Introduction to Astrophysical Fluids
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Lecture – 21
Accretion disks I

Hello, and welcome to another session of Introduction to Astrophysical Fluids. In the previous discussion, we talked about the generation of the stellar winds and also, we said that, this problem is equivalent to its inverse problem in the steady state and this inverse problem is the famous Bondi's problem of spherical accretion. We also mentioned that this is an ideal case; in reality no accretion, no mass accretion in an astrophysical system basically follows this type of isotropic accretion using spherical symmetry.

In general, what happens that, whenever a less dense star comes in the vicinity of a very dense and compact star; it loses mass, but this process actually leads to the formation of a disk type of thing and that is known as the famous accretion disk in astrophysics. In this discussion, in today's discussion, we will mainly try to understand qualitatively how the Accretion disk is formed and how due to the formation of the accretion disk, some energy can be released.

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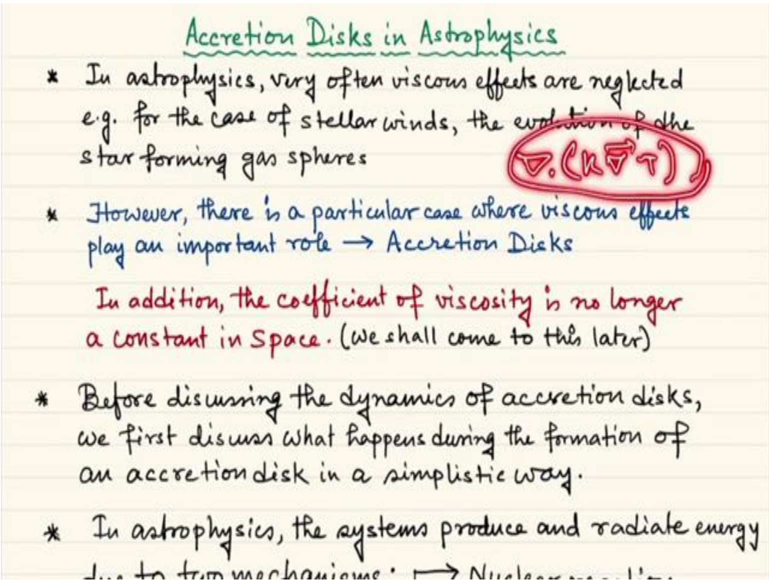

So, as I want to emphasize that, in astrophysics very often viscous effects are neglected. So, although we said that we have, I mean started discussing different astrophysical examples after deriving the real fluid equations, which includes different type of transfers. So, you can actually see that, very often viscous effects are neglected in astrophysical problems. But remember this does not say that, the fluid is perfectly ideal; all I mean there is always a non-negligible transfer of the thermal energy.

And that is why, if you remember that the divergence of K gradient of T term that is always non vanishing, almost always non vanishing. But however, I mean the thing which we are talking, that is the viscous effects; this is somehow neglected in the very usual problems of the stellar wind for example, or the evolution of the star forming gas spheres.

So, for the stellar wind we already saw that in Parker's model, the viscous effects are neglected, and for the evolution of the star forming gas spheres, maybe you know that we just considered mostly the equilibrium due to the interplay of the pressure gradient force and the self-gravity force.

However, there is one particular example where viscous effects must be considered and they play an important role in the formation of that astrophysical structure and that is the case of the accretion disks. So, most interestingly, I mean not only that we should include the effect of the viscosity in case of accretion disks but also, the coefficient of viscosity μ if you remember it is no longer a constant for a given fluid in space.

So, we shall come to this later, when we will talk about the analytical approach using basic equations. But before discussing this dynamics part using analytical approach or using equations, we first try to understand what happens during the formation of an accretion disk and actually very primarily why an accretion disk is formed, that we will try to discuss in a very simplistic way.

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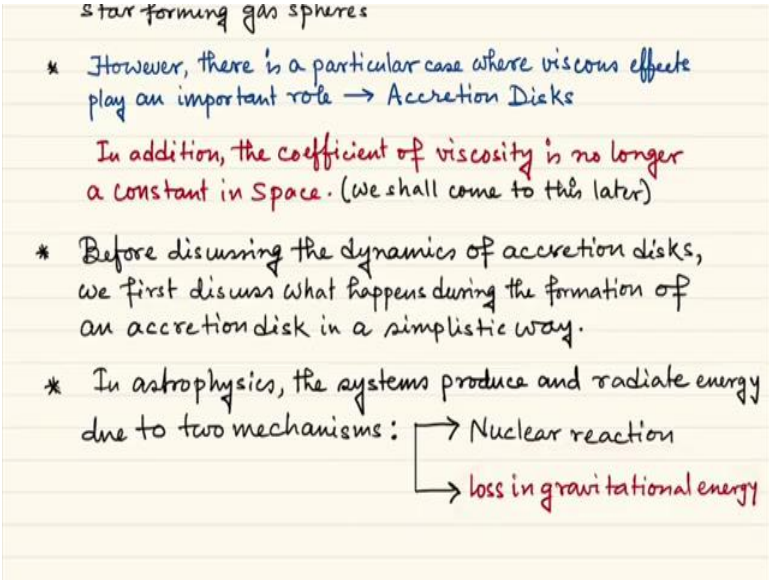

Now, in astrophysics it is true that, there are two type of mechanisms which principally produce energy, one is the famous nuclear reaction, you all know about that and the second one is the loss of gravitational potential energy. That lost energy basically gets transferred or transformed or converted into radiation, that is possible.

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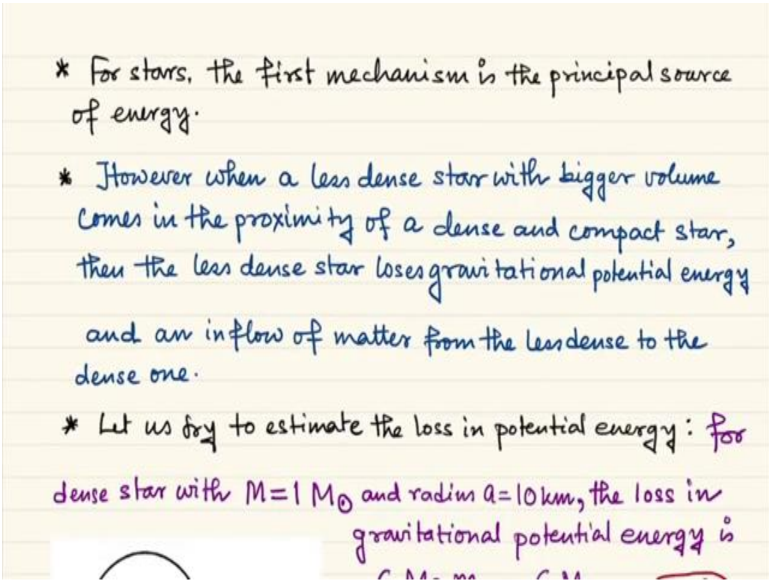

So, for example, when we are talking about the evolution of the stars then the first mechanism that is the nuclear reaction is the principal source of energy. For this type of system, the lost amount of the gravitational potential energy is actually negligible over the nuclear component.

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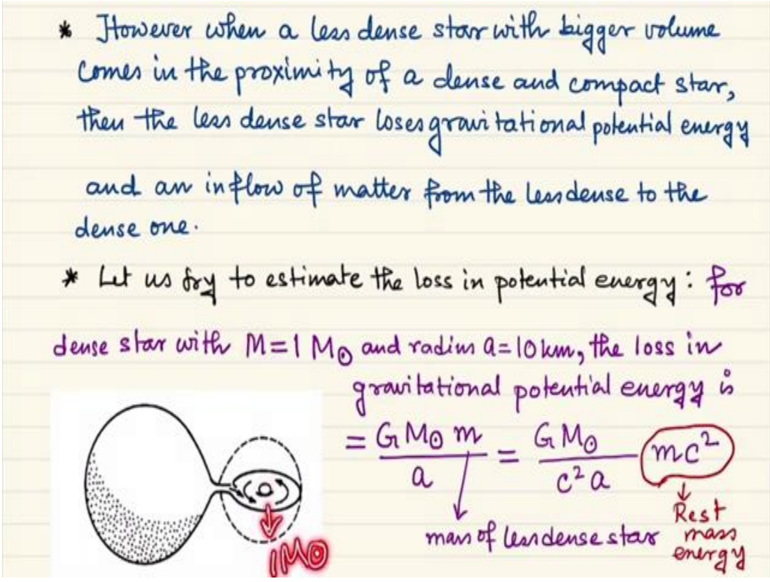

But when a less dense star with bigger volume, now you can see in the picture that is a good thing to see a schematic picture, here you will have this. So, this is the less dense star, but with bigger volume this one. When this comes in the proximity of a dense and compact star, the very small one Then the less dense star basically loses its gravitational potential energy. And what happens? An inflow of matter from the less dense to the dense star happens. So, basically the mass is coming from this one and going to this. But how does it, I mean goes towards this mass? Not like in a linear way or something in a spherical way but in a spiral way forming in a disk, that what is somehow I mean known phenomenon.

Now, we have to try to first of all understand why this is happening. But even before that we will do a very simple estimate to just show that, unlike the case of a star evolution, where the nuclear energy component basically dominates over the gravitational potential energy loss.

Here in the case of accretion disks, this is not the case. This is the primary reason of the energy production or primary source of energy production, because there is no source of nuclear energy. So, for that what we can do is that, we just can consider a typical dense star with mass which is equal to 1 solar mass $M_\odot$. So, maybe you should have an idea of what 1 solar mass is in terms of kilogram.

So, you just check and you will tell me whether it is nearly 10^{30} kilogram or 10^{31} kilogram or 10^{32} kilogram or what else. So, that is something you have to search it out. So, if it has a mass equal to 1 solar mass $M_\odot$ but the radius is only 10 kilometers. So, you can see that the total mass of the sun is only contained in a sphere of radius 10 kilometer then the density is enormous, that you can actually also calculate.

In this case, the loss in gravitational potential energy for that mass, so here we are talking about the mass of this one, this small star, this has one solar mass and this one is losing the mass. So, in this analysis if you see that, the loss in gravitational potential energy is given by $GM_{\odot}m$ divided by the radius. So, that I just wrote like as $M_{\odot}$. So, this is the unit of $M_{\odot}$, where m is the mass of this star, less dense star with bigger volume.

So, that is the amount of gravitational potential energy, which is lost when mass starts leaving this body and really reaches this body, of course.

You can easily just by multiplying c^2, where c is the light speed at vacuum. you can write the factor $\frac{GmM_{\odot}}{c^2}$ as mc^2 and mc^2 is nothing but roughly the rest mass energy of the star which is losing mass.

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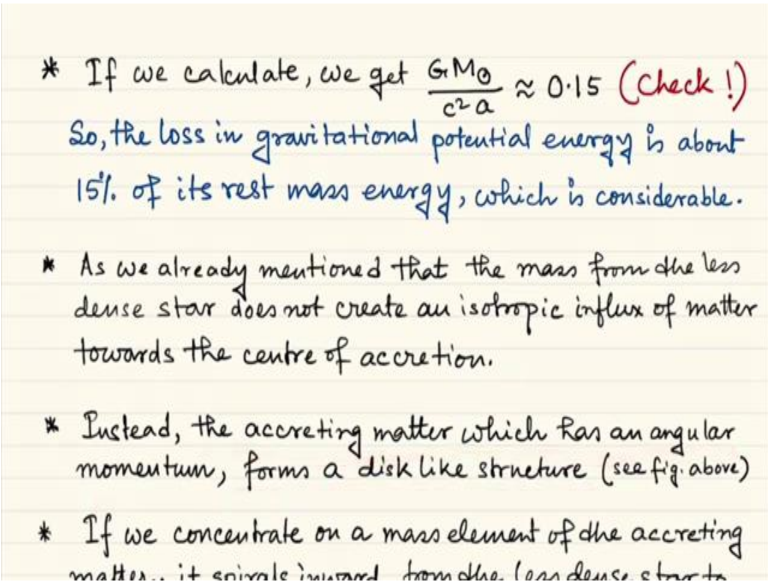

If you calculate this factor $\frac{GM_{\odot}}{c^2 a}$, you will see this will give you 0.15. Please check that. I hope there will be an exercise also for that like as assignment. So, that means that this loss in gravitational potential energy is about 15 percent of its rest mass energy which is considerable.

So, we finally could show that, in this case of the formation of this accretion disk, that means, in the case where the mass is lost from a binary system of a less dense star and a highly compact star. Basically, the loss in gravitational potential energy is a considerable source, although we could not compare this with nuclear reaction, but this is a very considerable source of production of energy.

So, now, as we already mentioned that the mass from the less dense star does not create an influx in an isotropic way that means like a spherical accretion.

So, this type of thing will never be accreted like a sphere but the accreting matter which has an angular momentum, indeed forms a disk like structure. Once again, you can just see this figure for reference. So, this is the disk which is formed over here, and the mass which is leaving this original volume is now going towards this mass in a spiral path.

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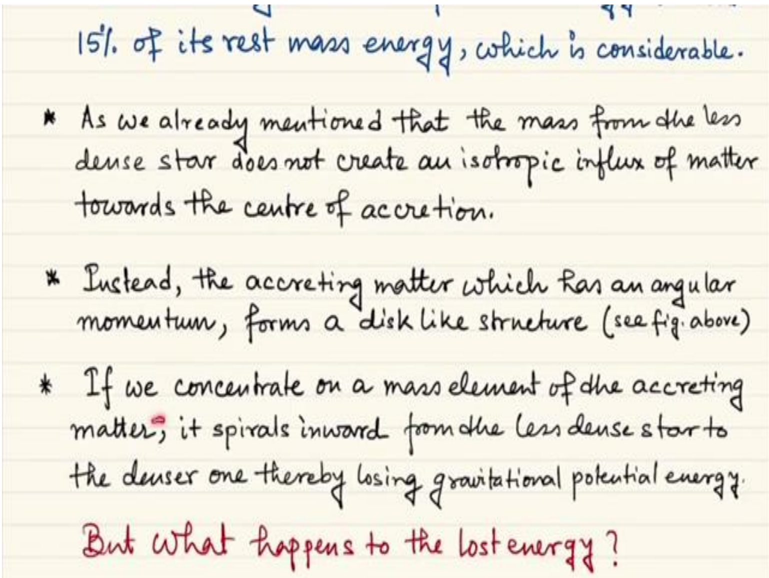

So, if we concentrate on a mass element on this accreting matter, we will just see that it spirals inward, from the less dense star to the denser one, thereby losing gravitational potential energy. Because, basically what is happening, these some amount of mass is now falling in the gravitational attraction.

I mean, gravitational field of this star and this field being attractive basically makes this system lose its gravitational potential energy because the system is doing work supporting the attractive field. So, basically the system is losing the gravitational potential energy. But the question is what happens to this lost energy?

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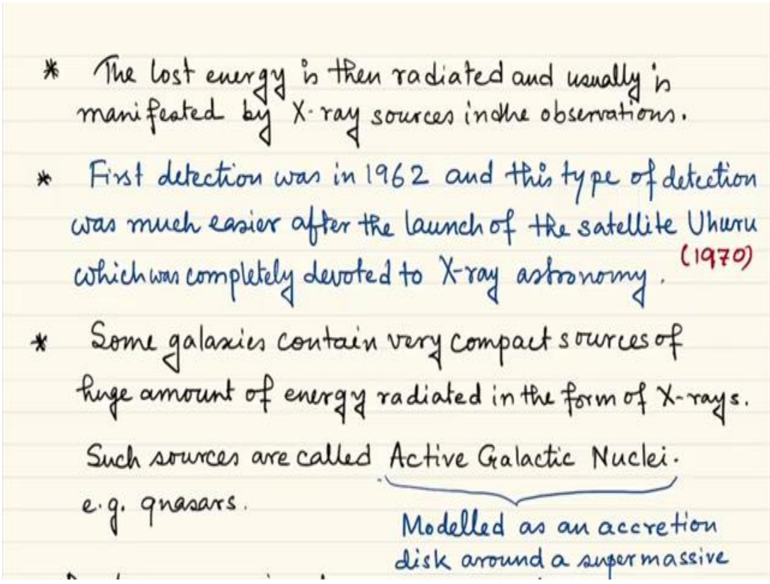

The lost energy is then radiated and usually is manifested by X-ray sources in the astronomical observations. Now, historical information one is that, the first detection was made in 1962 and this type of detection was much easier later when a satellite Uhuru in the year 1970 eight years after the first detection was launched and which was completely dedicated to X-ray astronomy.

Now, it is a question that, why we should study accretion disks in astrophysics? Why basically the accretion disks are at all important or at all interesting? Because some galaxies contain very compact sources of huge amount of energy radiated in the form of X-rays. Now, such sources are called active galactic nuclei and a very good example of that is the quasars.

You can search over internet to know what quasars are. So, what is active galactic nuclei? Active galactic nuclei are nothing but a very localized part of a galaxy which produces an enormous amount of energy, and the wavelength is almost always in the X-ray domain.

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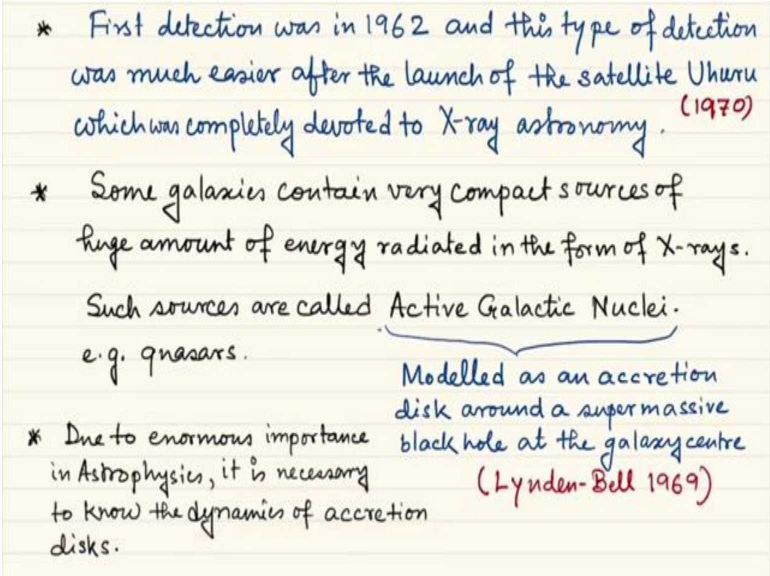

So, active galactic nuclei that is actually modeled as an accretion disk around a super massive black hole at the galaxy center. So, you understand what is the meaning of that, just I said that active galactic nuclei is a very local, I mean is a very compact space or a very local part with very small volume inside a galaxy let us say, we have a spiral galaxy like this and inside this galaxy near the center. Let us say it has a very short volume from where you can find an enormous ejection of energy or radiation. Let us say X-ray radiation.

Then people actually could study its property by modeling this as an accretion disk around the super massive black hole, that means, here this type of active galactic nuclei can be modeled when this compact star is nothing but a black hole itself. Which is situated at the center of the galaxy that is the famous Lynden Bell model of 1969. If you are interested, you can see the original work.

Of course, this is in the scope of this course, I encourage all of you to study different original research papers, to see that what these people have done, and that is also a very good occasion to cross check that what I have discussed here is really exactly the same of what is inside the paper.

So, due to enormous importance in astrophysics, for example, in modeling this type of active galactic nuclei, it is highly necessary to know the dynamics of accretion disks, that was the importance.

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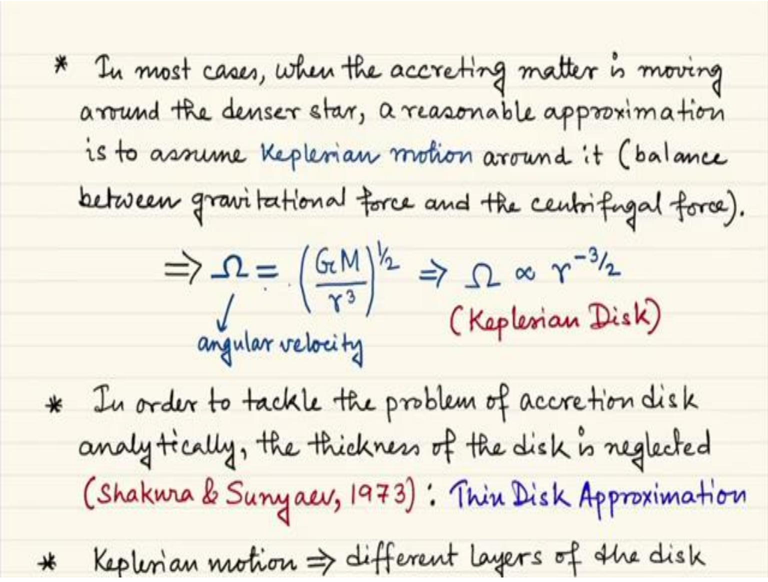

Now, before coming to the other points, one thing is very simple that in most cases, when the accreting matter is moving around the dense star a reasonable approximation is to assume that any mass element which is now getting accreted in a spiral path should follow a Keplerian motion around the dense star.

What is the meaning of Keplerian motion? The meaning of Keplerian motion is like there is a motion of a planet around the sun that means the balance. The force balance of this body will be just due to the interplay between the gravitational force, gravitational attraction force between that mass element and the dense star or the center of attraction here and which will be basically counteracted by the centrifugal force which will act radially outward.

So, gravitational force will be attractive, so it will be radially inward. So, if you follow this Keplerian type of motion then one thing is evident that due to Kepler's third law, the angular velocity or you can directly see from the force balance actually, both are equivalent, that angular velocity should have a dependence on the radial distance from the center of the attraction like this. So, that means Ω should be equal to some GM.

So, G is the universal constant, I mean gravitational constant and M is the mass of the center of attraction and this r is the instantaneous distance between the mass element and the center of the attraction. So, Ω should be should be written like this $(\frac{GM}{r^3})^{1/2}$, this is the angular velocity of the mass element. So, Ω has this $r^{-3/2}$ type of radial dependence, and this type of disk is known as Keplerian disk.

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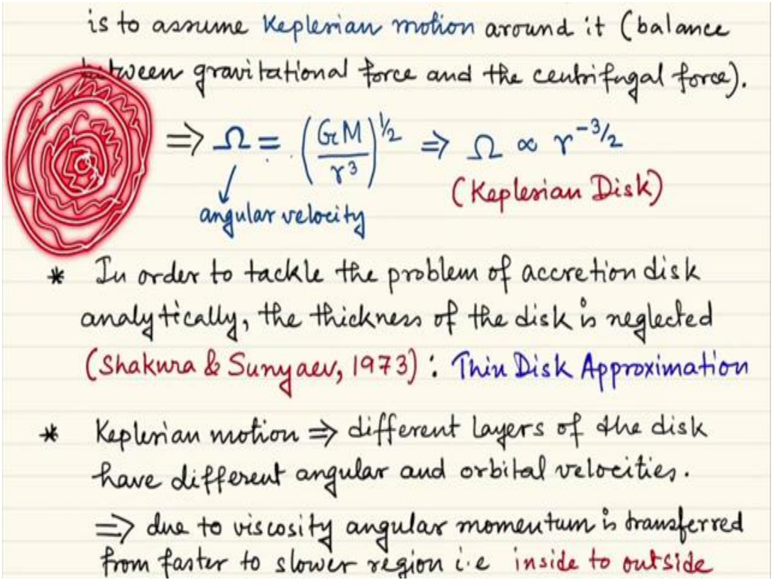

Now, in order to tackle the problem of accretion disk analytically, people try to do this, try to make this much more simpler using some reasonable approximation. So, one approximation was very handy, that was the approximation of a thin disk that is the thickness of the disk is neglected for considering the dynamics of the disk. That does not say that the disk is purely two dimensional but it simply says that the dynamics of the disk that means the mass motion can sufficiently be started in one layer.

If you know what is happening at one layer, this is sufficient. So, that is one possibility and then another point is, of course, if the disk is really thin, then, of course, the thickness is absolutely neglected, both structurally and also dynamically, and then trivially you can have only a 2D type of sheet type of thing and you can see that this type of thin disk approximation is very much valid.

This is useful specially, because this makes the mathematical approach a lot easier. That was proposed by two scientists Shakura and Sunyaev in the year 1973, actually you can see their paper as well. Now, Keplerian motion implies that, different layers of the disk have different angular and orbital velocities because angular velocities are already dependent on the radius.

So, it is you to think, how angular velocity has this type of dependence, then how the orbital velocity which is v and that should have a radial dependence. You will see that will also have a radial dependence such that, this will increase when r decreases. So, both the orbital and the angular velocities they are higher for inward orbits and they are lower for outward orbits.

Now, this is the very important point. Now, if we consider that the fluid is viscous then what happens? As the disk is made up of different layers having different angular velocity and different linear velocity, of course, as the same thing over here, I mean, this is somehow equivalent then what happens that they should have a relative orbital velocity and relative angular velocity as well.

And whenever they have a velocity gradient, they have viscosity, viscosity will try to reduce that gradient by exchanging either angular momentum or linear momentum. So, in order to reduce the linear velocity gradient, linear momentum is transported from the faster moving layers to the lower slower moving layers.

If there is a gradient to reduce the gradient of the angular velocities, angular momentum is transferred from faster to slower moving region. And what is the meaning of that? That means, from inside to outside, right.

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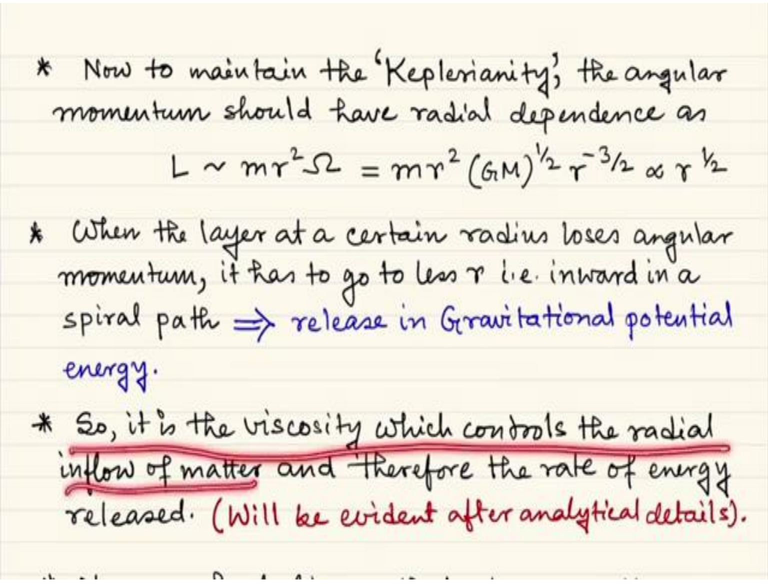

Now, the angular momentum is transferred from inside to the outside then the layer inside would lose angular momentum. Now, to maintain the Keplerianity, I mean that is very important to maintain because otherwise the force balance would be destroyed, the angular momentum should have radial dependence.

So, both Ω and v, they have such radial dependence, so that they are increasing when r is decreasing and vice versa but if we are considering about L, which is nothing but $mr^2\Omega$ roughly, then you can actually see L has a linear dependence of $\sqrt{r}$. So, that means when r increases, L is also increases.

So, in general, let us say some fluid layer in a given radius loses its angular momentum that means it has now a lesser angular momentum, so, with this angular momentum, this mass element will be eligible to be in an inner layer corresponding to another r which is less. Then the layer at a certain radius, as I just said it loses angular momentum, it has to go to less r, that is inward.

But how it goes inward? It goes in a spiral path. So, when it goes inwards in a spiral path what is the meaning of that? Spiral path is, of course, necessary because otherwise the Keplerianity it will not be there. The mass element is going closer to the central body. What is the outcome? There will be a release in gravitational potential energy.

Now, you see that, it is only the viscosity which controls the radial inflow of the matter. So, let say if the fluid does not have any viscosity, so then even if there is a velocity gradient or angular velocity gradient in two successive layers, there is no necessity for the system to transfer angular momentum from one layer to the other. It is only due to viscosity this is possible.

So, in the absence of viscosity, this mechanism would have been absent. So, it is the viscosity which actually controls the radial inflow of matter and therefore, it is controlling the rate of the energy released because of the whole mechanism. This will be much more evident when we talk about the analytical results.

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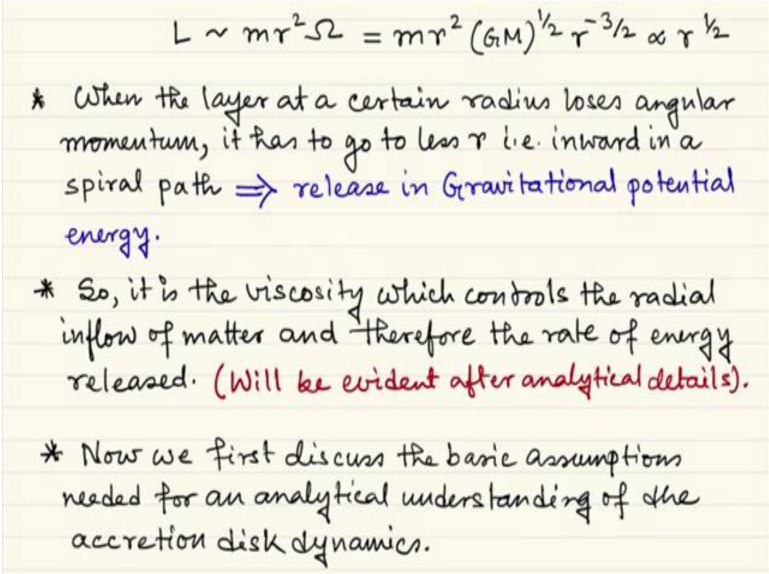

Now, finally, we will go to analytics. We have said that, we will do analytics after doing some qualitative discussion. So, now the qualitative discussion is over. So, we have somehow a rough idea, rough picture that how the accretion disks are formed and how due to the formation of the disk, some amount of gravitational potential energy is released.

We will now switch to do some analytical treatment and for that we have to write equations. But even before that, we have to talk about the basic assumptions. So, now, we first discuss the basic assumptions needed for an analytical understanding of the accretion disk dynamics.

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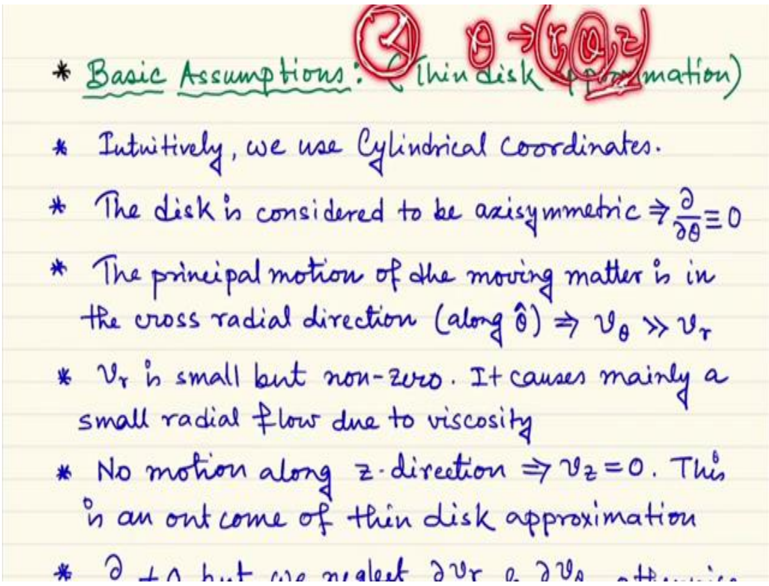

So, here there are a list of assumptions and all of them are equally important. First assumption is that, this is a disk, so this is not a sphere. So, intuitively this disk is axisymmetric. Now, which coordinate system should be the most appropriate one?

Of course, neither the Cartesian nor the spherical one, so it should be the cylindrical coordinate system. Now, the disk is considered to be axisymmetric that is somehow, sometimes it is coming from the disk itself.

So, in this plane, if you just fix your radius from the center then everything will be same. So, the coordinates of this cylindrical coordinate system are r, θ and z.

So, when we say about axisymmetric, it says that it does not depend on θ, no properties will depend on theta. So, in the plane of the disk everything can only change as a function of the radial distance from the center.

And that is why $\frac{\partial}{\partial\theta}$ of anything is 0. The third one is the principal motion of the moving matter is in the cross radial direction that is along $\hat{\theta}$. And what is the meaning of that? That means, that the v_θ is very greater than v_r and of course, obvious v_z as well.

Because we have not said anything about v_z, I am just saying v_θ is the dominating component. v_r is small, but non zero, that is also important because v_r is necessary to produce the small radial inflow of the matter, it causes mainly a small radial flow due to viscosity. So, v_r should be there, and no motion along z direction.

So, that is somehow when an immediate outcome of a thin disk approximation. So, if the disk is very, very thin then along z direction, no motion should be expected, no considerable motion. So, just a very good approximation, reasonable approximation is that v_z is equal to 0. So, v_θ and v_r is the smallest one but both of them are nonzero.

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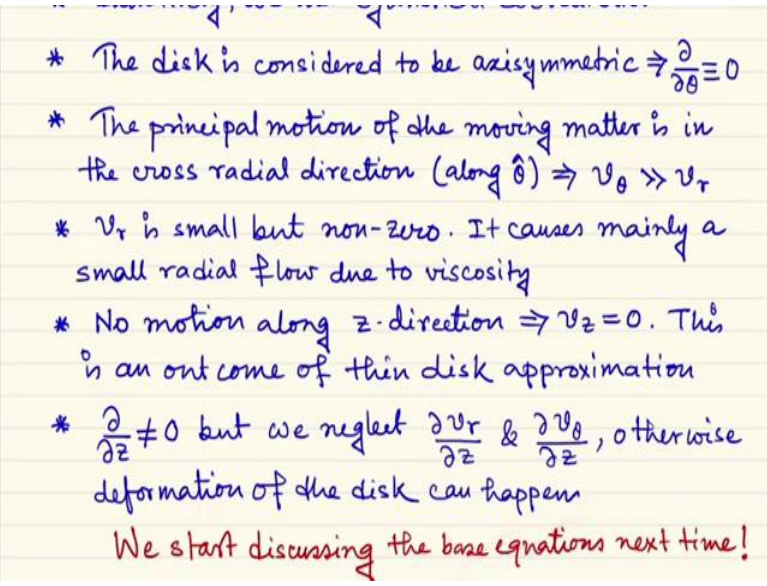

* The disk is considered to be axisymmetric $\Rightarrow \frac{\partial}{\partial \theta} \equiv 0$
* The principal motion of the moving matter is in the cross radial direction (along $\hat{\theta}$) $\Rightarrow v_\theta \gg v_r$
* v_r is small but non-zero. It causes mainly a small radial flow due to viscosity
* No motion along z-direction $\Rightarrow v_z = 0$. This is an out come of thin disk approximation
* $\frac{\partial}{\partial z} \neq 0$ but we neglect $\frac{\partial v_r}{\partial z}$ & $\frac{\partial v_\theta}{\partial z}$, otherwise deformation of the disk can happen

We start discussing the base equations next time!

Finally, $\frac{\partial}{\partial z}$ is non-zero although in the z direction there is no flow, no motion but there can be gradients. But we neglect somehow $\frac{\partial v_r}{\partial z}$ and $\frac{\partial v_\theta}{\partial z}$ because first of all both of them are small and if they are non-zero, you can actually think that there would be deformation of the disk and that is not really welcome in a thin disk approximation in general.

So, with all these approximations, finally we are all set for discussing the base equations of the accretion disk, and that we will do in the next discussion.

Thank you very much.