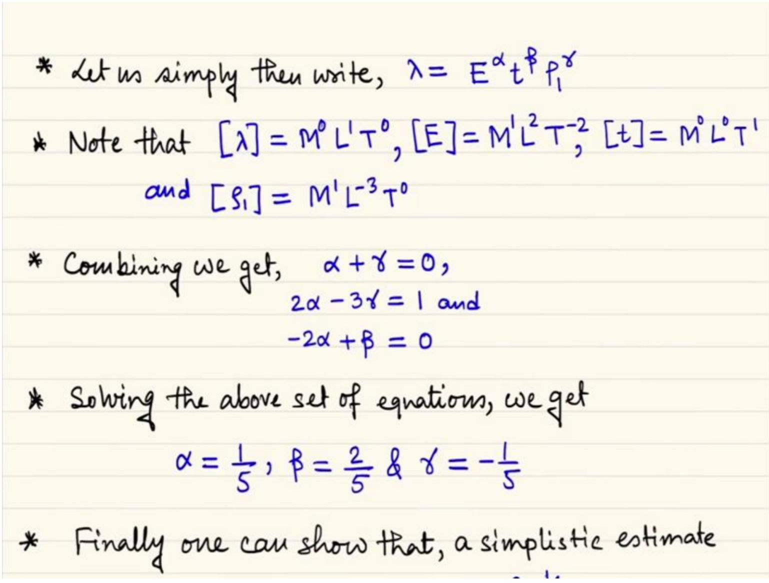

Now, let us simply write that, λ which is the size of the blast wave is equal to $E^{\alpha} t^{\beta} \rho_1^{\gamma}$. We just assume that; all these are the only three factors which can determine λ.

Now, it is true that, λ has a dimension of length. So, it will be $M^0 L^1 T^0$, E has a dimension of energy $M^1 L^2 T^{-2}$, it is like Mv^2, t will have the dimension of time purely $M^0 L^0 T^1$, and ρ_1 is nothing, but the mass density which is $M^1 L^{-3} T^0$.

So, if we combine all these dimensions according to this rule, finally we will have $\alpha + \gamma = 0$, $2\alpha - 3\gamma = 1$, and $-2\alpha + \beta = 0$. Now, if we solve this above set of equations; we can see that alpha is equal to $\frac{1}{5}$, beta is equal to $\frac{2}{5}$, and gamma is equal to $-\frac{1}{5}$.

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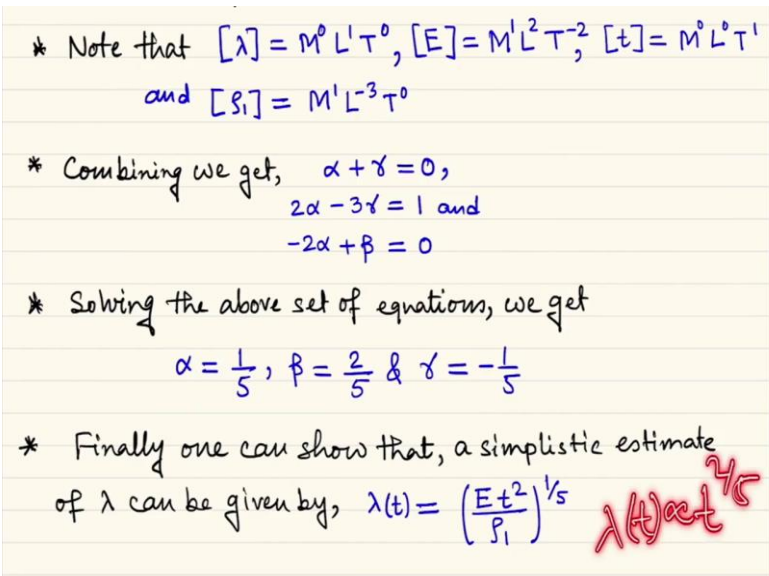

So, finally, one can show that, this simplistic estimate of λ just by using a dimensional analysis, which is not analytically rigorous but simply it gives us a rough form of the dependence of λ with as a function of time and with also energy and the density of the ambience.

It will be given by $(\frac{Et^2}{\rho_1})^{1/5}$. So, for a given explosion and given ambience E and ρ_1 they are fixed. So, $\lambda(t)$ will simply be proportional to $t^{2/5}$. So, that is the conclusion.

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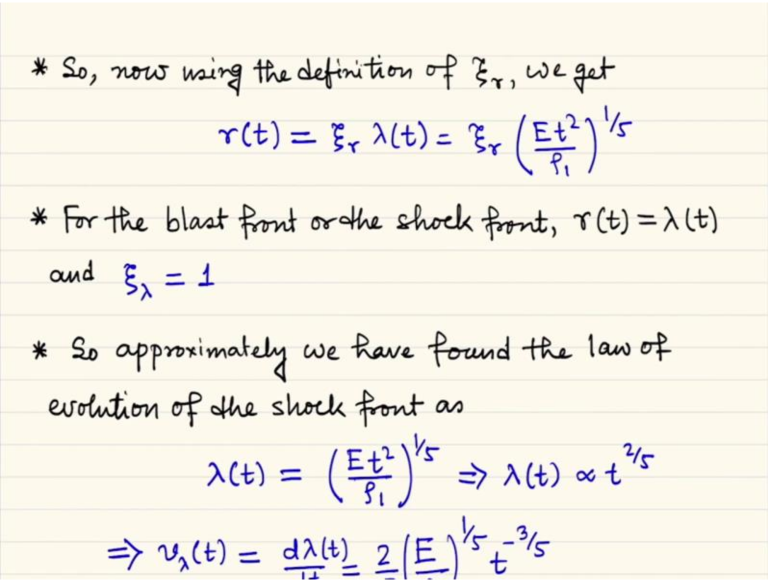

If that works well then, we will think that, our whole theory and this construction of the self-similarity, this works well. Now, of course, that was all about λ but to determine any arbitrary radius at some given instant t, then this will be simply equal to $\xi_r\lambda(t)$ and that is equal to $\xi_r(\frac{Et^2}{\rho_1})^{1/5}$.

So, in order to have information about $r(t)$ in addition to $\lambda(t)$, we have to know ξ_r as well. So, that information is needed. Now, coming back to the size of the blast wave then we have to concentrate on the blast front and this $r(t)$ is nothing, but $\lambda(t)$ itself.

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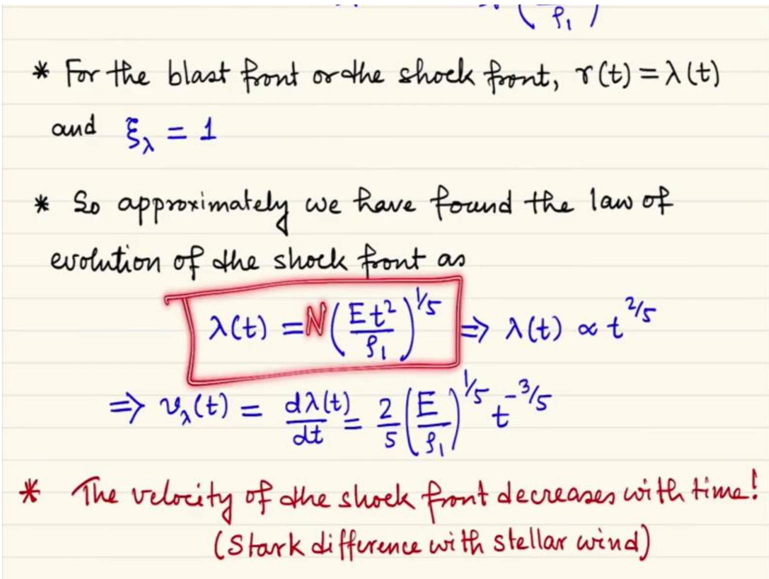

So, at blast front, the $r(t)$ is $\lambda(t)$, right. So, ξ_λ is equal to 1 and we know what it is. So, then $\lambda(t)$ or $r(t)$ whatever this is the same thing, they are approximately equal to this one, that is equal to simply $N(\frac{Et^2}{\rho_1})^{1/5}$.

So, $\lambda(t)$ is proportional to $t^{2/5}$, and $v_\lambda(t)$ the velocity with which the blast wave front expands in the surroundings is given by $\frac{d\lambda(t)}{dt}$ and which is $\frac{2}{5}(\frac{E}{\rho_1})^{1/5}t^{-3/5}$.

Hey, can you feel something strange, well that means that, although λ changes or λ actually increases with time, v_λ actually decreases with time, and here you see that is a very stark difference with the velocity of the shock front decreases with time and this is a very stark difference with this stellar wind expansion.

Where, for example, when we were talking about the solar wind, we saw that this is basically becoming from subsonic to supersonic and that is the observed fact. But here this is the opposite thing which is happening, the velocity is actually reducing.

Now, the question is that, it is true that, whenever we are talking about this type of blast wave expansion or something, so, we are thinking that the shock wave is expanding and the shock waves are not exactly the same as the solar wind plasma expansion.

But it is true that, once the solar wind becomes supersonic, can somehow this type of shock wave formation be possible there. So, is it like that maybe after gaining the supersonic speed maybe there is a reduction of the stellar wind velocity with time, you have to search, this is the research part.

So, there is also something which is expanding and we do not know whether the speed is actually. So, we know that there is an acceleration from its origin near sun for the stellar wind and from its subsonic regime to supersonic regime but what really happens after attending the supersonic regime that, we have to think, and only then we can think the analogy or we can somehow try to compare them with this type of shock front.

Of course, this shock front is a very, very strong shock. So, just check and see that, which type of researches are being done on that. So, what I can tell you that, this is a very active domain of research.

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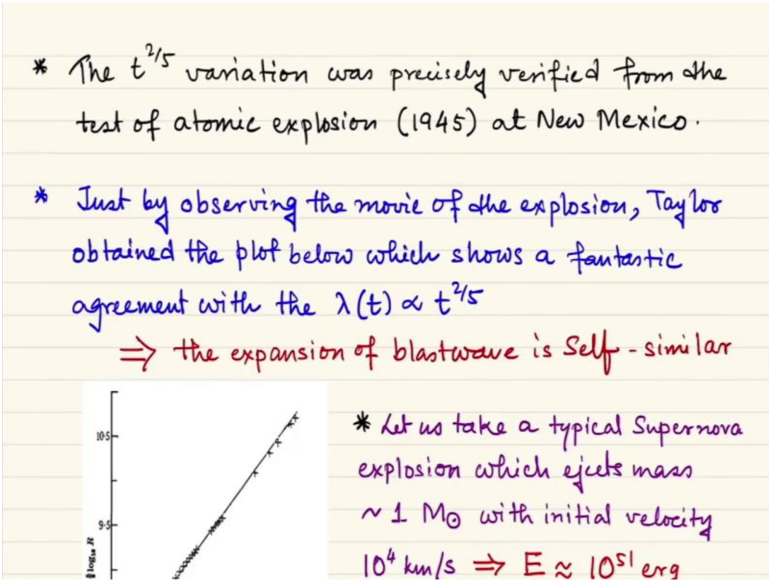

So, coming back to this one that, we predicted I mean Sedov and Taylor they predicted $t^{2/5}$ variation of the size of the blast wave explosion as a function of time and that was actually precisely verified from the test movie of the atomic explosion, so by Taylor.

So, Taylor estimated the sizes of the blast wave from the different instants taken from a movie of the atomic bomb explosion in 1945 at New Mexico. So, just by observing the movie at the explosion, Taylor obtained the plot which showed a fantastic agreement with the $\lambda(t)$ proportional to $t^{2/5}$ and the plot was like that.

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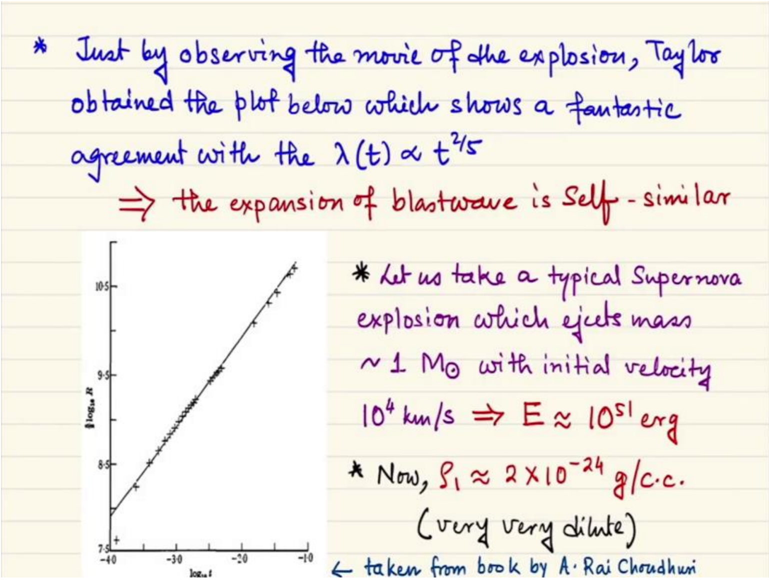

So, it was a $log\ log$ plot if you see. So, of course, the plots are not $log\ log$ but the quantities which are plotted they are logarithmic quantities. So, $\log R$ versus $log\ t$ actually $\frac{5}{2}\frac{\log R}{log\ t}$ and you can see that finally, there is a fantastic straight line with slope 45 degrees. So, finally, you can see that this one is holding excellent.

Now, what is the conclusion? The conclusion is that, our basic hypothesis or the basic assumption that the expansion of blast wave is self-similar is actually true. Now, let us just take a typical supernova explosion for the order of magnitude, and for such explosion, actually the mass ejected is of the order of one solar mass and the initial velocity is 10^4 kilometer per second.

That means the velocity with which the system or the blast wave tries to starts expanding, just after the explosion. It gives us an initial kinetic energy of 10^{51} erg. Now, just by taking a typical very, very dilute interstellar medium type of thing, you can see that the density is 2×10^{24} gram per cubic centimeter.

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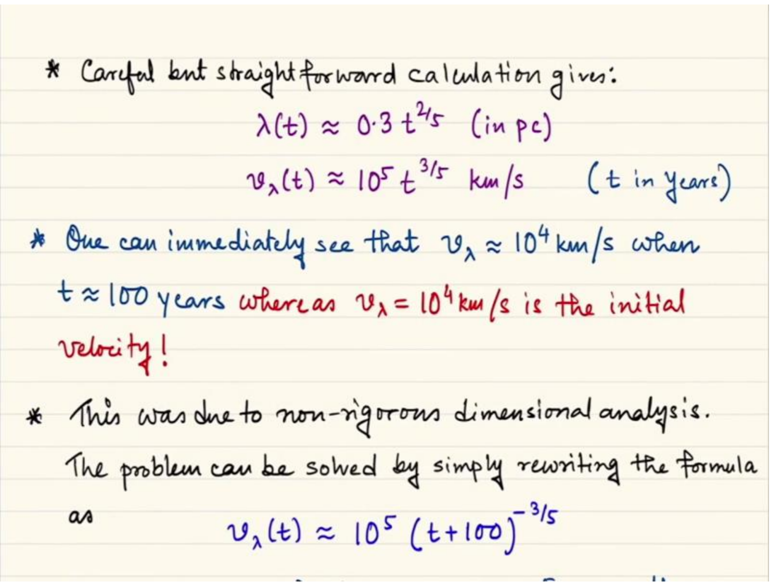

If you can actually see that, some straightforward calculation gives us $\lambda(t)$ is equal to almost equal to 0.3 $t^{2/5}$, where the λ is in per sec, and v_λ is equal to almost 10^5 $t^{3/5}$ in kilometers per second and in both the cases, the time is in years, that is much more practical.

Now, what is the strange point? The strange point is that, if you see that you notice carefully that, v_λ is equal to 10^4 kilometer per second when t is almost equal to 100 years. But we know from our knowledge that, we supplied v_λ is equal to 10^4 kilometer per second at the time of the explosion that means it was the initial velocity.

So, why is this discrepancy? That is simply because, the method by which we determine this formula was not rigorous, it was very hand waving dimensional analysis, and this dimensional analysis cannot give you, for example, if let us say that if in reality there is some dimensionless number N for example, then dimensional analysis cannot give you any news of that, that is something drawback.

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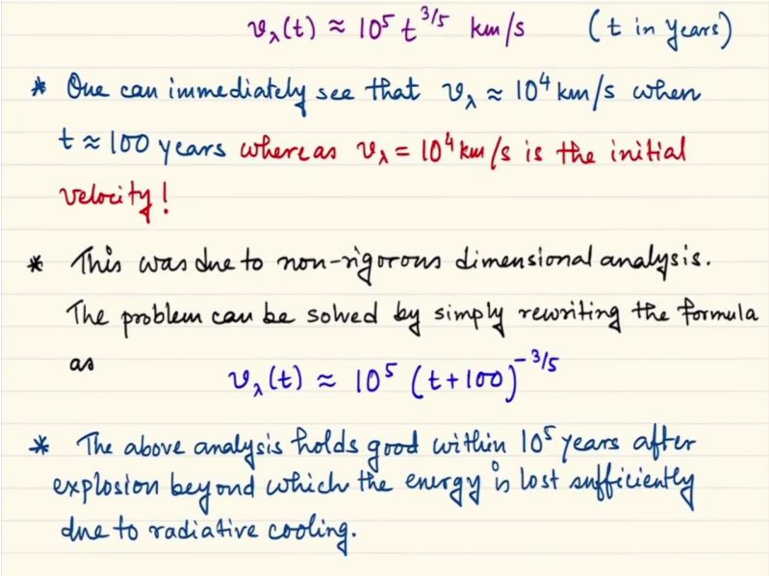

Another drawback which we will understand here now. So, we can see that the initial velocity actually is calculated from the final form as a velocity after t is equal to 100 years. So, how to solve that? That is simply to say that this is not the correct formula, the correct form is like this $v_\lambda(t)$ is almost equal to $10^5(t + 100)^{-3/5}$.

So, it simply says that, there is a shift in origin, in the original formula. So, let us say if someone now today derives this type of formula from analytical point, analytical considerations, he or she should get this form, and that actually tells you that, when t is equal to 0 then this part is actually 100 and so we have v_λ is equal to 10^4 kilometers per second and we agree with that.

So, that was the solution of this discrepancy. Now, the final thing is that, in this whole discussion, we just used some very simplistic approximations and assumptions and dimensional analysis method to understand how the blast wave evolves with time. So, it is true that, here we did not do any formal analysis of the shock.

So, that we will do in the next lecture, but here we just showed that, by simple things you can do very, very interesting thing in astrophysical premise and the last thing is that. So, all this analysis holds very good within 10^5 years after explosion beyond which it is usually found that the energy is lost sufficiently due to radiative cooling.

Then this type of law will not hold any longer because E will be no longer a constant. So, there will be radiative cooling from the blast wave so, then you have to think and calculate accordingly.

So, that was all about the introductory part of the blast wave and also, we showed that, how simplistic things give us an idea of the blast waves development with time and how actually we could cross check the very hypothesis of self-similarity, both for supernova explosion and also a very important terrestrial application, that is the application of the explosion of the atom bombs.

So, this is also very much interesting thing that, we have to think that which type of correspondences we can draw every day with astrophysics to our everyday life. There are not so many, but if we can somehow bring some of the correspondence, I mean this is always very much recommended.

So, in the next lecture, we will use the basic conditions Rankine Hugoniot conditions of shocks, and we will do something much more analytical to see the structure and the jumps over the blast waves.

Thank you very much.