Introduction to Astrophysical Fluids
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Lecture - 01
General Introduction

Hello, welcome to the lecture number 1 of Introduction to Astrophysical Fluids, I am Supratik Banerjee from physics department of Indian Institute of Technology, Kanpur. So, as you know that of all branches of physics the branch which is like remembered with the most of the motion is the branch of astrophysics. Because every time we talk about astrophysics every time we can see in our mind that stars, planets, nebula, galaxy, all the exciting things.

Now, the question is that how to really know formally what is happening in terms of physics inside those things. Can we somehow structurize or schematize all the phenomena or at least some of the phenomena much more formally in terms of the physics or mathematics.

This course is mostly dedicated to the students who are really interested in the advanced physics or can pursue a career in the research of astrophysics or astronomy etc. Let me start by defining astrophysics in a much more formal way.

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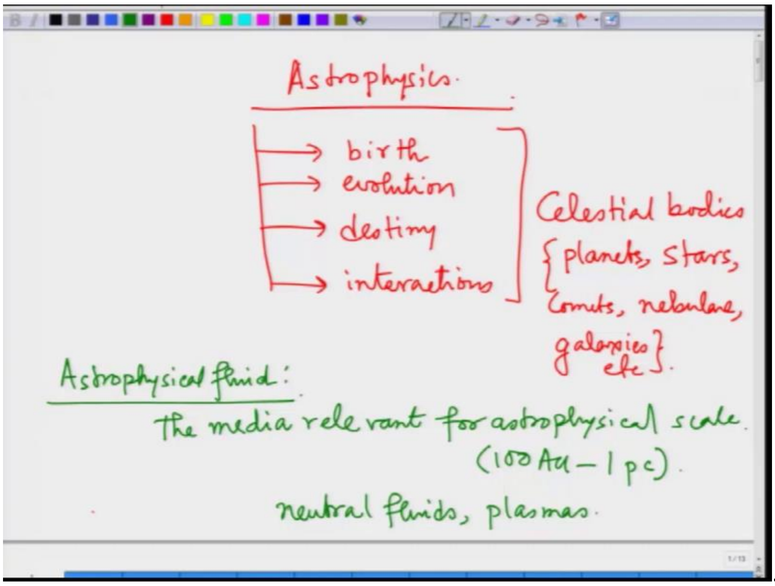

So, astrophysics is a branch of physics which is simply used to understand the birth, the evolution, the destiny and also the mutual interactions of celestial bodies. What are those celestial bodies? For example, you have planets, you have stars, you have comets, you have nebulas, you have galaxies and many others.

So, here basically when we will talk about celestial bodies, it is not really about a specific type of body or a specific type of mechanism which we will talking about, but rather we will be mostly interested in understanding a specific type of phenomena. And to understand how we can simply in terms of the basic laws of physics, predict that or we can understand what is going on even at a very simple level.

Now, in this course content basically not the evolution of the celestial objects will be discussed, on the other hand the astrophysical fluid aspects or rather the structural aspects of the astrophysical fluids or astrophysical media will be discussed.

Now, the question is that what is an astrophysical fluid? So, astrophysical fluid is roughly defined as the fluid media rather the media relevant for astrophysical scale. Now astrophysical scale simply means that a specific length scale which varies from 100 astronomical unit to the order of 1 parsec. In terms of kilometers as you know like 1 astronomical unit is of the order of 10^8 kilometers right and 1 parsec is 10^{13} kilometers.

So, when we are talking about 100 astronomical units this simply means that you will have 10^{10} kilometers. So, our astrophysical length scale will vary from roughly 10^{10} kilometers to 10^{13} kilometers. I assume that all of you know the basic definition of astronomical unit this is the average distance between earth and the sun, and parsec is almost roughly about 3.26 light year.

Now, I leave this as an exercise to you the students to search and learn what are the basic definition of parsec is, this is also very interesting. Now the question is that in this astrophysical scale which is varying between 100 astronomical unit to let us say 1 parsec. What are they? they can be simple neutral fluids they can be plasmas.

Now, in practice basically in most of the cases we can see the fluids are ionized gases. So, normal hydrodynamic fluid cannot be a suitable description for such fluids, so we have to learn plasmas. And plasma, what is plasma? Plasma is basically a gas or a fluid with charged constituents. Let us say ions, electrons a plasma can also contain neutral atoms or neutral molecules, but it is also containing essentially charged particles.

Now coming back to the question why astrophysics is so attractive to physics students or rather to students who are interested in science. Astrophysics is not a formalism-based science, rather astrophysics is developed in order to explain interesting phenomena which we observe every day, maybe not with our naked eye, but maybe with the help of telescopes or with the help of other direct and indirect observational methods.

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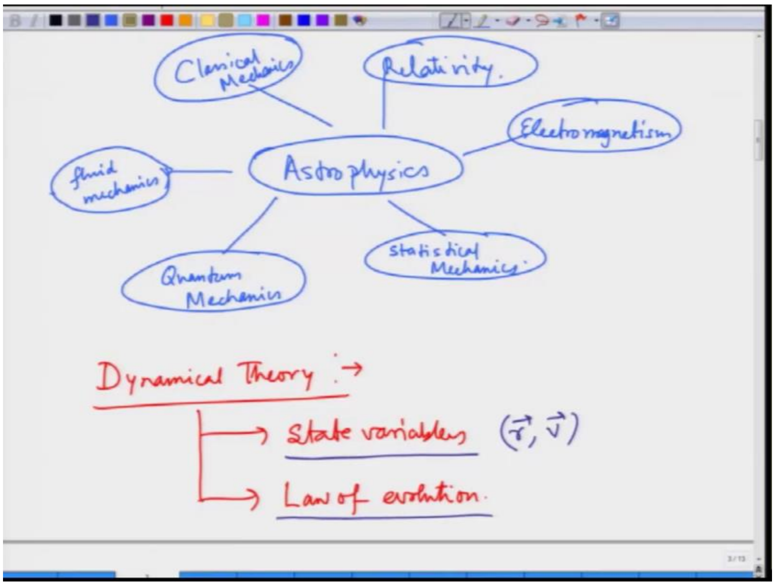

So, in a nutshell if we try to write where astrophysics really lies we can see that astrophysics is the confluence or a meeting point of all the 6 very interesting domains of physics, one is of course classical mechanics. Because we will be mainly be interested in the motions of the particles. So, we have to understand how the Newton’s law can be used there. So, classical mechanics is an essential tool.

Then fluid dynamics, so fluid dynamics is essentially based on Newtonian mechanics of course, but it has on it is own some different value. So, that is why I am writing it in a different case fluid mechanics and that is the base objective of this course.

Then comes quantum mechanics. Quantum mechanics will not really be covered in this course, however, just for your information in the compact objects in astrophysics let us say in a neutron star, in a white dwarf quantum effects cannot be avoided. For example, the degeneracy pressure of the electrons.

Then comes statistical mechanics, now the statistical mechanics is also included in the effect of this degeneracy pressure of the electrons which I just said. So, this was a combination of quantum mechanics and statistical mechanics. Here in this course roughly I will be using statistical mechanics at its base, but after once the fluid equations are derived, we will not be using statistical mechanics explicitly.

Now, the final two things are electromagnetism and relativity. Relativity is essential when the fluids velocity is very very high, close to the velocity of light, then we have to take the relativistic effects into account, like the relativistic jets. Or if you have a very massive object which can curve the space time around it then the general relativistic effects must be taken into account, but both these effects will also not be taken into account in this course.

What about the last one electromagnetism? This is a must because as I said that the most of the astrophysical fluid is a plasma and plasma is nothing but a charged fluid. So, we have to understand the physics of plasmas which is the superposition of fluid dynamics and electromagnetism.

So, that was the standing of astrophysics which is the confluence of these 6 domains and now we, out of all these things in the scope of the current course, what we will be trying to do to form a formal theory in scientific language, what we call a dynamical theory.

Now, dynamical theory simply says I hope all of you know basically what a dynamical theory is, but maybe this is the first time for some people you have learnt this word or come across this word. Dynamical theory is a formal theory to study the evolution of a system. Any system which is changing in time dynamical theory is the appropriate theory to study the corresponding evolution.

Now a dynamical theory should have two components; one is the state variables, another is the law of evolution. This is very much intuitive to understand that using state variables you can designate or you can label the systems state at an instant of time. Let us say for a classical particle you just designate the motion of the particle at an instant of time by its position and its velocity. But that is just the state at one point.

Now how to understand with time how it changes, that will be given by a set of either ordinary differential equation or a set of partial differential equation or in some cases it can be a combination of two. These are known as law of evolution. So, what is the law of evolution for classical mechanics? Let me ask you this question. All of you know this is nothing, but famous Newton's laws.

Now, think what can be the corresponding state variables and the law of evolution for the other very popular branches of physics, let us say quantum mechanics. So, the state function ψ, you all know may be that is the state variable and the Schrodinger equation for example, is the law of evolution.

For electrodynamics, you have electric field and magnetic field, they are the state variables and you have the laws of evolution as the equations of Maxwell right. So, previously we have discussed the different type of dynamical theories in physics like classical mechanics, electrodynamics, quantum mechanics and their corresponding state variables and the law of evolution right.

Now, here in this course the basic formal objective of us is to derive or develop a dynamical theory for the fluids. Now you can tell me sir fluids are nothing but the composition or this is composed of classical particles, it is true, but they are not the same thing. Why they are not the same thing? We will now be talking in detail that aspect how basically from the particle picture we should switch under certain restrictions to the fluid picture.

So, let us start with a very fundamental picture. So, you know like every continuum basically is made up of particles. Now you can say at first point they are classical particles, classical colliding particles, like a bowl of gas molecules which are colliding with themselves every time and even if you go further in, even in the smaller scale you can also then see the quantum effects right. So, then you will have some wave particle dual entities they are moving right, so this type of picture can be there.

Now how to get promoted from that very I mean very small scale quantum picture to a continuum picture that is the question.

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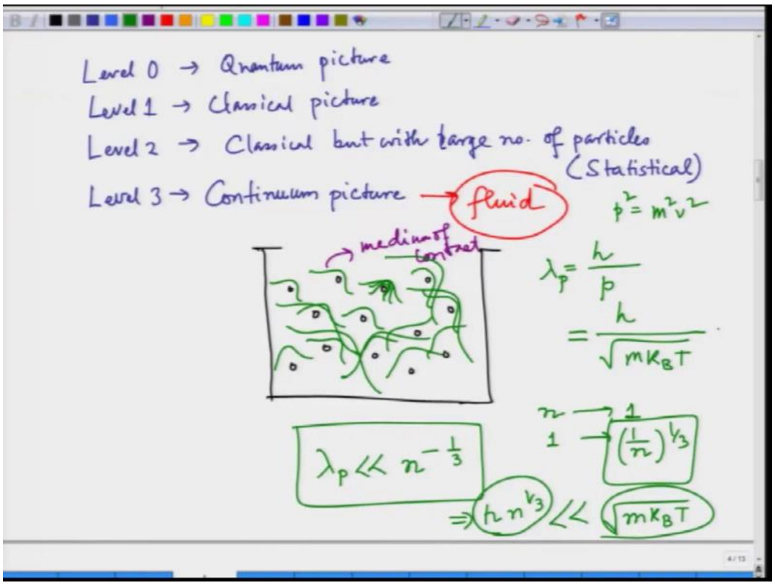

So, we start by saying that for our case level 0 is the quantum picture, level 1 is the classical picture, level 2 is also classical picture, but with large number of particles. So, it will be a classical picture, but a statistical description will be associated. And finally, level 3 that is the continuum picture, we call that fluid picture.

Now, if you understand what the quantum effect is, basically it is nothing but to consider a container which is full of particles having wave nature, you do not have to go to very complicated details of quantum mechanics just remember de Broglie's principle that every particle has a wave particle duality and every particle has a wavelength associated with it.

So, some wave packet associated with every particle right. That wave packet can be of different nature or for one particle to another they can be identical as well we do not know in general. Now what de Broglie says is that the wavelength will be h or the Planck's constant by p.

Now, p is the momentum of the particle and by doing some very easy calculation as you know that $p^2 = m^2v^2$ and $\frac{1}{2}mv^2$ is basically of the order of K_BT, then you can simply say p will be of course, of the order of $\sqrt{mK_BT}$, where m is the mass of one single particle, K_B is the Boltzmann constant and T is the kinetic temperature.

Now, the question is at under what circumstances these particles will interact in a quantum way. Of course, you can say that when these particles when these black points that is their classical structure, they will not intersect or they will not touch each other, but this green wave packets type of thing they will touch each other and can interact. That is that is the way when we can say that even the particles are not colliding classically or not touching each other classically, but by the help of their wave packets they are touching each other. So, that is their medium of contact.

Now, try to understand one simple thing, if we can just make the bowl very very large or the container very large. Then the particles, they are thermally excited particles, so actually they can have much more space available for their motion and they can move further from each other. Or in some other way if we can say if we just now contract the whole container then these particles will be coming very close to each other.

So, in the first case even the quantum effects will be negligible and in the second case the quantum effects will be important another thing is that if in some way we can make these wave packets more and more spread out, then you can see that even for a given distance the quantum interaction cannot be avoided.

So, this is the case when we say that the de Broglie's wavelength is somehow greater than the mean inter-particle distance. Now how to measure mean inter-particle distance in some system like this. Let us consider n is the number density of the particles, that means, n molecules or n particles will be in unit volume then what will be the effective volume for one molecule.

You all know that is high school mathematics, in fact primary school mathematics, $\frac{1}{n}$ will be the effective volume for one simple molecule and then what will be the linear distance between two particles in average that will simply be the cube root of that.

So, the intermolecular distance or inter-particle distance will be in average roughly, that is a rough estimation of course, will be like that and that should be very very greater than the de Broglie's wavelength in order to get rid of the quantum effect. So, once again if the de Broglie's wavelength is greater than the intermolecular distance, then we cannot get rid of quantum effects, for the inverse case where the de Broglie's wavelength is smaller than the inter particle distance, then basically you can say whether they have wave packets or they are without wave packets I really don't care because in both cases the interaction in the classical level and the interaction with the wave packets are all almost the same.

So, the that is the case which is translated by, $\lambda_p \ll n^{-\frac{1}{3}}$ which can also be written like,

$hn^{\frac{1}{3}} \ll \sqrt{mK_BT}.$

So, now you understand there are two very direct ways to get rid of this quantum effects; one is you just make the density very low, so n will be smaller and smaller or you increase the temperature.

So, what is the meaning of increasing the temperature, that is the exactly the technique which I said you that to make the de Broglie's wavelength shorter and shorter, so that means the size of the de Broglie's wavelength then be like small and small and then basically whether a particle is really having a de Broglie's wave packet or not we cannot understand.

So this is the condition by which we can get rid of quantum effects to first switch to classical systems. Now you have a number of particles or molecules which are classical that means, we do not have any wave particle related to this, but we simply have this type of particles.

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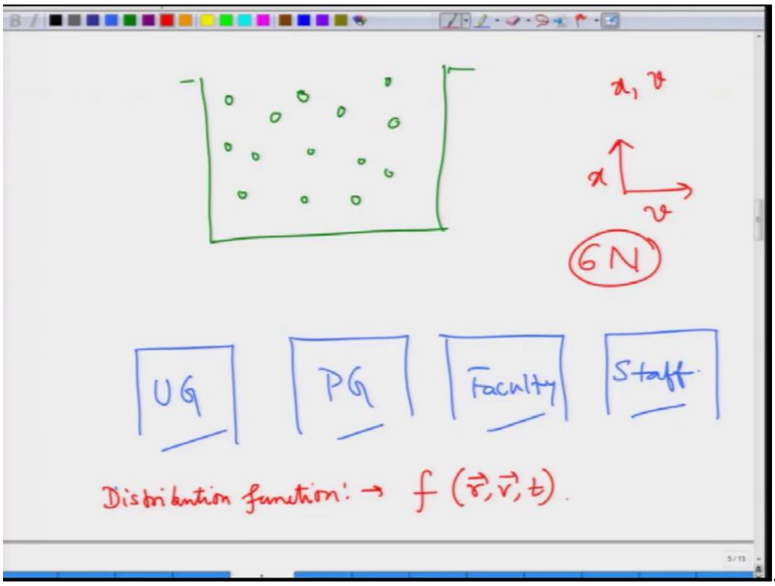

Now, we can also think them as rigid spheres. They are moving, they are colliding with each other, but other than the collision, let us think that there is no interaction. Let us say these particles are not charged. So, that there is no Coulombic interaction or these particles are not moving really under any very strong external any force field or potential. So, that other than their thermal collision, they do not have any other source of interaction.

In this case what you can do? You can simply write the Newton's equation for the systems and you know that you have to take the all the collisions into account and you know that also this these are assumed to be rigid spheres. So, all the collisions will be elastic collisions. So, you know linear momentum as a vector is conserved, that kinetic energy is conserved.

So, you have so many conserved things, you can solve for the equations right. Now the question is that if you have a large number of molecules or large number of particles then what you can do?

So, basically treating a moderate number of particles is not very difficult, even if you just treat maybe analytically this is not possible, but numerically you can simulate them. Now the question is when you have a very very high number of particles, then what to do! Let us say you have a 10^6 number of particles the question is can I simulate them? So that at any instant of time I can see where they are, if I know their mutual interaction force.

I can actually tell you this is not even the good question to ask. The good question is that should we be interested in knowing that which particle is where at any instant of time. Let me explain this thing by using an interesting example. Let us say your university of college is organizing a yearly festival, you all are interested in yearly festivals I know and the festival organizers they have made 4 entry gates; one is for undergrad students, one is for PG students, one is for faculties and one is for the other staffs.

Now, let us say for some reason the weather was very very bad and unfortunately the organizers could assume that only 10 people could go there. Then will they be interested to maintain all the 4 entry gates or to know that how many people are coming through every gates? They will just know who came because we have only 10 people. So, we can we can be interested in who finally came. Now let us say the weather is beautiful, everyone is free and is coming to enjoy the festival then you have the whole institute.

Now, what will be our objective to trace one single UG student or to trace one single faculty? no. Then we will count how many UG students are entering through their gates, how many PG students are entering through their gates, how many faculties and how many staffs they are entering through the corresponding gates right.

So, the objective is now changed. That is exactly the essence of introducing distribution functions when we have a classical system having a large number of molecules we are no longer interested in the position and velocity of the individual particles, rather we will be interested in the gates right and that means, that then we will fix mentally, an imaginary gate at every point of the phase space.

I assume you know what phase space is constituted by, phase space is constituted to show the state of a system. So, all the coordinates of the phase space are given by the positions and the velocity of the system at an instant of time.

So, this is exactly the same situation where for a large number of particles are there in a classical system, we will be interested in designating the number of particles per unit volume or the density of particles for a given position and for a given velocity at a given time instant. And what is that? This is nothing but the so called distribution function. We call it as f and this is a function of $\boldsymbol{r}$, $\boldsymbol{v}$ and t.

Now, let me just tell you for recapitulation, in case you know that, even in case you do not know just have a look. So, for example, for one dimensional case you have one particle which has a velocity v and position x basically. So, the phase space will contain a plot of x vs. v.

Now, if you have a system containing N particles in 3D space then basically you can understand that then every particle will then have 3 space coordinates or 3 position coordinates and 3 velocity coordinates. So, 6 coordinates for N particles they will have the system globally we will have $6N$ coordinates and then you also have time.

So, then you can say my phase space is a space of $6N$ dimensions.

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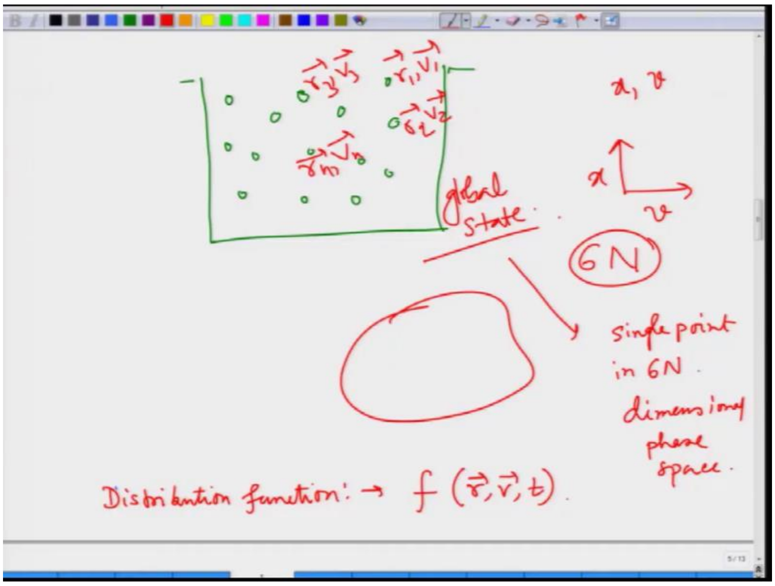

If I define the state of the system, like we can denote the coordinates like $(\boldsymbol{r_1}, \boldsymbol{r_2}, \boldsymbol{r_3}, \boldsymbol{r_n})$and $(\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}, \boldsymbol{v_n})$, then if I just take a snapshot at a given instant of time I will get an ensemble of $6N$ values, which gives me a global state of the system at any instant and that will be represented by a single point single point in $6N$ dimensional phase space. So, we have somehow introduced the concept and the basic philosophy behind introducing the distribution function.

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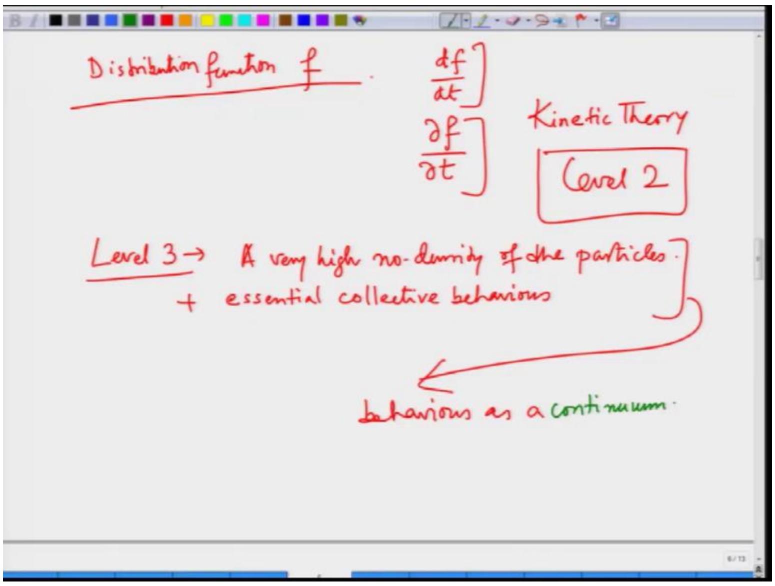

Now, when we have a distribution function f then we will not be simply happy to know the distribution function itself because once again the f basically gives us the information of the density of number of particles at a given point in phase space

Now, this basically is not really describing in real sense the evolution this is the picture at one instant of time. To know how it changes with time or how it evolves we have to know $\frac{df}{dt}$ or $\frac{\partial f}{\partial t}$ whatever, but something like $\frac{d}{dt}$'s. Now this a theory of knowing this is known as kinetic theory. So, that is our level 2.

So, we started by saying 0-th level which was quantum level then we said for classical level we just have to use Newton's laws of motion and then basically when you have a very large number of particles your objective changes and you are now much more interested in the statistical distribution of the particles in the function of their position and velocity and also time.

And then you define some quantity which is the density of number of molecules in the phase space and which is known as the distribution function. So, the distribution function is a function of position, velocity and time.

Now, the last thing is that if you have a container containing a large number of molecules. Now if the large number is so large that you cannot see the particles are having some gaps in between them, let us say an overcrowded bus for example. Can you see there is a space to stand between two persons? No.

So, this is like an overcrowded system. If you have an overcrowded system of molecules or particles which needs to be described by it is collective behaviour. So, that is very important, not only an overcrowded system is sufficient, but an overcrowded system which is having very strong interaction between them and the behaviour of the system cannot be described just by individual particles, but a collective description of them. Then the whole system can move as a continuum or can be seen as a continuum and this is basically the essence of fluid or continuum theory.

So, you can see that you can easily achieve some type of this level of theory when you are just increasing the scale of interest, that means you are basically diminishing your resolution power right that is one thing possible that you can see that there will be less and less gaps between two molecules or two particles, but that is why the second point comes into play that their collective effect due to inter-particle interactions will be inevitable or will be essential to include.

So, level 3 needs a very high number density of the particles plus essential collective behaviour, that gives us the behaviour of the system as a continuum.

Now, like we said for particles, we need their individual positions velocities, what will be the corresponding state variables? Have you thought about that? Because we now have a continuum system. So, we do not have the particle type of description. So, you cannot really think of the position and velocity of a fluid particle.

So, what will be the corresponding state variables if one wishes to develop here the dynamical theory of fluid? Well, the state variables will be defined in the later lecture when we will develop formally the fluid theory and we will call them fluid variables.

They will also be the function of the position, velocity and time, just like the distribution function. But in general, as you will see that, when we do the macroscopic theory basically the macroscopic or the traditional fluid variables, they are obtained by integrating them over the velocity space. So, finally as a result we have all the state variables for fluids.

Let us say I am now giving you some examples which we will define much more formally later let us say the fluid density, the fluid velocity, this is not the normal particle velocity or phase space velocity what we are talking about, that is fluid velocity or fluid pressure.

All these things will come there and as we will obtain them by integrating the distribution functions or some specific type of mathematical forms of this relating distribution functions in velocity space. So, in the final expression the velocity dependence will not be there. So, all the macroscopic or fluid variables will just be a function of position and time.

So, we already discussed the different levels of dynamical theory. So, the level zero was quantum level, level one was normal classical level, level two was statistical classical level and the level three I just mentioned I have not talked in detail about that and that is the objective of our next lectures to develop fluid equations, so the theory of the continuum or the fluid theory.

So, I already said that the state variables for such a dynamical theory will be the fluid variables which will be a function of space and time only. Now what will be the corresponding law of evolution because a dynamical theory needs both. So, that will also be derived later.

But for instance, you can just be interested to know the those names; one we call the equation of continuity, one we call the equation of momentum or most more popularly it is known as the Euler's equation when there is no viscous effect with viscous effect we call Navier-Stokes equation, we will all derive that much more formally.

Thank you.