Physics of Turbulence
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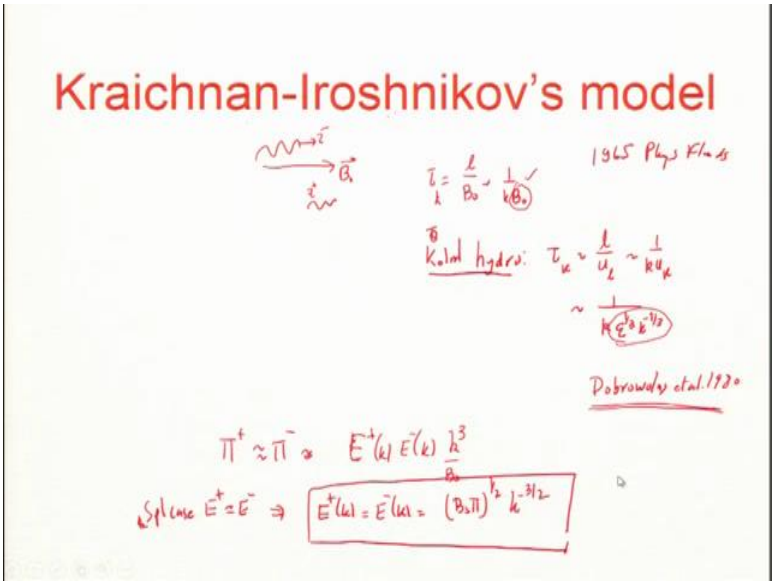
Lecture – 43
MHD Turbulence
Turbulence Models

So, in the last slides we discussed energy transfers and define fluxes. So, these are all coming from non-linear terms ok. Now, we can use these ideas and also earlier ideas done by various researchers; to talk about energy turbulence models for MHD. A Kolmogorov model is great for hydro; it is very good as long as we force at large scales. Of course, we need modification if the energy is injected at different scales. And, there was generalized for buoyancy driven flows.

So, we see an MHD what kind of generalized models exist. But at the outset I say that there are still unsolved problems, there is no convergence. Though I am a believer of some theory which I am strong believer but there are there is no convergence, among everybody does not say well this is a correct model ok. There is no one correct model because there are large number of variables and so on. But there are still lot of miss I mean divergences.

So, one thing I can say it at the outset that this not a passive vector. So, five third is not expected really for this fb and fu which are active know. So, they can change the fluxes. So, we cannot use five third theory ideally. So, we let argue from different angles and we have to do that. So, in this slides at present some leading models, which are 50 year old model, some are 30 year old models. So, I will just present them very scaly sketchily.



So, first model is my Kraichnan -Iroshnikov's model. Kraichnan paper is one-page paper in 1965; Physics of Fluids it is just one-page paper that is why it is, I think it has been misinterpreted and is hugely sighted but it is incorrect ok. So, what does Kraichnan say? So, according to Kraichnan these are mean magnetic field and that leads to Alfven waves right. So, we discussed Alfven waves. So, that is why Alfven wave is a key for modeling imaginary turbulence.

So, remember so, let us assume that $\vec{B}_0$ is in this direction and Alfven waves travel. And, the two does Alfven waves z^+ and z^- and one of them travel in fact, they travel in opposite direction. So, one with u B in opposite direction travel in along B_o and one with ub parallel travel along u minus. So, z^+ should travel backward in minus B_o or minus $B_o \cos\theta$ it may not be in the same direction ok.

So, these are is one-page paper. So, this huge amount of hand waving you know the slight of approximations and, but the ideas are an interesting. So, there are two waves or two packets going in opposite direction. So, they will interact briefly, according to Kraichnan. And what is the time scale for interaction? So, length scale is l will come from wave number and the time the velocity is $v_{o,}$ $v_o \cos\theta$. But there is one model which tries to model cos theta as well; but let us assume that we ignore theta effects. So, l by B_o will give the time scale right, from divisional arguments a time scale for a wave number k will be 1 by k B_o a B_o is a constant ok.

Remember for Kolmogorov what is the time scale Kolmogorov theory? Tau k is Kolmogorov hydro tau this is please remember this ha; So l by u(k) u(l). So, 1 by k u(k). And what is u(k) in Kolmogorov theory? Epsilon one third k minus one third ok, this comes from some initial arguments from five third we can easily derive this which I have done in the class. So, this will give you k2.

So, this is k dependent and in Kolmogorov in Kraichnan theory is k independent; because of Alfven effect. So, B_o cannot be B_o effects are real what about u_o can somebody may say well. If I apply mean velocity field for hydrodynamic turbulence, then I should get similar feature right. Why is in the time scale 1 by k u_o for hydro ok?

So, it turns out velocity field can be illuminated by Galilean transformation know, we can go to a frame where u_o is 0; but we cannot go to a frame where B_o is 0 B_o cannot be gotten rid of ok. So, that cannot be gotten rid of by Galilean transformation. So, these effects exist and they are real. Now we can do divisional analysis which I will not do it here. This is a nice paper by Dobrowolny et al in 1980, 1980 or I think.

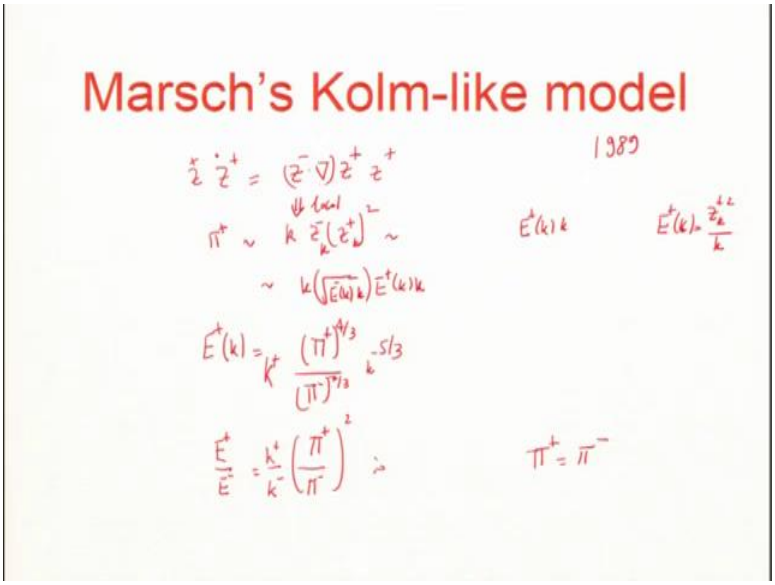
So, around that and there it is derived in more systematic manner a Kraichnan idea because it is one-page paper. So, in this theory, it has been argued so, there is z^+ and z^- are basic variables. So, I am basically studying Dobrowolny's result; by Iroshnikov's also very short paper a similar idea. So, that is why it Kraichnan Iroshnikov's Russian and American 1965.

So, in this theory pi plus a flux of z^+ approximately equal to pi minus and that goes as there approximately equal ok. And so, we can write well I mean there is a formula for in terms of $E^+(k)$ $E^-(k)$. So, it's a product of E^+ and E^- this generalization by Dobrowolny; Kraichnan assumed that if it is E^+ equal to E^-. But it need not be equal there they are unequal these are B_o below in k^3 ok. So, these proved by Dobrowolny.

Now, if you assume E^+. So, one thing you can see that if E^+ and E^- are not equal it could be any number it is a product but the fluxes are equal right. So, fluxes pi plus and pi minus are always equal irrespective of ratio of E^+ and E^-. These are prediction of these phenomenology ok; the proof I am not giving you I mean these are bit detailed proofs.

Now, let us take a special case, when E^+ equal to E^- then what happens $E^+(k)$ both are equal. So, I can write down equal to E^- (k). So, B_o will go there, B_o pi square root right because there is a square k minus 3 half these are theory of Kraichnan and iroshnikovs.

So, it is not five third is minus 3 half ok. So, there is a difference in exponent, but the numbers are quite close 1.5 and 1.67 now let us go to. So, this is old theory and people believed it very strongly.



So, there is a paper by Marsch in 1989 and it says the following says, well, let us ignore the B_o effect ok. So, you can assume the B_o is 0 or some of it is just trial error and we just let us ignore B_o effect. If you ignore the B_o effect, then we have z^+ dot is z^- dot grad z^+ right these are some. Now, for scaling arguments you do not keep all the terms you keep only this. So, if I look at the fluxes what happens and multiply by z^+ both sides.

So, this left-hand side is like flux, you can also argue from what the flux would derived but this is easier than by dynamics. So, this $\prod^+$ under steady state you can put a force in the right side. $\prod^+$ and this one is grad will give a k z^- give $z^-(k)$ $z^+(k)$ squared. Now what is $z^+(k)$ squared? $E^+(k)$ times k is that correct? This is a definition of, what is E^+ (k)? Is z plus square divided by k dk is k right I mean that is what we said is power law physics so, d k is k.

Student: sir there is a star and

So, is a real space. So, there is no star.

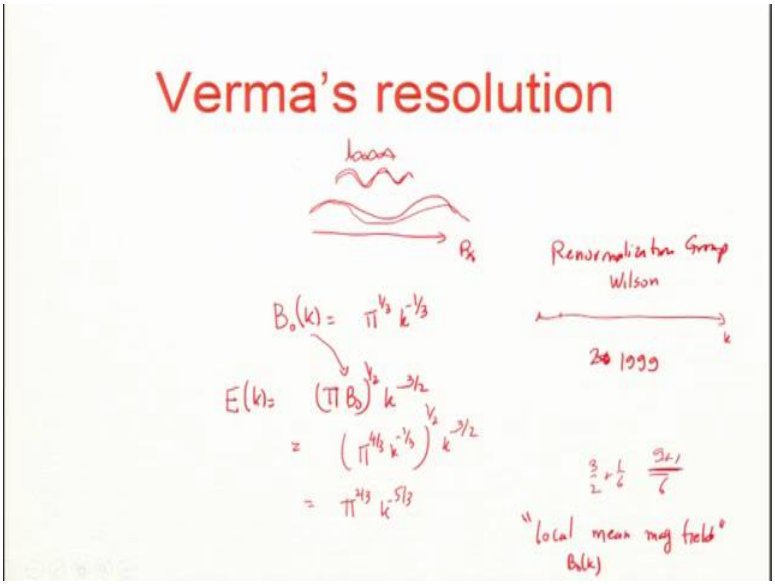
Student: Ok.

First, we so, unless you have to put the star in fact, you have to, but I am doing an estimating the scaling argument. So, you are doing a real space so, where the Kolmogorov theory can be derived exactly in the same manner.

So, from here this from here I am going to real space a Fourier space. So, here I assume local approximation local interactions. So, modes of similar sizes are interacting. So, first equation is in real space second equation is a Fourier space. So, let us write this is k $E^{+}(k)$ k and the other one will be square root of $E^{-}(k)$ times k. So, the second term is z^{-} square root. So, there is a asymmetry in E^{+} and E^{-}.

So, if I so, this equation for $\prod^{+}$ $\prod^{-}$ will be similar equation with plus minus interchange. So, if you workout for E^{+} and E^{-} inverted you get the following. $\prod^{+}$ four third $\prod^{-}$ two third k^{-} five third; there is a constant k^{+}. So, you get five third law, but the two fluxes $\prod^{+}$ and $\prod^{-}$ this is for $\prod^{+}$ E^{+} and you can get equation for E^{-} as well right. So, what will happen to E^{-}? Just replace plus and minus.

Now, you can also derive E^{+} by E^{-} is $\prod$ so, five third will cancel. So, $\prod^{+}$ by $\prod^{-}$ square k^{+} by k^{-}. So, here if E^{+} you know and E- are not equal, then $\prod^{+}$ and $\prod^{-}$ are not also not equal if they are connected by square. So, this relation differs from Kraichnan and Iroshnikov's pi plus and pi minus are equal ok. So, this is a second phenomenology given by Marsch in 1989. Now, looks kind of odd know that we have five third and three half which of them is correct



So, this is 5 by 3 and this is otherwise three half which is correct. So, that is where I come. So, I said let us try to understand this bothered me for it still bothers me. So, there is an intuitive idea is a mean magnetic field B_o and there Alfven waves are various scales. So, it is turbulence so; that means, there will be Alfven waves have many many scales right.

So, the B_o there is a one big Alfven wave a smaller Alfven wave smaller Alfven wave like this they travel in all directions. Now, for this Alfven wave does this c only this B_o or does it see collecting effect of all waves it should be collective effect right I mean. So, if you are trekking in the Himalayas or in a mountain you are not looking at the mean slope of the or the mountain local ups and downs.

So, same way the wave should be seen local mean magnetic fields. So, this is are you convinced with this. So, at least intuitively it seems possible that though there is a mean magnetic field effect which you will probably effect of the phase being changing like Bo effect. But, the non-linear interaction the local mean local fields must also affect the Alfven waves.

So, for these Alfven waves what should matter is undulation here and for these Alfven wave what should matter is b coming from here and so on. So, this procedure is called is addressed by a theory called renormalization group ok. So, this is a scale by scale theory. In fact, if you look at quantum will low dynamics. So, if you have seen this you might have heard this there are beard charge of electron is infinity heard of this term or no?

So, electron has infinite charge and infinite mass, but what we see is not the infinite charge and infinite mass why do not we see this? Whether there is no single electron, there is electron will create a electric field; electric field will create positive electron pair because of Heisenberg. So, electric field will if you quantize it you will get various virtual particles we call virtual particle know.

So, electron is covered by these lots of particles which are being created. So, what we see the single electron or what we think is a single electron is collection of centered electron plus lots of virtual particle. So, like this horse riding in a very dusty environment. So, lot of dust collected around the horse so, that is the analogy some people make. So, there are lot of virtual particles around it. So, if we go close to the electron, you are not going to the center of the electron, but you are seeing collective effect shielding of the infinite charge by this lot of virtual pairs am I making myself clear or not.

So, what we seeing are if collective effect. So, if a charge will increase, if you go closer and closer in you understand. So, log so, what we are seeing is a charge at some scale. So, similarly here my magnetic field effects will change when I go to different and different scales. So, the B_o which is postulate by Kraichnan is not a single constant B_o is function of k. So, when I go to inertia so, if I make this wave number scale k; then I have small wave number here if you see the mean magnetic field large scale now there is mean magnetic field.

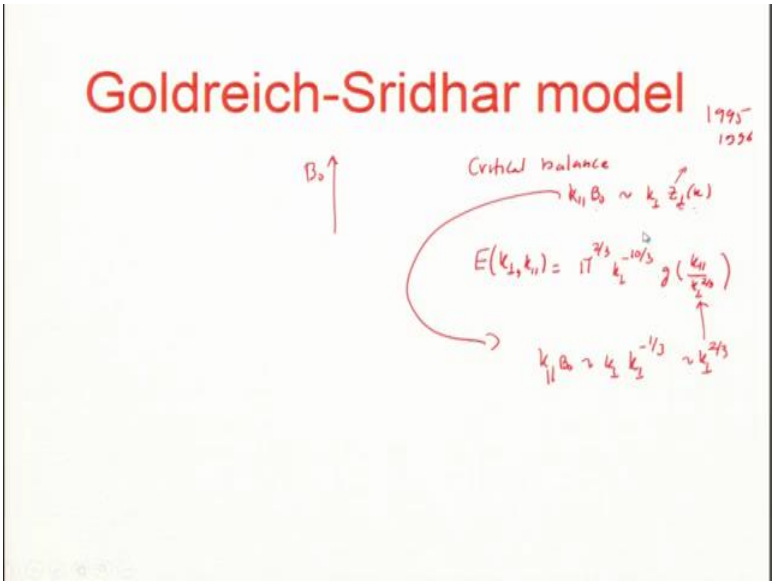
If you keep going down then you see effects of other fields like by going closer to closer electron the effects are changing of the shielding ok. So, this is solving the problem scale by scale and we follow this procedure by Wilson. And, you can show whether this is a big calculation it published in 2000 1999.

So, B_o is k dependent and it is $\prod$ one third k^- one third of this kind of nice know it is same as E(k) of Kolmogorov theory. Kolmogorov theory is $\prod$ one third k- one third I wrote that. So, B_o what is affecting the wave waves nonlinearly is this. So, now if I use this formula for Kraichnan so, what is Kraichnan formula a Kraichnan and Iroshnikov's formula k- three half. So, let us put this $\prod$ half $\prod$ one third.

So, B_o let us put the B_o here, but I is replace B_o of Kraichnan by $B_o(k)$ yes. So, they will give you so, inside let us put inside. So, this $\prod$ and $\prod$ one third will give you $\prod$ four third right; $\prod$ four third k^- one third here to the square root k^- three half. So, this will give $\prod$ two third k^- one six and one six and three half is three half plus 1 6, 6, 9 plus 1. 10 by 6 is.

5 by 3 ok. So, we recover Kolmogorov theory, by just replacing B_o by k dependent B_o, but here assume z^+ and z^- say equal. Giving you run equal and calculation is very complicated which I try to do it there is some work, but is more detailed I am not confident of that calculation. Now this calculation is reasonably correct and clear. So, this is how we make a consistent theory of Kraichnan theory and five third theory.

And, show that mean magnetic field is not b_o, but it is a local mean magnetic field. And, this is a word which is used quite often now in literature local mean magnetic field which is $B_o(k)$ ok. So, this I would like to verify this numerically ok, this has not been done in a computer simulation. So, there are ways to measure B_o and we need to do it ok. So, we move on so, three models done. So, let us go to fourth one Goldreich Sridhar model, which is highly sited paper, but which is weaker than my B_o effect paper.



But what this Goldreich says is, if this B_o then it brings a non-isotropy. Its true I agree B_o will give, but if you make it too strong B_o, then there are some more complications. And a very strong Bo will make it 2 dimensional. So, all these theory will just disappear. Now, each one bring in 2 D inverse cascade all that thing comes. So, B_o we assume model one and then isotropy is reasonably good approximation I do not know.

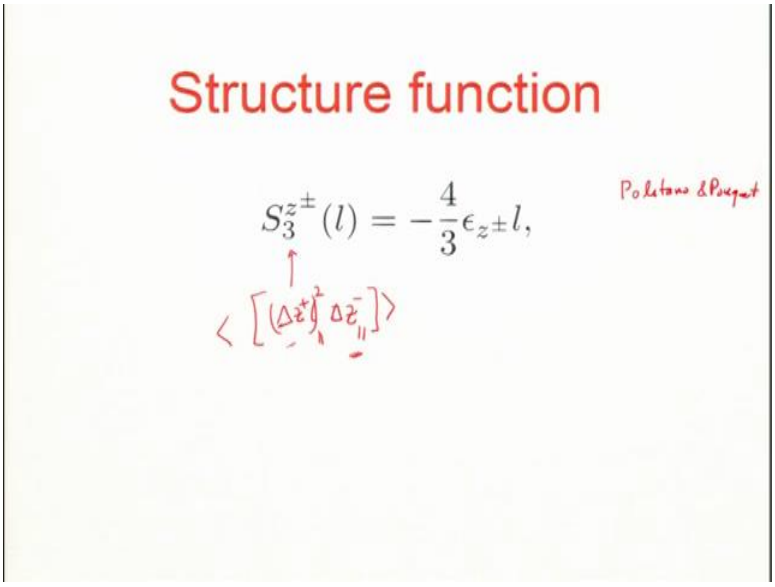
So, this is where I think there are lot of divergence. So, B_o effect and this is a highly influential paper accepted by lot of people. So, according to them this is for critical balance. So, I am just stating them ok; so I am not taking any side except my side of my $B_o(k)$ ok. So, critical balance is k parallel B_o is k per z per k. So, these are relation of the Alfven wave with B_o is also correcting k perp and k parallel.

Now, this is a postulate some people argue against it of course, many people believe it as well and this also should be tested. If you put that then you get different spectrum for k parallel and k parallel k per. So, you get basically formula for E(k) perp E(k) parallel. So, this is a formula with anisotropy right, which is a good formula, but I mean this some people claim that it has been verified. But I mean there are still issues specially with very strong bo.

And, now if I just average over k parallel is integral over k parallel, then you get again minus five third. The formula is bit complicated if you like I can write that formula, but it's combining both I do not know I mean you can look at this there is a formula with k parallel k perp and is there in my in my notes. So, it is here ∏ two third k perp minus 10

by 3 g which is a non this g is a function this is coming from here if you put k perp z perp is k, but minus one third you will get this right ok.

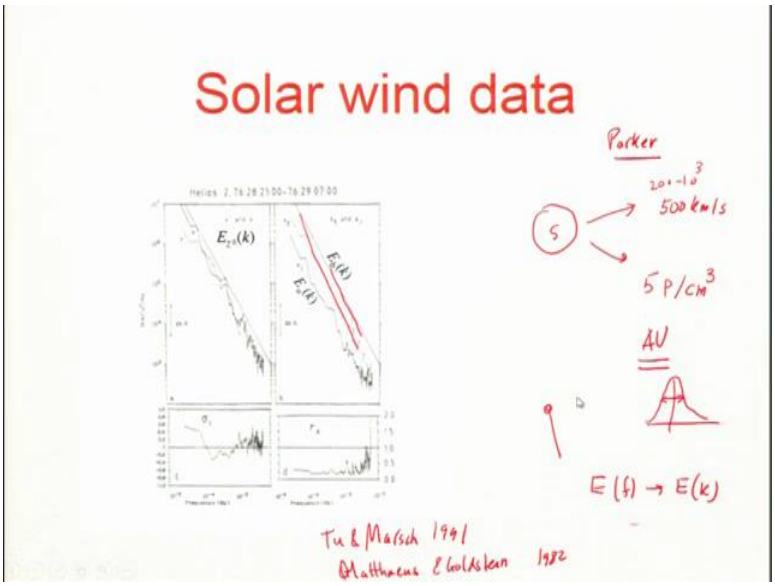
So, because is five third z perp. So, z perp will give you from here k parallel Bo(k) perp k perp minus one third know five third will give this. So, this gives you k perp two third. So, this is this is exactly come from here ok. So, this is a formula from this you can derive 5 by 3 and various term. And this is paper in 1995, 1996 ok. So, this is fourth model a quite a few more I will skip the paper by Gaultier there is a paper by Boldyrev, but I will skip there is a endless story there is lot of models



Unless look at structure function. Now this is kind of nice if you ignore the Bo effect and you get five third. So, this means what is Kolmogorov theory? Is minus 4 by 5 epsilon l; now here we get this S_3 it is a interesting combination of z^+ and z^-. So, it is z^+ square delta z^- take the component parallel components. So, this is average this is S 3. So, is a combination of z^+ and z^-.

So, z^- is a mediator and z^+ and z^+ are receiver and giver. So, mediator is coming with power 1 and giver and receiver are coming together with power 2 and this is exactly same as passive scalar formula which I discussed. So, the derivation is identical for passive scalar for this derivation and this is by Politano Politano and Pouqat this is their paper. So, this is a bit later in 2000 you find them in the in the book ok.

So, this is 2000 something or 99 late 1990. So, this is telling us that structure function of course, with Bo effect is not included in this, gives you a five third. If I go to spectrum we expect five third I mean not quite derive it, but five third.



Now, So, how to prove this? It turns out doing lab experiments, though some of it has been done, but very few getting turbulent MHD is very difficult; the reason being a that u as. So, eta is large there is a diffusivity. So, you need very large l. So, like solar wind so, this is one good setup which lot of people I mean including I will also I mean I did little work on this. So, Supratik does lot of work and I think you may be doing some work. And suns atmosphere is not static so, there is a sun.

So, this was proved by parker, nobody believed him for many years many years means several decades I think they did not believe. So, he says earth atmosphere is not stationary like earth at sorry suns at atmosphere like earth has a study site atmosphere, but nothing is blowing out of the earth. But according to parker suns is emitting electrons and protons and some helium from which is atmosphere and there is a wind blowing and this is called solar wind.

And its mean speed is around 500 kilometer per second huge speed know 500 kilometer per second. Of course, it ranges from 2000 to 1000 there is lot of variations, but there is huge speed. And the density of this plasma is how much is, very rarified is 5 particles, 5 protons per cubic centimeter is very very verified ok. I mean if you take one once when you cube out the gas here, there are I mean I think ten power twenty atoms molecules.

But still so, one question is can you apply MHD model. So, MHD model requires that. So, remember I made one remark that you need continuum approximation. Then what is continuum approximation that the mean free path length is much smaller than the system size then only we can ignore these collisions. Now for the solar wind the mean free path length is, I mean I mean the interaction is there is something called one AU Astronomical Unit.

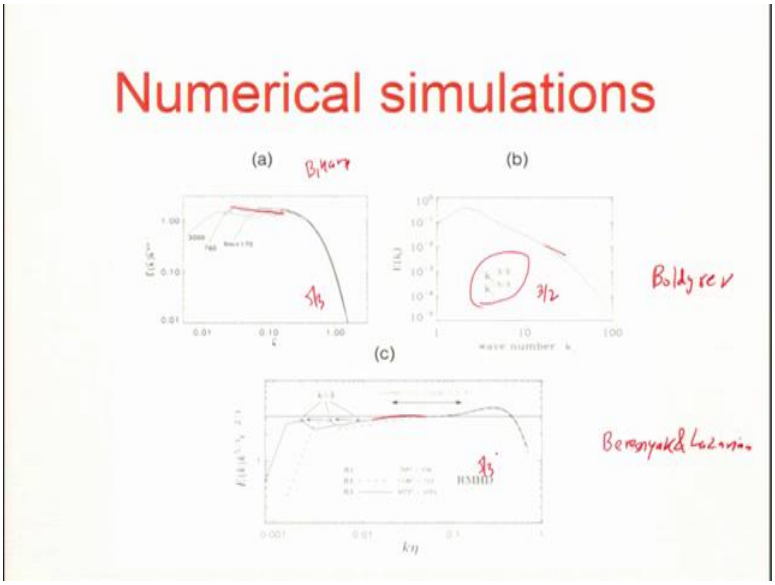
So, distance from the sun to earth is called one AU. So, one collision is verified when it is on the average from sun to earth it hardly collides. But still if you look at the probability distribution function is not too far away from actual length. If you look at so, this way to check whether it is thermalized; so, what is Maxwellian distribution we have this Gaussian distribution the Vrms know.

So, how does it become Maxwellian, if it is not colliding. So, reason it is becomes Maxwellian I think I agree with this, but mostly argued by researchers, there is ambient magnetic field. So, the ambient magnetic field acts like a scatter is a random magnetic field. So, that is scattering there you do not require a magnetic field to scattering. Just imagine that there is electron which is going in a magnetic field and for turbulent magnetic field you do not need another electron to scattering. So, magnetic field will scatter and if it is turbulent then it can make it Maxwellian stop ok.

So, this is what we will make it Maxwellian. So, we see this solar wind this is paper by T u and Marsch, I think in 1991 this is a paper by Goldstein and Matthews and Goldstein 1982. So, all of them show five third to good approximation. So, these lines are five third lines spectrum ha. So, this line I am did not draw it properly. So, these are so, that is well I mean. So, spacecrafts have been sent. So, there are spacecrafts like voyager pioneer sent way back in 60s 70s 70s 1970s and they collect data.

So, what does they do? They put a probe so; they are just going in the free space outer space. And they are probed which measures magnetic field speed temperature and so on. And I but the speed of the probe is much smaller compared to 500 kilometer per second. So, you can use Taylor approximation Taylor hypothesis according to which if I am stationary here, the wind is blowing then you can treat as if the wind is stationary and I am scanning along the line. So, you need to convert the frequency spectrum to wave number spectrum I made this remark before know.

So, you can convert E_f which is what is measured by the space craft's E_f; you can make it to convert to $E_f(k)$. And, these are Eff not $E_f(k)$, but using this Taylor hypothesis we make this conversion. And, they show reasonably reasonable convergence with five third though 5 by 3 and 3 f are very close one point the difference is 1.6 and then enough error bars. So, people can get I mean if a believer then you say well I do not believe you. So, we can plus there error bars are when the difference is too small ok; so, but this has been reported many years.



So in fact, Kraichnan was believed, but you see there are solar wind was saying that it is not numerical simulations. People have measured this especially after 2000 spectrum, but this may be not very clearly visible, but this is minus three half theory by Boldyrev; this is by the Biskamp he believes its five third five third line. But these lines are not very this is by Beresnyak and Lazarian and these are not very convincing either way so, these three half five third, five third. And now so, what we did was we said by let us look at indirect predictions.

Indirect prediction was look at this $\prod^+$ and $\prod^-$ this we wrote in 1996 see this $\prod^+$ and $\prod^-$ and E^+ and E^- are unequal; this is the way to test know if E^+ and E^-. In fact, their issue was roughly 10 and look at $\prod^+$ and $\prod^-$. If they are equal then Kraichnan is right.

If unequal then Kolmogorov theory should work. And we found that Kolmogorov works better ok. So, Kolmogorov works better this is 1996. So, these are indirect proofs which are more convincing than looking trying to fit a line with this five third or three half. Now I think more convincing will be this B_o effect whether we can show that this Alfven wave scattering is by B_o. And there are ways to do it, but nobody has done it so far ok.

Thank you.