Introduction to Astrophysical Fluids
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Lecture - 10
Application of moment equations in collisionless systems

Hello, as promised, in this lecture I will tell you about a very interesting Application of the moment equations for collision less Boltzmann equation.

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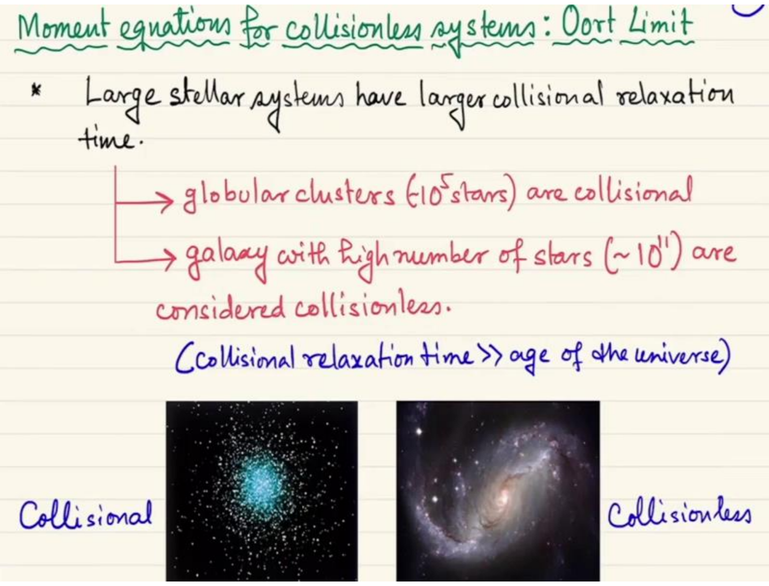

So, first of all let me tell you something about the large stellar systems. So, in general you know like when you are doing astrophysics, basically we have to know at least roughly about several stellar systems and their sizes. So, one is very popular is called the globular clusters.

Now, in these days of internet, you can search over internet and you can have a lot of knowledge about globular clusters. I am just giving you the pathway, please search and if you still do not understand please let me know, we will discuss. So, one is globular clusters which contain roughly 10^5 stars.

Another is very interesting which is called the galaxy. Galaxy we all know and it contains a high number of stars which is of the order of 10^{11}. In the scope of this course, I will not go into much details of that, but just as a thumb rule, I should say that large stellar systems have larger collisional relaxation time.

And what collisional relaxation time is? So that is also something you can search, but roughly saying if you have some system which has, let us say τ as the collisional relaxation time, then anything which is happening inside the system within this time τ will not have any effect of the collision.

If we study some phenomena during a reasonable time interval, let us say t which is less than τ, we can simply say that no clear effect or no prominent effect of collision is visible and for that case we say that the system is collision less and that is exactly the case for this galactic system.

So, the galaxy for example, a spiral galaxy with this 10^{11} number of stars some crude approximations, but somehow trustworthy approximations give that the collision relaxation time is much greater than the age of the universe. So, I have taken place of course, where for those systems, we can say that collisional effects are negligible. So, for these systems, we can somehow apply the collision less Boltzmann equation.

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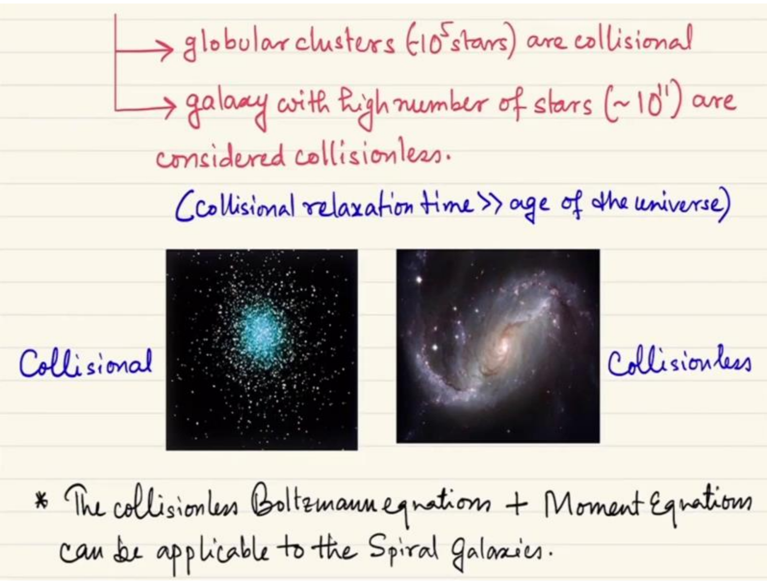

Now, it is true that the Boltzmann equation can be applied for microscopic systems, right. But when we are talking of the stars, then the question is can we apply that well? If the total system is much larger with respect to the individual stars or individual components or constituents basically, here stars or the constituents, then why not, they can be thought as the molecule of that large system.

That is exactly the case for both the clusters and the galaxies. The only thing is that the globular clusters, they are having relatively small number of stars. So, their τ or the collisional relaxation time is much smaller. For example, the galactic dynamics, some phenomena, they are happening during such a time scale which is of the order or even greater than that collisional relaxation time and that is why the collisional effects cannot be neglected.

But a priory, there is no restriction to apply Boltzmann equation for a stellar system. The only thing is that when you can, both collisional and collision less Boltzmann equations can be applied in principle. But the thing we have to keep in mind is that when we will try to apply collisional Boltzmann equation, then of course, we have to properly model the collision integral starting from our knowledge of the nature of interaction.

If the interactions are not very simple like binary elastic collision, and instantaneous collisions then it is a very hard task. For the stars, this is the case where we have a long range and known instantaneous gravitational interactions all the time.

That is why whereas, for collision less stellar systems, collision less Boltzmann equations can immediately be applied to derive the corresponding moment equations. In principle the collisional moment Boltzmann equation can be applied to derive an analytical expression for the collision integral and to check which things are conserved in that collision and this type of thing.

And from that deriving the corresponding moment equation in order to get rid of that collision integral, is not an easy task. However, there are very good books of galactic dynamics, you can search over internet and you can just read about that in depth. You can also go through the books for introductory discussion, which I mentioned in the reference books in the course handout.

Now, here, this is a typical picture of globular cluster, this is a picture of a spiral galaxy. So, the collision less Boltzmann equations plus the moment equations can be applicable to spiral galaxies, and that is true.

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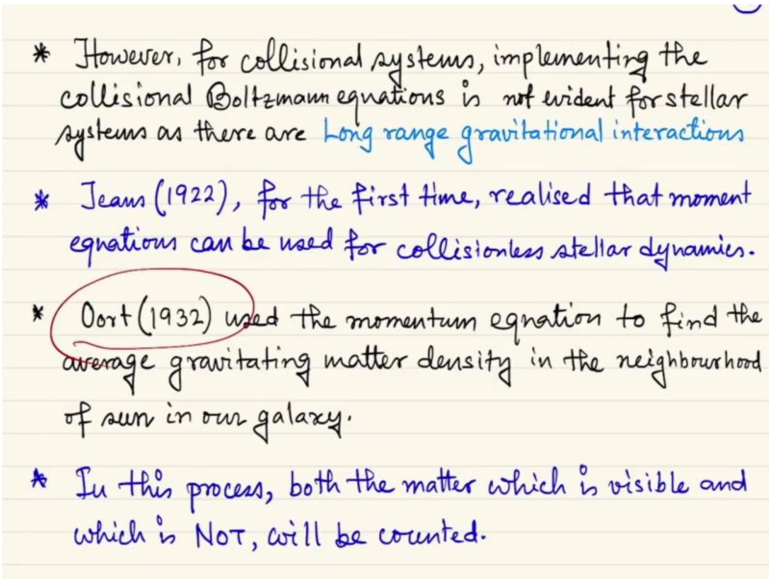

However, for collisional systems, implementing the collisional Boltzmann equation is not evident due to the long-range gravitational interactions. Jeans is the famous astrophysicist, the famous astronomer as well after whose name we have Jeans instability, Jeans mass, Jeans (Refer Time: 08:31) type of things.

For the first time, in the year 1922, he realized that the moment equation can be used for collision less stellar dynamics and for doing very interesting job. So, one of those jobs, I will discuss here. Now, Oort in the 1932, used the momentum equation to find the average gravitating matter density in the neighborhood of the sun in our galaxy milky-way.

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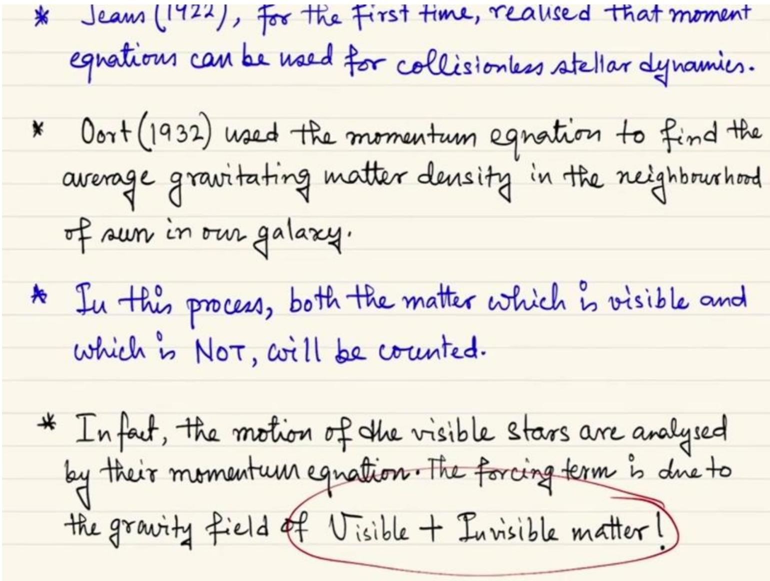

Now, in this process, both the matter which is visible and which is not, will be counted, ok. For example, some matter which is not radiating any light or any electromagnetic radiation, and whether you are using visible telescope or light telescope or visible ray telescope or X-ray telescope or UV telescope, you cannot detect them.

But it is true that when we estimate the matter density in general, by calculating the luminosity, we cannot detect them, but he proposed a method by which that can also be detected.

So, in fact, the motion of the visible stars is analyzed by their momentum equation. So, we can easily see the visible stars and we can study their momentum equation, there is no problem. The forcing term is due to the gravity field of visible plus invisible matter and that is the smart point. Try to understand, what it is said over here, that although we are seeing the dynamics of the visible stars, but what is the forcing?

Remember, there is a forcing term $\langle a \rangle$ and I said this is the body force, other than the pressure term. And I said that this is the body force and usually this is the gravitational force and the gravitational force comes due to all the matters, whether this is radiating or not. So, you understand? We are studying the dynamics of visible matters, but the visible matters they are forced due to all type of matters visible plus invisible.

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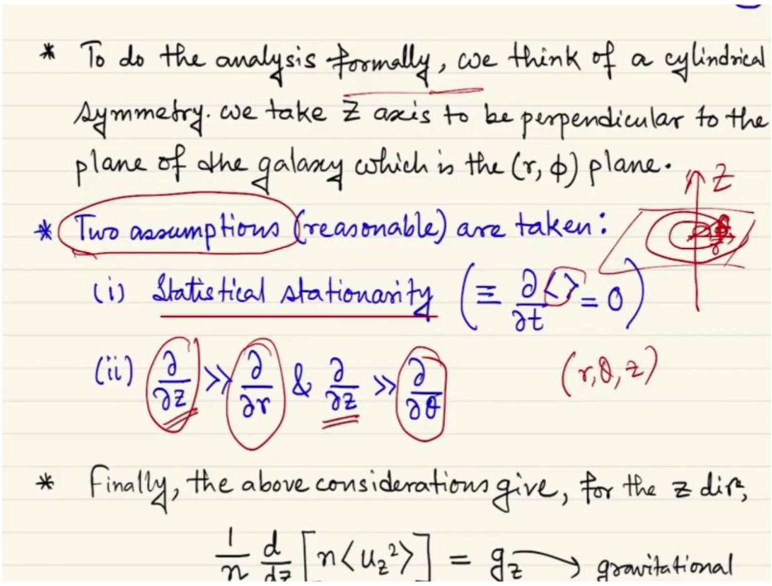

So, to do the analysis formally, we think of a cylindrical symmetry, and we take the z axis to be perpendicular to the plane of the galaxy which is the $r - \varphi$ plane for us.

And we also use following Jeans's assumptions, which is statistical stationarity. That means, $\frac{\partial}{\partial t}$ of any average quantity is 0, and $\frac{\partial}{\partial z}$ is very, very greater than $\frac{\partial}{\partial r}$, and $\frac{\partial}{\partial \varphi}$. So, basically the only special partial derivative survives is that $\frac{\partial}{\partial z}$.

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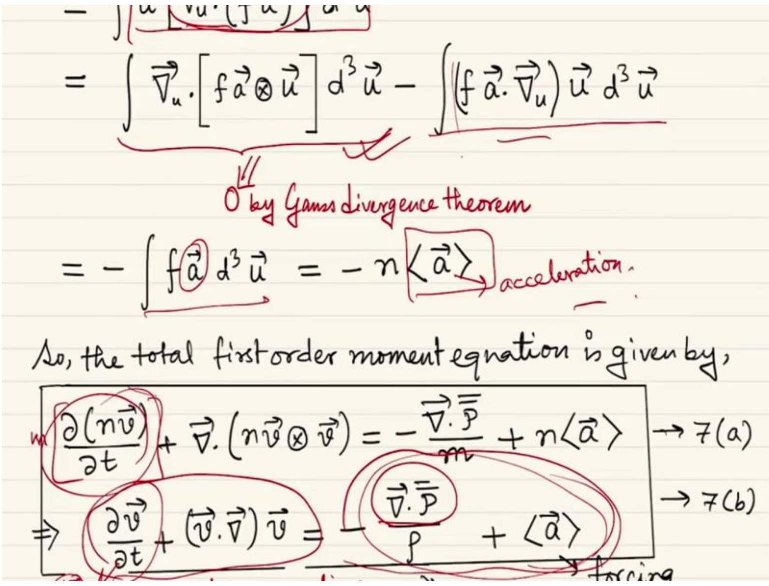

So, if you now go to the momentum equation, which I talked number of slides earlier (equation 7(a)).

From equation 7(a), you can just extract its z component and you will simply see this is nothing but $\frac{1}{n}\frac{d}{dz}[n < u_z^2 >]$ and it is equal to g_z and g_z is nothing but the z component of gravitational acceleration. This is the corresponding force. The only variations occurring along z direction is considered here, all the other variations are neglected in space.

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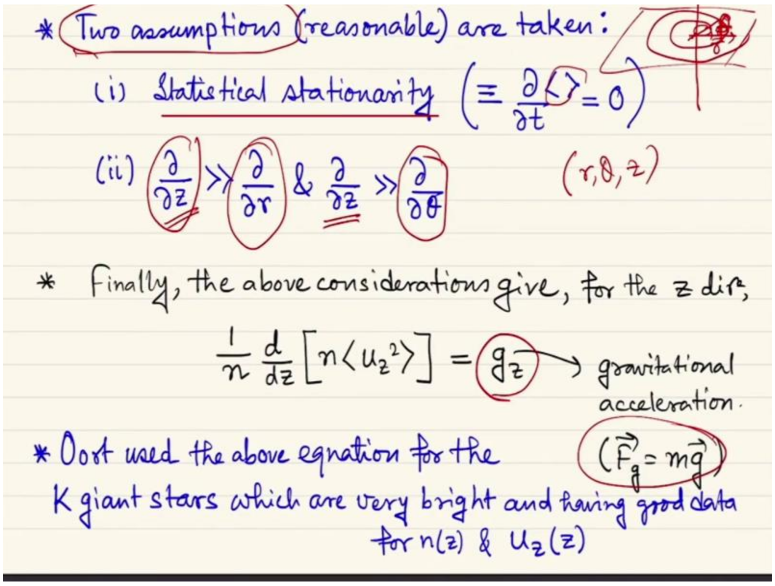

Now, Oort basically used this above equation, for a specific type of stars K giant stars which are very bright and easily visible. He had also another reason for that because at that time, when he did this experiment, astronomers had some good data for the z profile of the density and the u_z of those K giant stars. Once again, go to internet and search what the K giant stars.

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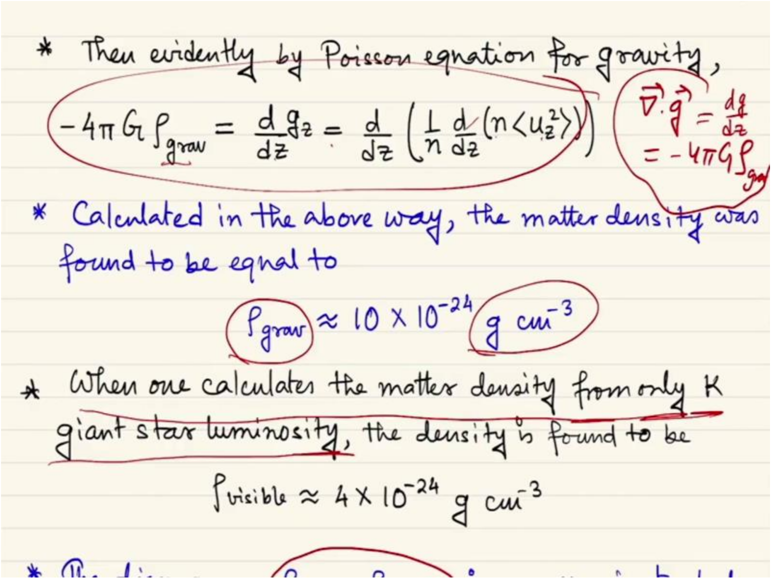

Actually, we will see they are the stars which can be seen from very far in the galactic plane. Now, although we are only concentrating on the dynamics of the K giant stars, basically the forcing is just a small population maybe of the whole mass. The forcing which they are experiencing that is due to the whole matter density, which is called as the gravitating matter. The gravitating matter is the matter, which is responsible for generating the gravitational force.

So, we will classify two type of matter, one is visible matter, and one is gravitating matter. The gravitating matter includes visible, as well as the invisible matter. So, finally, the poisson equation for gravity, which I expect that all of you know that is nothing but in vectorial form.

Once again, I can write it as $\vec{\nabla}.\vec{g} = -4\pi G\rho$ and this is the ρ of the total mass, which is responsible for creating the gravitational field. Here for our case, this divergence of g is nothing, but $\frac{dg}{dz}$.

So, $-4\pi G\rho_{grav}$ is equal to $\frac{dg_z}{dz}$. So, we write everything, we know the z dependence of n, from our data. They do it by calibration and we also know u_z as a function of z from our data and from that Oort did this.

Nowadays, we have much more sophisticated data of that, but at that time even for K giant stars, they had very good data. They have calculated, using Poisson equation of gravity,

ρ_{grav}. This is roughly $10 \times 10^{-24}\ g\ cm^{-3}$, and it is a very small mass. So, it is a very dilute matter density. You try to understand.

But when one calculates the matter density from only K giant star luminosity. Another homework for you because all the assignments are consisting of only MCQ questions, and that is not much interesting calculations or much innovative things to ask. So, this is my personal request to you that the things which I ask you to search for, during the lectures, please do that. That is for your thing, that is how you can really learn astrophysics.

So, this is actually astronomy and astrophysics board that how using the luminosity. Luminosity can be used to infer or conclude about the size of planet of a stellar system, not planet actually star. All these things are very interesting. Please learn about that.

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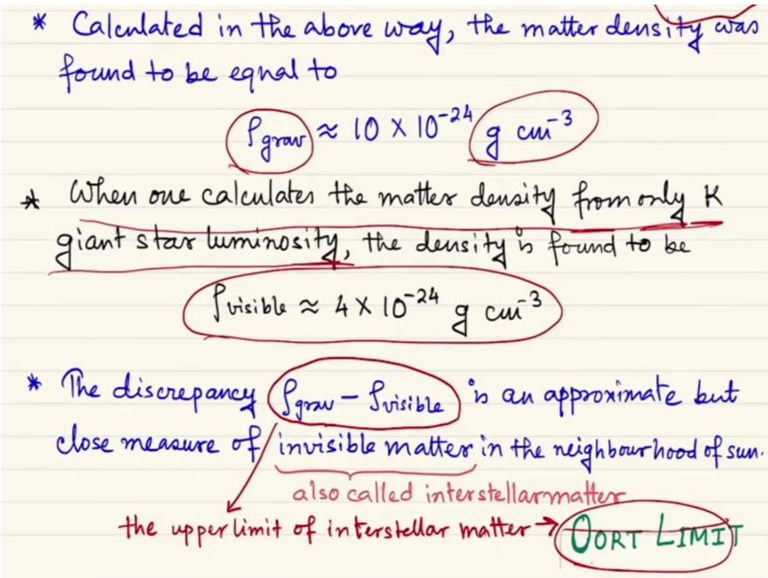

Now, if you do that then the visible density comes out to be $4 \times 10^{-24}\ g\ cm^{-3}$. Now, you can tell, sir, these both things are of the same order and even we do not know that whether they were calculated, without mistake or not.

No, actually it is true that they are very, very small, but actually you see that the one is the half of the other. So, this one is the half of the other matters, and 10^{-24} is not the matter. At this scale, by that time Oort did the experiment that data were so precise that these discrepancies was something which sought proper physics to be explained.

Now, you see these discrepancies, $\rho_{grav} - \rho_{visible}$ basically is an approximation, but very close measure of the invisible matter in the neighborhood of the sun. And this invisible matter in common parlance of astrophysics, we call that interstellar matter.

That means, mostly the visible matter is the stars themselves because they are radiating. But between two stellar bodies what is there, that we cannot see, and that is why we say that they are interstellar matter. Of course, the interstellar matters can also be stellar bodies but because they are not detected as stellar bodies, we are just calling them interstellar matters.

And by this way, you can see that we have some approximated measure of the amount of the invisible or interstellar matter. Now, it is true that here the discrepancy is of the same order, but the discrepancy can actually be much, much larger in the case of very, very bigger galaxies. When we are talking about the cosmological scales, then basically people are talking about the famous problem of dark matter.

Now, another point is that when you calculate ρ_{grav} by this way, actually you are somehow calculating or Oort calculated the upper possible limit for the gravitating mass.

So, if someone can come out, and say that I have another method, by which I found $100 \times 10^{-24}\ g\ cm^{-3}$ matter density. This is not possible. This is only possible if that type of thing is not a normal gravitating thing.

Basically, when we are talking about the matter must have gravitational power, and that is something very important. So, whenever you just learn about dark matter, you will see that dark matter cannot interact electromagnetically, but gravitationally interacting. So that is the only way of interaction of the dark matter.

So, only how you understand that they are the dark matters? So, their contribution to the production of the gravity field is only by their gravitation. So, you see this value, I mean the higher value of the matter which can include dark matter as well. So, I would not call at this moment dark matter, but just visible and invisible matter, ok. And this is known as the famous Oort limit.

So, in the year 1932, I mean to be very honest, the formal astrophysics was not that developed, well it was developed somehow, but not that much developed. In the year 1932, they did not have very much sophisticated observational tool and instruments, but you see that Oort came out with a revolutionary idea just using collisional, and collision less moment equations.

So, that was one of the very interesting applications of the moment equations of a collision less system. Now, from the next lecture, we will again concentrate on the moment equations of a collisional system and starting from its equilibrium distribution, which is nothing but the Maxwell Boltzmann distribution, we will derive a dynamical theory. That will be the theory of an ideal fluid or equation of an ideal fluid.

Thank you very much.