Introduction to Astrophysical Fluids

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Lecture – 20
Stellar/solar wind

Hi. In the last discussion, we basically showed that the hydrostatic model of the solar corona or more generally the corona of any star cannot be valid only because it gives us a pressure which is non-zero and considerable at infinity.

Now, it basically led to think the people that the corona is not hydrostatically stable. That means, that corona is dynamic and it is flowing, and after that various spacecrafts measured in measurements.

Finally, nowadays we understand that the coronal plasma or the coronal fluid is basically expanding in the radially outward direction from the sun, almost in an isotropic way.

There is a small anisotropy due to magnetic field that is true, but almost in an isotropic way, and this is known as the so called stellar wind, and for sun we talk about solar wind correspondingly. So, in this lecture, we will discuss this stellar wind or solar wind.

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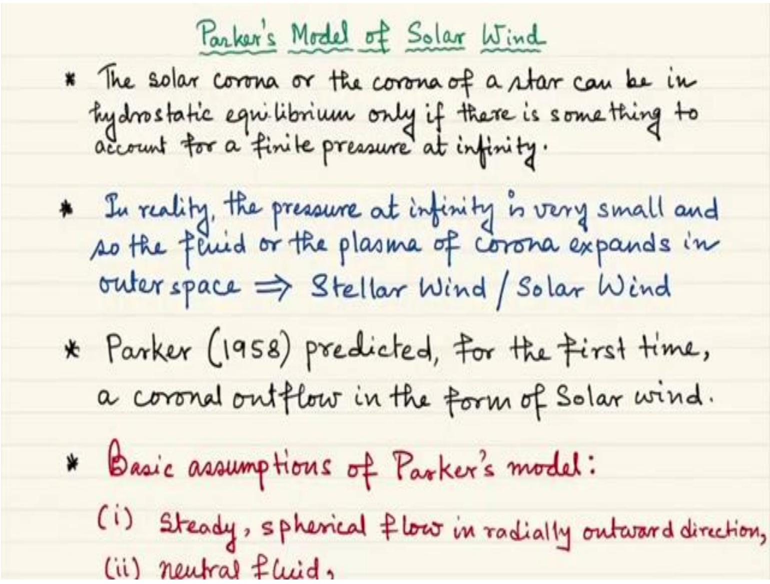

So, now Parker was the first one who concluded that the solar corona or the corona of any star cannot be kept in hydrostatic equilibrium only by the interplay of gravity and hydrostatic pressure. Or in an alternative way, one can say that the corona of a star can be in hydrostatic equilibrium only if there is something to account for a finite pressure at infinity.

And we all know that in reality the pressure at infinity should be very small unless there is some agent to account for the pressure. So, infinity means that there is no mass, there is no plasma, I mean ideally. Of course, if there is a source which is a distant source that is also sometimes possible that we will come at the very end of this discussion then it is possible.

Now, in general the pressure at infinity is quite small and actually it is very near to 0, and then the fluid or the plasma of the corona expands in outer space and this is known as the stellar wind or solar wind. So, in the year 1958, Parker predicted for the first time this coronal outflow in the form of solar wind.

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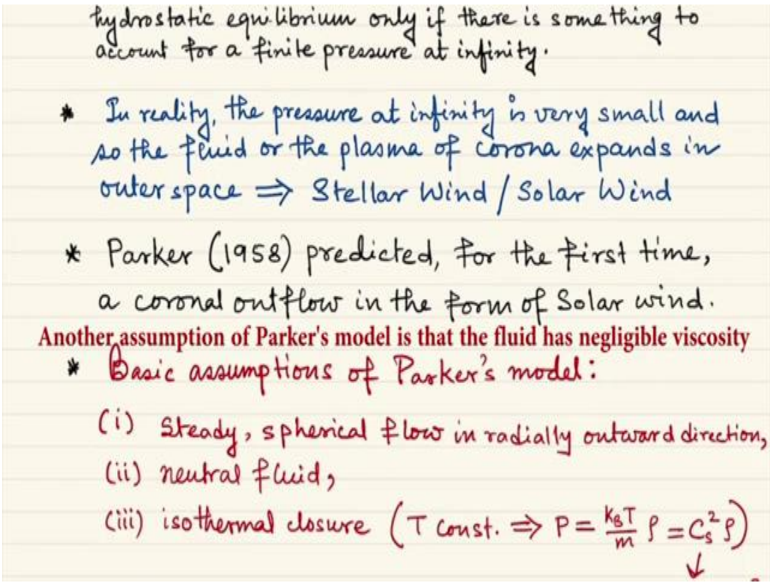

But the model with which he started his work was a very simplified model and with respect to today’s consideration the model has one or two things which are not completely correct. But even with this very simplified model Parker reached to a number of important conclusions that we will discuss.

So, the basic assumptions of Parker's model were of course, a steady spherical flow in radially outward direction of the coronal plasma. Although, he realized very well that this is the plasma, this is an ionized charged fluid with charged particles or ionized particles.

So, ions, electrons will be there mostly, but as a first approximation he assumed his model to be constituted of neutral fluids, and that is why we are talking in this context, otherwise I would have talked about this after introducing plasma and the plasma fluids.

And finally, he said that we do not have to think about the energy equation because we will assume a simplified isothermal closure for the solar wind, and this is also not a very good approximation for the solar wind because most of the cases the temperature is not constant. So, some people do it with polytropic closure, that is somehow much closer to reality than isothermal one, but he did this just to start off.

So, polytropic closure means T is constant. So, P is nothing, but $\frac{k_B T}{m}\rho$. So, this is another way of writing. So, $\frac{\rho}{m}$ is n, so $nk_B T$. So, P is equal to $nk_B T$, and if you write in this way then $\frac{k_B T}{m}$ all are constants for an isothermal case, so you can say this is the constant times ρ. So, pressure should be proportional to density in an isothermal fluid.

And the proportionality constant you can easily remember what is this, this is nothing but $\frac{k_B T}{m}$ which is equal to the sound speed square. So, already people knew about the sound speed in a medium. So, we all know that for a medium, I will also come into the detailed derivation of sound wave for compressible fluids, but much before that people used to know that sound wave is produced by the pressure and density variation of a compressible fluid.

The $\frac{dp}{d\rho}$ gives the sound wave speed. Mostly this gives the square of the sound wave speed to be precise, and for an isothermal case $\frac{dp}{d\rho}$ is nothing but $\frac{p}{\rho}$ and which is equal to C_s^2, but only for isothermal case your sound speed is a constant. For non-isothermal polytrophic fluid this is no longer true.

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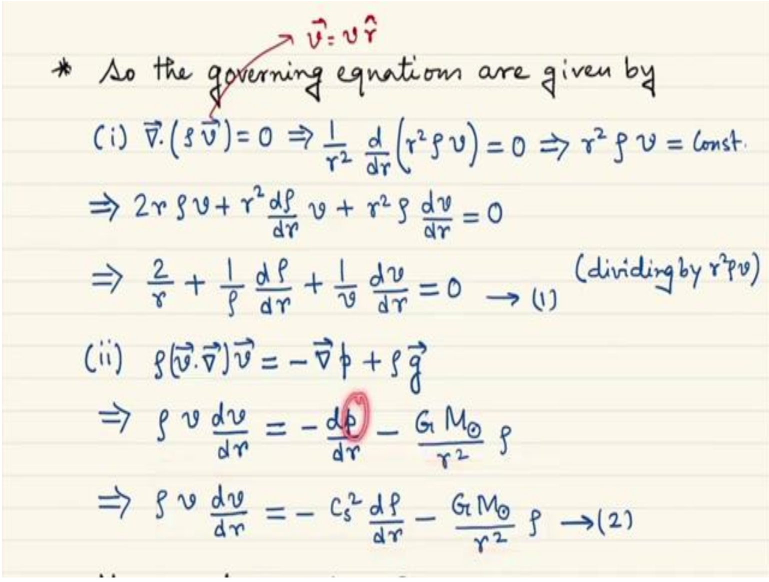

The governing equations, we can now write in this way. So, first of all we said this is steady flow, so that means, all the $\frac{\partial}{\partial t}$ will go away, and this is the residual part of the continuity equation $\vec{\nabla}.(\rho\vec{v})$ is equal to 0. Now, when we are talking about the spherical symmetry case with only radially outward flow, then this will be I mean the divergence will be written in a spherically symmetric way. And the only non-vanishing velocity will be the radial part of the velocity.

So, let us say here basically when we write this divergence term, we assume that $\vec{v}$ is nothing but $v\hat{r}$. If you do that, finally, you can show that $r^2\rho v$ is a constant of the flow.

Now, these things are the function of r only. So, ∂ or d they are both the same. So, now, you differentiate both sides with respect to r, you will have $2r\rho v + r^2\frac{d\rho}{dr}v + r^2\rho\frac{dv}{dr}$ that is equal to 0.

And then again dividing the whole thing by $r^2\rho v$, of course, here we should assume that $r^2\rho v$ is never equal to 0 then you can say that this is the final equation comes to be $\frac{2}{r} + \frac{1}{\rho}\frac{d\rho}{dr} + \frac{1}{v}\frac{dv}{dr}$ is equal to 0. Now, this is the residual part of the momentum equation for spherically symmetric case, I am sorry steady flow.

So, $\frac{\partial v}{\partial t}$, $\frac{\partial \rho v}{\partial t}$ is going away or rather it should be $\rho\frac{\partial v}{\partial t}$ which is now 0 because $\frac{\partial}{\partial t}$ is 0 and this part we have to write in spherical coordinates. So, this will simply be $\rho v\frac{dv}{dr}$, because only the radial component of velocity survives. Gradient of p is $\frac{dp}{dr}$, so minus $\frac{dp}{dr}$ and what about this one? So, this one is simply minus $\frac{GM_{solar}}{r^2}$.

Again, we are assuming that the coronal plasma is inside the gravitational field of the sun. I mean excluding the mass of the solar plasma if you want, in any case the addition or subtraction of the coronal plasma does not change much to the solar mass.

So, that is something to know that if you search the rough ratio of the coronal plasma mass in general and the total solar mass then it will be clear. Then finally, you can just substitute p by $C_s^2\rho$. There is no other change other than this substituting p by $C_s^2\rho$ and C_s is a constant of the system.

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(i) $\vec{\nabla}\cdot(\rho\vec{v})=0 \Rightarrow \frac{1}{r^2}\frac{d}{dr}\left(r^2\rho v\right)=0 \Rightarrow r^2\rho v = Const.$

$\Rightarrow 2r\rho v + r^2\frac{d\rho}{dr}v + r^2\rho\frac{dv}{dr} = 0$

$\Rightarrow \frac{2}{r}+\frac{1}{\rho}\frac{d\rho}{dr}+\frac{1}{v}\frac{dv}{dr}=0 \rightarrow (1)$ (dividing by $r^2\rho v$)

(ii) $\rho(\vec{v}\cdot\vec{\nabla})\vec{v} = -\vec{\nabla}p + \rho\vec{g}$

$\Rightarrow \rho v\frac{dv}{dr} = -\frac{dp}{dr} - \frac{GM_\odot}{r^2}\rho$

$\Rightarrow \rho v\frac{dv}{dr} = -C_s^2\frac{d\rho}{dr} - \frac{GM_\odot}{r^2}\rho \rightarrow (2)$

* Now we eliminate $\frac{1}{\rho}\frac{d\rho}{dr}$ from (1) and (2) and get

$$\frac{2C_s^2}{r}+\frac{C_s^2}{v}\frac{dv}{dr} = v\frac{dv}{dr} + \frac{GM_\odot}{r^2}$$

So, we can simply write this equation number 2. Now, we can actually divide the whole thing of equation 2 by $C_s^2\rho$.

So, check at home you should obtain this $\frac{2C_s^2}{r}+\frac{C_s^2}{v}\frac{dv}{dr} = v\frac{dv}{dr}+\frac{GM_{solar}}{r^2}$, which after simplification comes out to be $\left(v-\frac{C_s^2}{v}\right)\frac{dv}{dr} = \frac{2C_s^2}{r} - \frac{GM_{solar}}{r^2}$. So, these two terms.

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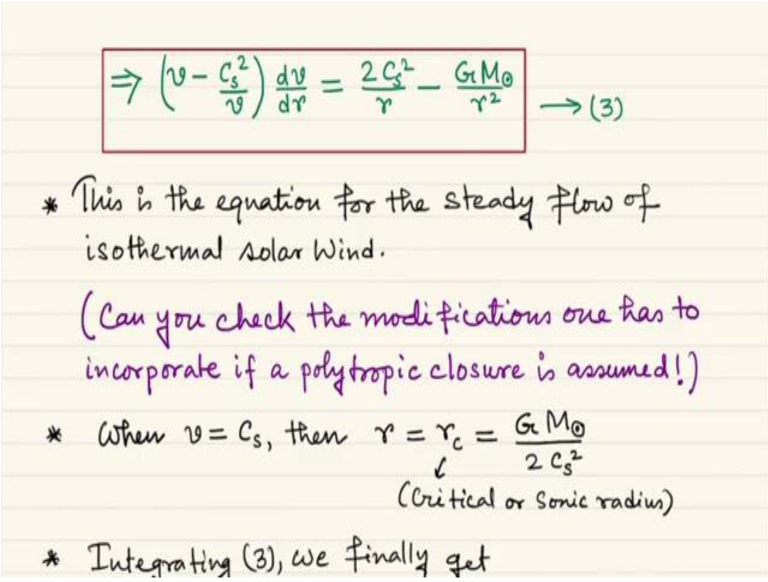

So, this $\left(v - \frac{C_s^2}{v}\right)\frac{dv}{dr}$ is the term related to the velocity gradient. Now, this is the Parker's equation for the isothermal solar winds considering this as a steady flow. This is the flow equation. So, this is the steady flow but velocity is non-zero, so velocity should have a gradient with respect to the radial coordinates, right.

Now, it is a very good exercise, you can do at home yourself if you are blocked then I can help or you can search over internet for the other literatures that what will be the modified version of this equation if instead of an isothermal fluid you take a polytropic fluid. Remember at that point C_s^2 is no longer a constant of the system. So, do that. I am not saying that you must get a different form, maybe you get the similar form or a different form just do it yourself and let me know.

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* This is the equation for the steady flow of isothermal solar Wind.

(Can you check the modifications one has to incorporate if a polytropic closure is assumed!)

* When $v = C_s$, then $r = r_c = \frac{GM_\odot}{2C_s^2}$

(Critical or Sonic radius)

* Integrating (3), we finally get

$\left(\frac{v}{C_s} = \text{Mach no.}\right)$ $$\boxed{\left(\frac{v}{C_s}\right)^2 - \ln\left(\frac{v}{C_s}\right)^2 = 4\ln\left(\frac{r}{r_c}\right) + \frac{2GM_\odot}{C_s^2 r} + C_0}$$

If you do not do the exercises then this will be difficult to follow these things. So, it is really recommended that you do the exercises small exercises which I suggest you during the lectures. Now, when you see in this total equation $\left(v - \frac{C_s^2}{v}\right)\frac{dv}{dr} = \frac{2C_s^2}{r} - \frac{GM_{solar}}{r^2}$ you can simply see that this term $\left(v - \frac{C_s^2}{v}\right)\frac{dv}{dr}$ is 0, when v is equal to C_s, then this term should also be 0 and that gives us a specific value of the radius which is called the r_c that is critical radius.

Sometimes people also say sonic radius because this is the radius at which the flow velocity is just equal to the isothermal sound speed, and that is equal to $\frac{GM_{solar}}{2C_s^2}$. So, what is the meaning of that?

So, Parker thought that the solar wind basically after getting originated from the solar corona when it expands, first it starts with a very small velocity then it achieves the sound speed, at this critical radius or sonic radius and after that it becomes supersonic.

Of course, when we talk about supersonic or subsonic all are with respect to the isothermal sound speed. But in general, whether this is really supersonic or subsonic you have to do with a proper polytropic closure. So, that is another thing. So, you can do that for fun.

Now, in this course, I will just discuss Parker's model which is based on isothermal closure. So, you can see that this $\frac{GM_{solar}}{2C_s^2}$ is the critical radius which changes the nature of the solar wind from subsonic to supersonic that was the Parker's initial conjecture somehow; that solar wind is getting energy somehow and then it accelerates and when it accelerates it gets in a further speed of it, of course, should increase.

So, what is really the reason of the acceleration. This is another question which maybe we should come when we will discuss about turbulence, but at this point just think that it is somehow very reasonable consideration that the solar wind basically accelerates after generation. So, this is much reasonable to see that the velocity first starts to be less than C_s then reaches C_s and then overshoots C_s.

Now finally, you have this $\left(v - \frac{C_s^2}{v}\right)\frac{dv}{dr} = \frac{2C_s^2}{r} - \frac{GM_{solar}}{r^2}$ relation but how to do that? Now, you have to just integrate v to see the solution of this whole problem. So, in the final equation, v will be a function of r and that is what we have to find. So, if you do that correctly, one bonus is C_s is your constant. You will see the solution is given like this, $(\frac{v}{C_s})^2 - ln\left(\frac{v}{C_s}\right)^2 = 4\ln\frac{r}{r_c} + \frac{2GM_{solar}}{C_s^2 r} + C_0$.

This $\frac{v}{C_s}$ is known as Mach number. So, that is a vocabulary I think I expect all of you know. So, if Mach number is greater than 1, the flow is supersonic; if it is equal to 1 this is called transonic or sonic; and if it is less than 1 then it is called subsonic.

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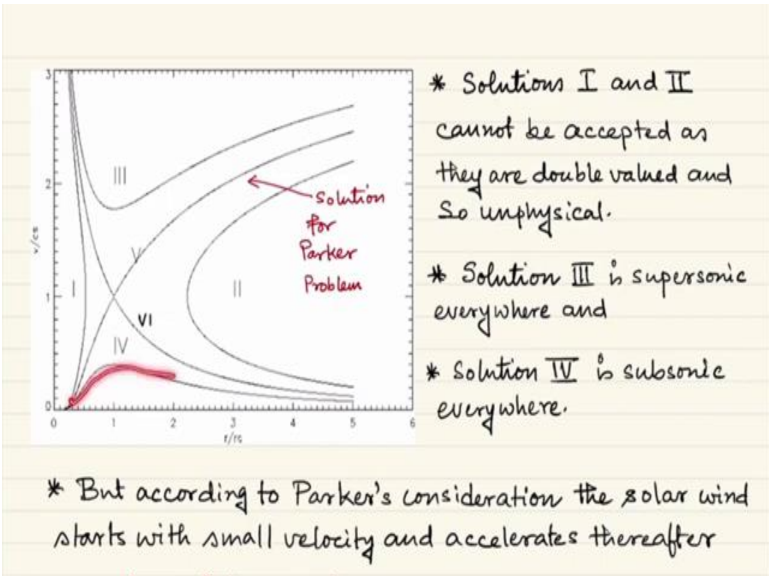

So, then you can simply plot the Mach number with this dimensionless number $\frac{r}{r_c}$, so Mach number is also a dimensionless number and you will see that this type of equation actually permits 6 type of solutions. So, these are all denoted by 1, 2, 3, 4, 5, 6, so, now which solution to take?

Now, solution 1 and 2, they have a problem. So, this is the solution number 1. If you see this 1 and 2, they are actually double valued; that means, for a given value of $\frac{r}{r_c}$ or that is given value of r, you can actually have two possible values of corresponding solar wind speed and that is not possible. This is unphysical, that is not possible. So, that is why these two are not accepted physically.

Now, solution 3 is this one which says that the solar wind is always supersonic because Mach number is greater than 1, so this is the sonic line. So, 4 says the solar wind is subsonic always. It is true that there are other considerations by which one can actually show that there are evidences.

One of course, from the spacecraft data that solar wind starts being subsonic that is just by calculating the initial coronal temperature and this type of thing. So, if it starts being subsonic then there are two possibilities, either it becomes supersonic or it retains subsonic, right. So, in any case 3 is not possible.

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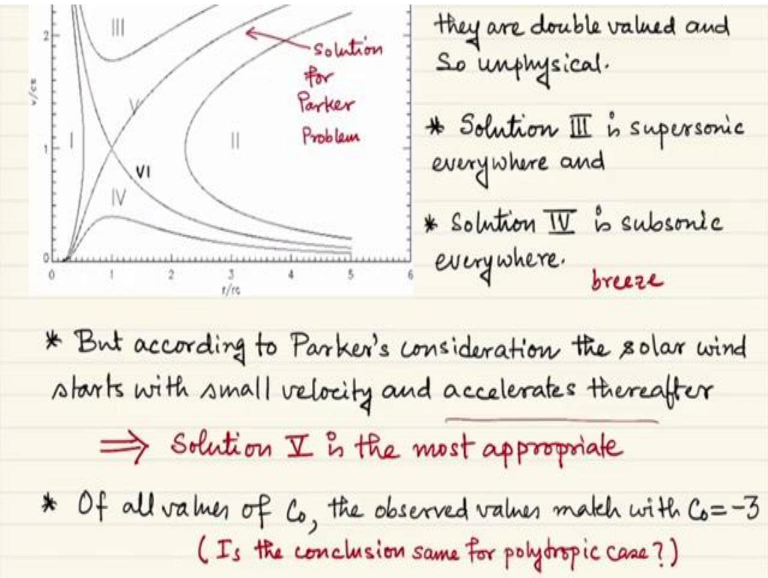

Now, 4 may or may not be possible, but once again then the Parker's consideration says that 4 is a valid solution for solar wind. So, this is called the breeze solution.

Now, Parker considered solar wind to start with small velocity and then accelerates thereafter and that is why finally, for Parker's problem solution 5 is the most appropriate because if you see 5 it starts being subsonic, and finally, gets supersonic and this is the sonic radius or critical radius and that is exactly; that means, r is equal to r_c that is $\frac{r}{r_c}$ is equal to 1.

And, why not 6? 6 is something where the wind decelerates actually. It starts being supersonic and it then becomes subsonic which is not also possible. Now, after that just fitting the observed values with this solution, one can actually show that the realistic values of the solar wind speed basically matches closely when C_0 or the constant of integration is nearly equal to minus 3.

So, this is also my question to you when you will do the polytropic case then just think and do, is the conclusion same for polytropic case; that means, also if there are these 6 types of solutions and you have also one type of solution which is perfect and the other not. Does the same story continue there or this is the different story? So, that is the research part. So, something I will do here, but you have to think a bit I mean beyond what is done in the lecture.

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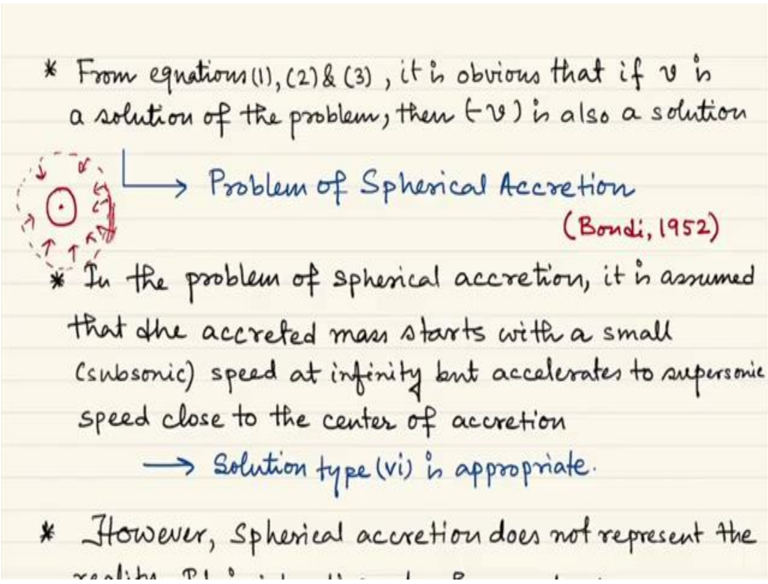

Now, finally, there is another thing, we can see that even with this simplistic neutral fluid spherical, steady, isothermal consideration, I mean Parker's model was powerful to at least give the nature of the solar wind in a very I mean, how to say as a first approximation. So, this was not bad at least the transition from super subsonic to supersonic becomes here.

So, I just check equation 3 $\left(v-\frac{C_s^2}{v}\right)\frac{dv}{dr}=\frac{2C_s^2}{r}-\frac{GM_{solar}}{r^2}$ that if v is a solution of the problem then minus v is also a solution, because in this term there is $v\ dv$. So, minus v will not affect here.

So that means, that if this is the equation for some plasma which is flowing outward that can also be the equation of a problem where some plasma or some material is coming spherically inward. This is nothing, but the popular problem of spherical accretion which was addressed by Bondi in the year 1952.

So, mass accretion in astrophysics is a very deep problem and at that time people started, so they found some observational evidences of mass accretion, but they did not know how to proceed analytically. Bondi started by very simplistic model of spherical accretion, and then finally, it was shown that this model of stellar wind the equation of stellar wind and the equation of spherical accretion, they are exactly same when we talk about the steady condition.

Of course, if the condition is not steady then they are not equal because $\frac{\partial v}{\partial t}$ term when it comes then v and minus v makes a difference. But for steady case spherical accretion problem and stellar wind problem is the same thing just for the opposite direction.

So, in the problem of spherical accretion, it is assumed that the accreted mass starts with a small subsonic speed at infinity. That means, at very large distance there is a source of some mass. For example, let us say some star comes in the neighborhood of another star.

So, when it starts to come, it was at the large distance and then it comes really closer. So, it simply says that some mass is there actually at a large distance which is theoretically at infinity, and that mass starts with a small subsonic speed.

So, if you see that the schematic figure is like that, let us say I have some mass source which is isotopically distributed around this star, and from this mass source mass is coming isotopically inside to accrete to this star.

Then with respect to this star at very large distance, if it starts from very large distance from the star, you can say that there is a source of mass and actually source of pressure at infinity. So, in that case, actually infinity pressure, pressure at infinity will be something considerably bigger and then it basically leads to an inward flow. So, again the flow will be from higher pressure to the lower pressure region, as we all know.

So, this is the problem of spherical accretion, of course, which starts with subsonic speed at infinity, but finally, accelerates to supersonic speed close to the center of the accretion.

So, now, you see it starts with subsonic speed at infinity and, so here for example, this one and then it accelerates to have supersonic speed at very close to r is equal to 0. That simply says that solution number 6 will be an appropriate solution for this Bondi's spherical accretion problem. This problem is also called Bondi's problem.

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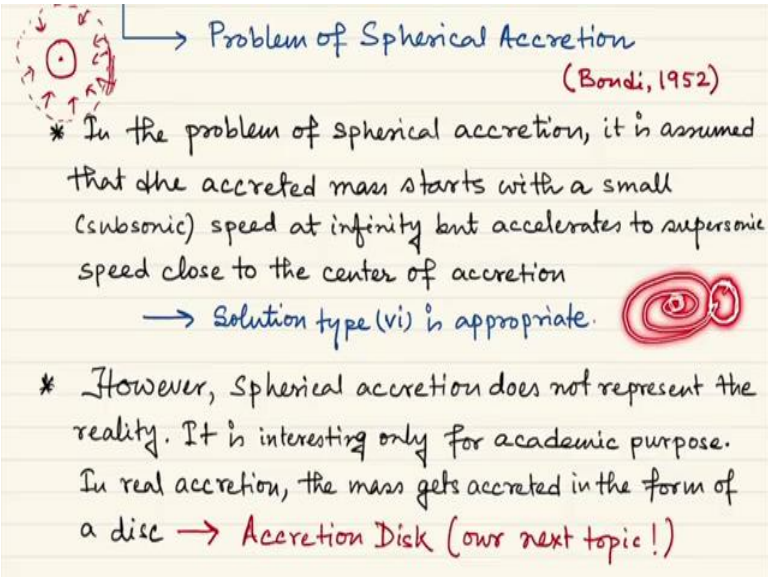

Now, however, as I said that this was just a first way out to address the accretion problem in astrophysics theoretically, but spherical accretion does not represent the reality. Actually it is interesting only for academic purpose. But in real accretion the mass gets accreted in the form of a disc, which basically becomes because you have a star and another star comes.

There is a star and another, so it is a very dense and small star is at the center and the less dense star less massive star, but the bigger with a bigger volume comes and then it starts to leave mass and this mass with an angular velocity actually starts accreting this mass in the form of a disc, and that is the famous accretion disc of astrophysics. That is exactly our next topic.

Thank you very much.