Physics of Turbulence
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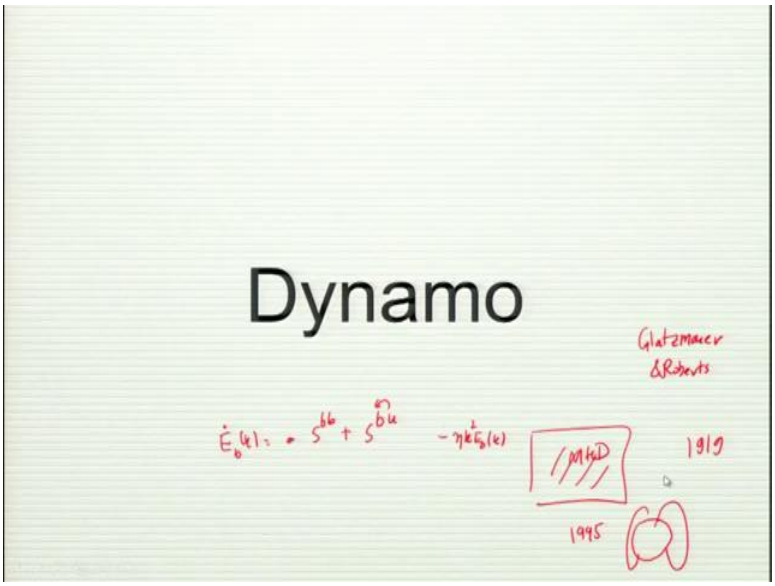
Lecture – 44
MHD Turbulence
Dynamo

So, this is the last part for MHD Turbulence is Dynamo ok.



So, what is dynamo? So, it is not like cycle dynamo. So, cycle dynamo you have seen know so in fact, I opened it up. So, you put a so we in generator electricity, but there is a magnet already. So, is a magnet and there is armature moves in the magnet and generates current. But you should generate current without any external magnetic field.

So, you only have a plasma or MHD plasma and do some stirring or do something on it. But do not put any external magnetic field or no bar magnet and like the cycle dynamo and now generate magnetic field. So, is possible so it was for proposed in 1990 for the sun.

Suns magnetic field has been a puzzle or even earth magnetic field. So, earth has no bar magnet inside anyway it is too hot. So, any bar magnet will melt inside the sun, or the earth. So, that it has been a big puzzle for long, but most we believe in this MHD self induced. So, it has induced by on it is own. And how it is induced by own? Can you I mean you can easily so, let us let us figure out how it is induced on it is own.

So, if you look at equation for E_b, E_b dot $E_b(k)$ let us let us put wave number. So, there is one term which is S^{bb} and there is a much. So, which is b to b it turns out as I said remark S^{bb} does not get any energy from u, it is among themselves. So, two sets of people and once that are be just exchange things among themselves. So, that is not going to increase b and always remember that there is a diffusion term. So, if you do not get anything from other source.

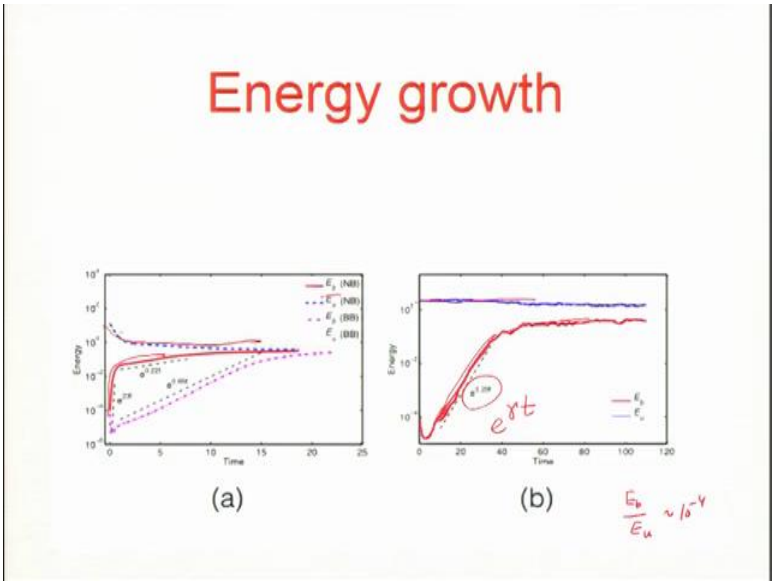
Student: E_b.

E_b will die, but there is other source which is S_{bu}. So, u gives to b and this is the source for magnetic energy which is the mode to mode transfer which I derived. So, from the mode to mode transfer there is a flux and this is what leads to dynamo which you understand at least on numerical simulations, lot of analytical work and this can produce magnetic field.

And so, there lots of evidences that this is the theory and I think one of the let me just make a remark that one of the leading which tilted the believers or well which tilted. I mean so if so well fine MHD will work these are idealized equations. But there may be other things in inside the earth, but in 1995 there is a paper. There is a simulation done after the earth magnetic field.

Somewhat realistic the numbers are not same as earth parameters. Now earth has nu, eta density and so on. But not quite same realistic parameters, but as best as possible and they found this is by Glatzmaier and Roberts; Glatzmaier and Roberts in 1995. He has found that the magnetic field of the simulation in a spherical simulation it is switched, is flipped.

Now, unfortunately I do not have a slide here, but is a very nice simulation where the magnetic field of the earth in the simulation flipped similar to what happens in the observations. So, then people said well I mean this is really the model to try and after that there is huge amount of work on planetary dynamo solar dynamo from simulations and these kind of stuff. So, it is a huge field with lot of activity and I will only show some well I am only going to show some applications based on energy transfers ok.

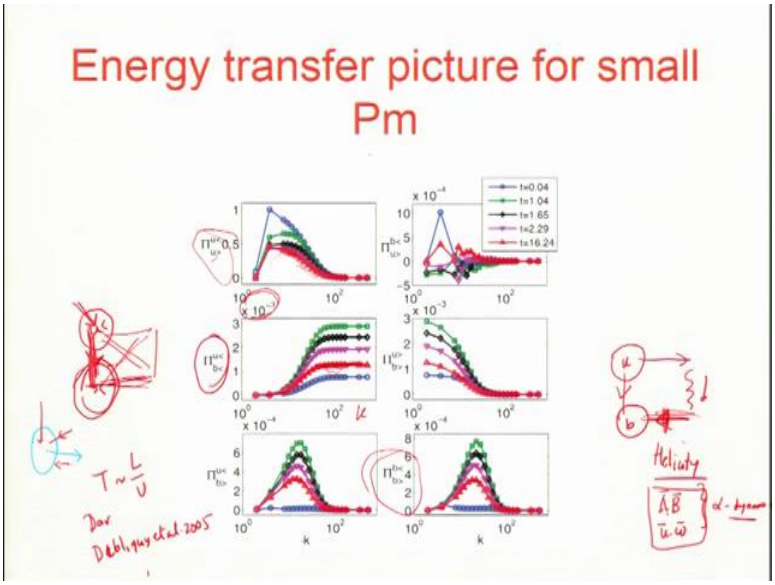


So, these are the simulations done in our lab. So, in these simulations we keep lot of kinetic energy u(k) and some seed magnetic field very small magnetic field. So, E_b could be much less than E_u, you can a spectral simulation. So, we can scatter it in different range of waver numbers E_b by E_u is may be 10^{-4}.

But we need a seed field we can show that if E_b 0 it will remain 0 throughout. You need some small seed field and that is also one of the puzzles in the universe. What cause the seed field and for me well any small fluctuation is enough. So, there are so many charge particles there is small fluctuation we will create that seed field ok.

And anyway there are debates on that as well. So, you can see that magnetic in this these are this b, his red line is a magnetic energy E_b. Now there are various plots you do not need to worry about all of them. So, E_b is increasing with time see in fact, there is a similar plot. So, it grows exponentially e to the power γ t. So, γ is called growth rate and then it saturates.

A kinetic energy look this is very tiny E_b is 10^{-3} E power minus 4. So, E_u does not change very much u is approximately it can decrease by factor 2. So, here it is decreasing somewhat u, but u is approximately same. E_b only grows from nearly 0 to finite value. Some simulation show it because equipartition, some simulations find that they are not equipartition ok. Now question is what makes it grow? Now this has been I mean you want final models what is making in grow it you need is growing. These are box simulation you also do in spheres you find that simulation.



Now, the earlier models now I think I need to say this for completeness. So, this is a u field at a large scales. Now remember is five-third theory you believe that at least without magnetic field it is five-third large scale forcing five-third. Now one set of people said well energy will cascade like what Kolmogorov says. So, it will go to low wave numbers and so low small scales high wave numbers then it will cascade to b field b.

Let us small scales from here it will transfer then there is inverse cascade to b. And these require some helicity which we did not discuss in my MHD discussion as well. So, helicity is A dot B magnetic helicity, and kinetic helicity which is not conserved for MHD. But that also plays a important role u dot omega and there is a papers called alpha dynamo ok. So, these are big thing in solar dynamo, alpha dynamo these relates the fluctuation with the mean field.

So, these use to be the models this is highly popular model called alpha dynamo model, you get my point you need this inverse cascade. So, you needless magnetic energy large scales how can you get energy in large scales? So, well there is cascade of kinetic energy is small scale this thing then it grows backward. Now in our lab you have done lots of simulation without helicity ok, and you find the magnetic energy always not only in our lab. I mean the everybody if you turn off this helicity still get a magnetic field to grow and saturate.

Now, question is what makes it grow if the helicity is so, important, where the I must also say that there are people who say well this is not required. But this helps, but the question

is if there is no helicity what makes it grow. Now it turns out that we can easily answer that question by energy transfers. In fact, this is what we discovered first.

So, you remember this flux formulas u less than b less. So, this is a simulation done by Gaurav in the same paper which we wrote in 2001, this is a huge transfer. So, kinetic energy was forced here so which way and will it transfer. So, there are 6 of them know here, here, here, here, here, here. So, there is one guy which was very very good very powerful and that was this one.

So, u small to u less to b less which is large scale transfer. So, if nobody had talked about. So, this is a transfer which is large scale u to large scale b. So, we do not need these kind of transfers we just need these transfer right. I mean and you using this we can make models of galactic dynamo and estimate the time scale.

So, time scale from these models we can say is l by u; l is a system size, and u is a RMS and you get reasonable numbers, I mean I am not really deep into galactic dynamo. What you can get numbers which is one it written over time which is a 10^8 years or so, and so.

So we can make interesting predictions by this flux arguments, now we computed this flux in for I mean we have done this flux many many times. So, there is one paper by there there is one paper by Debliquy, this is our Belgian collaboration Debliquy et al 2005 ok. Now, these are simulation for dynamo first with seed field.

Now what I would like you to now there is another parameter called small Prandtl number, magnetic Prandtl number. Now for the earth and the sun may be Prandtl number is small and this is for really small is 10^{-7} or so and that cannot be simulated. So, best simulation we could get is 0.2 and people have done some tricks they got to 10^{-2}; 2 but I think that using lot of modeling, but you can seal this u less to b less this one.

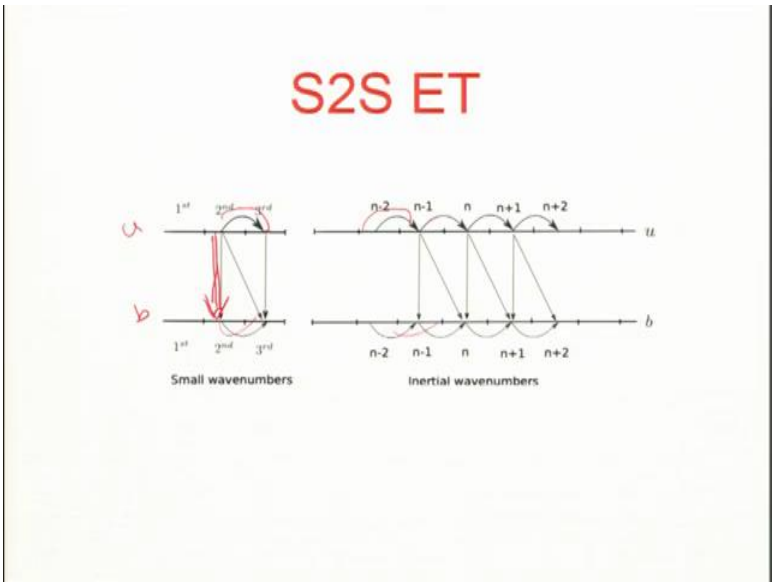
Now, this is wave number now x axis is wave number k k. Now this is at final time u less to b less is one of the most significant transfers among b. Whether a u less to u greater this kolmogorov flux which is quite significant, because kinetic energy is still dominant right. Kinetic energy as I said is giving tiny bit you know this guy rich guy is just giving some small things to magnetic field.

So, u less to u greater is stilled a large, but u less to b less is the most significant transfer for small Prandtl number dynamos. And this is that final time which is quite large ok. Now these are details which you can look at paper, but I mean we are believers that this is a significant thing to look at for dynamo. And of course, we should look at other things like these and these and it turns out one more point that b less to b greater this one. So, b less to b greater is which one this one is positive.

Student: Right.

Right all of them they have come with positive sign. So, when this those are the inverse cascade of b. If b large is losing all the time then there must be somebody to give that otherwise it cannot be sustained cannot be grown. So, this is the critical part which is well there is one option is so you see there is not much choice. So, this has to grow I just want to be emphasize my point if this has to grow and if this is positive. Then how can it grow impossible know.

So, one way is to something comes from the top and something comes like that ok. So, these are the things which come and that, not that is not inverse case. So, this is this sign is definitely incorrect these are helicity. With helicity this sign may come with negative there is evidences are there is a contribution from helicity which makes it this plus helicity effect that gives negative ok. So, these are observations one can make from fluxes.



You can also look at shell to shell transfers. Now I am talking about the small Prandtl number now which is good for modeling sun and earth. So, these are the picture of shell

to shell so u less so there is u to b. So, u less to b less is like this and u to u is forward, like forward is going down in Fourier space a magnetic is also forward. But this is what is sustained in the magnetic field and now these are critical input from fluxes. So, the flux formula are very useful for these computation.

Thank you.