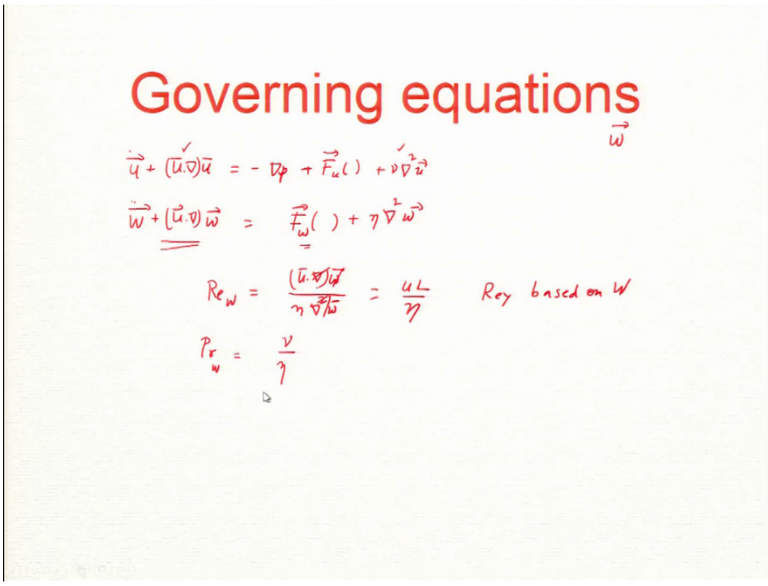
Physics of Turbulence
Prof. Mahendra K. Verma
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 40
Flow with a Vector

We will deal with Flows with Vectors now. So, velocity is a vector, but it is advecting another vector. So, example being magnetic field, you know magnetic field is advected by velocity field. Of course, magnetic field does something to the velocity field too. So, that is what we are going to solve. So, I will put as a general framework, ok. So, flow with a vector. It could also vector need not be only magnetic field. It could be dipoles, electric dipoles or some vector particle which is travelling. So, one example you say active or like fish population. So, that is like a vector you know going around, ok. So, let us look at the equations.

(Refer Slide Time: 01:08)

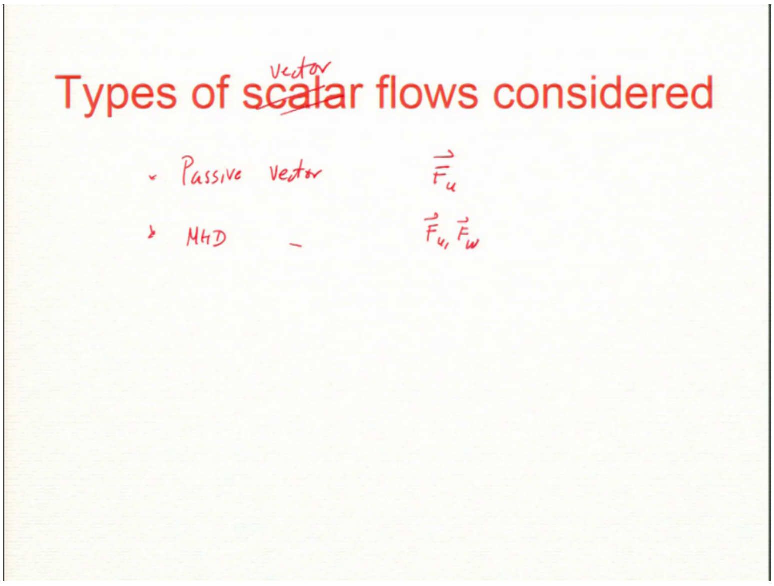


So in fact, is very similar to what you do for scalar, there is a very similar framework. So, let us write down those equations. So, velocity field will be same as before and I am going to put another vector field $\boldsymbol{\omega}$. We write equation for $\boldsymbol{\omega}$. So, $\boldsymbol{\omega}$ is a vector field, ok. So, $(\mathbf{u} \cdot \nabla)\boldsymbol{\omega}$ is the advection of $\boldsymbol{\omega}$. There is no pressure gradient in this, ok, so pressure gradient only acts from the velocity field. So, there could be force on $\boldsymbol{\omega}$. Let us keep a general framework. It could depend on $\mathbf{u}$, no, or it could depend on time it could be general.

Of course, $\boldsymbol{\omega}$ can back react on $\mathbf{u}$ or $\mathbf{u}$ can also influence $\boldsymbol{\omega}$, $\mathbf{u}$ is influencing $\boldsymbol{\omega}$ here, but $\mathbf{u}$ can also influence $\boldsymbol{\omega}$ here, ok. So, we define few quantities Re_w, Reynolds number based on $\boldsymbol{\omega}$, is the Peclet number, so is the ratio of the non-linear term, $(\mathbf{u} \cdot \nabla)\boldsymbol{\omega}$ by $\eta\nabla^2\boldsymbol{\omega}$, right. So, usual non-linear by diffusion, which is $\frac{UL}{\eta}$.

So, this Reynolds number based on $\boldsymbol{\omega}$, is different then, Reynolds number for velocity field, which is $\frac{UL}{\nu}$. So, you can also define Prandtl number, which is $\frac{\nu}{\eta}$, but this Prandtl number of $\boldsymbol{\omega}$, so, it turns out which will not do in this course but their times when we have both temperature and $\boldsymbol{\omega}$. So, magneto convection, you know which some of your working on. So, then we will have two Prandtl numbers, $\frac{\nu}{\kappa}$ and $\frac{\nu}{\eta}$. So, you need to differentiate that time, also Re. So, there are Peclet number, which is different than Re_ω, ok. Now, so this is clear. So, these are the equation nothing more complicated.

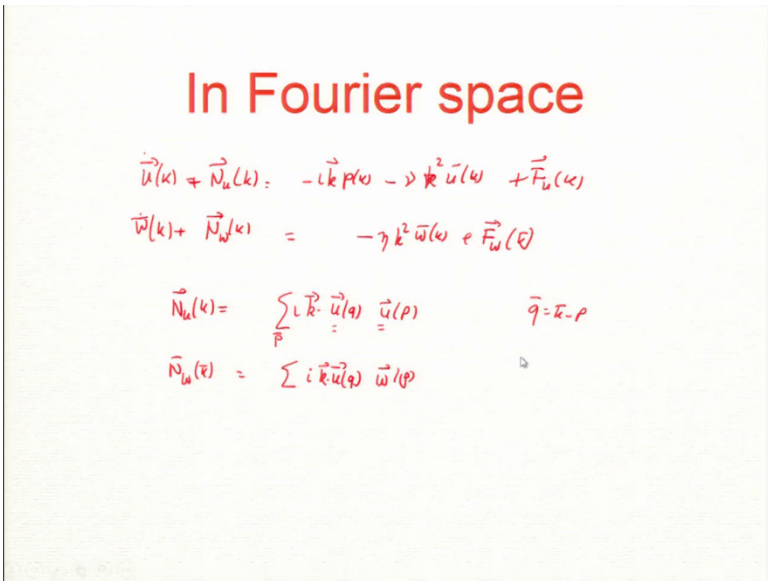
(Refer Slide Time: 04:27)



So, what type of you know? I am sorry this is vector flows considered. So, right now for today's lecture I will consider two types of flows, one is a passive vector. Passive vector is $\mathbf{F}_u$ is independent of $\boldsymbol{\omega}$. So, force is, velocity field is not affected by $\boldsymbol{\omega}$, so it is passive, no. So, $\boldsymbol{\omega}$ is driven by $\mathbf{u}$, but not $\mathbf{u}$ is not driven by $\boldsymbol{\omega}$. So, it is called passive, is same thing is passive scalar, ok.

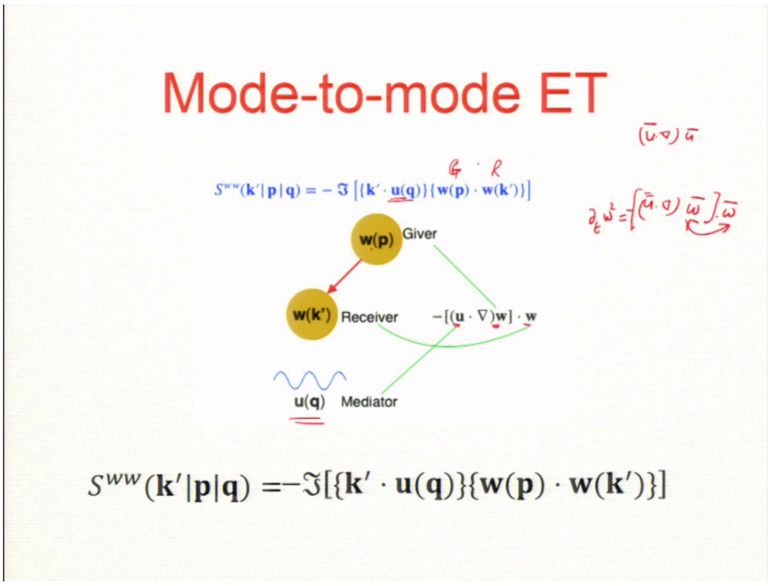
And second, I think some of you will be interested is MHD, magneto hydrodynamics, ok, where $\mathbf{F}_\mathrm{u}$ and $\mathbf{F}_\omega$ are quite complex and which I will try to do later.

(Refer Slide Time: 05:29)



So, your Fourier space how does the equation look like? Please refer to the above slide foe equations in the Fourier space.

(Refer Slide Time: 07:22)



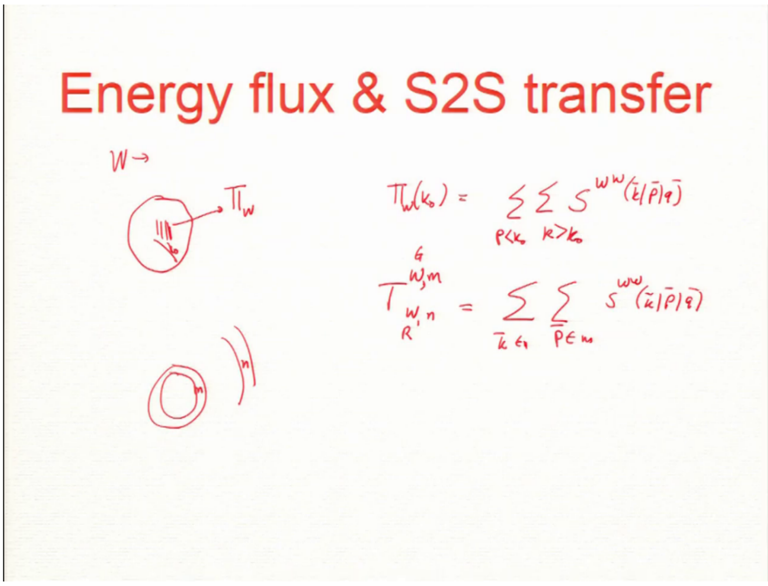
Now, mode to mode transfer. So, for, so there will be quite a few, but $\mathbf{u}$ part is same as before. So, $(\mathbf{u} \cdot \nabla)\mathbf{u}$ will induce energy transfer among u modes, that we done before. So, I will not worry about it, ok, right. I will not discuss here.

But now there will be a transfer from $\boldsymbol{\omega}$ to $\boldsymbol{\omega}$ because there is a term, so $(\mathbf{u} \cdot \nabla)\boldsymbol{\omega}$.

$\boldsymbol{\omega}(\mathbf{p})$ gives energy to $\boldsymbol{\omega}(\mathbf{k'})$ and $\mathbf{u}(\mathbf{q})$acts as a mediator, ok. So, this is what I have written here. This $\boldsymbol{\omega}$ is the receiver, this $\boldsymbol{\omega}$ is the giver and this $\mathbf{u}$ is a mediator. So, what is the formula?. Please see the above slide for the formula.

Now, this is coming from advection. Now, we will have $\mathbf{F}_u$ and $\mathbf{F}_\omega$that can also lead to energy transfer.

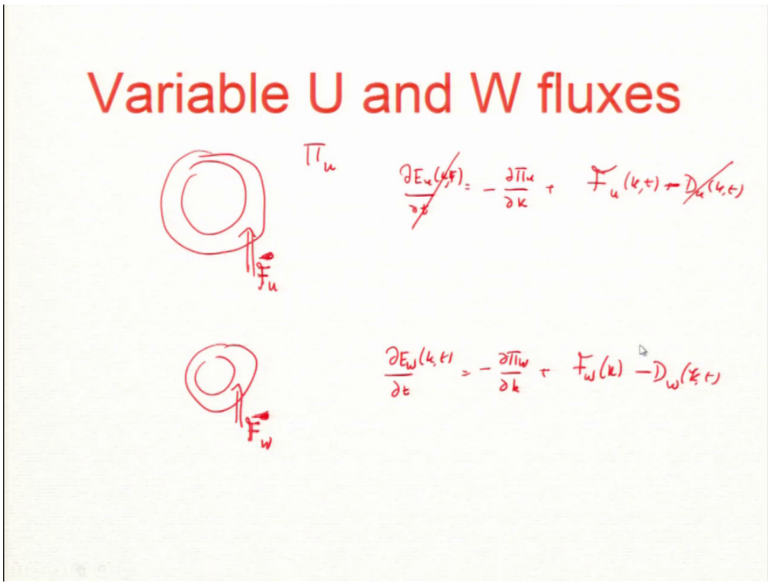
(Refer Slide Time: 09:42)



Now, come to the energy flux, Kinetic energy flux same is before, but now we need to worry about is there is a flux for $\boldsymbol{\omega}$ because there is a there is a mode to mode transfer, so there will be flux. In fact, can easily see the flux of $\boldsymbol{\omega}$, Π_ω which is energy going from modes inside to modes outside, please refer to the slide for the formula. from w to w.

We can also define shell to shell transfer. So, T_ω^ω, so this is my notation. So, giver is above, receiver is below. So, this is a giver, this is a receiver. So, shell, which shell? So, there two shells, no, shell m and shell n. So, shell m is a giver, shell n is a receiver. So, what should be the formula. Please refer to the above slide.

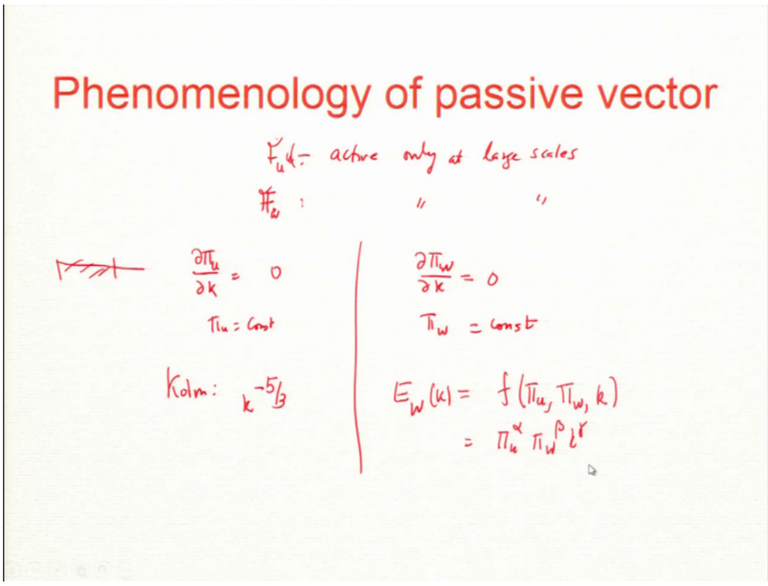
(Refer Slide Time: 11:43)



Now, let us work out what happens to the flux. So, what can change flux? We did in the past for $\boldsymbol{\omega}, \boldsymbol{u}$ and for θ as well. So, if we have a shell is this is injection by external force in the inertia range which will come from $\mathbf{F}_u$, then it can change the flux, this for Π_u. similarly, we can do for $\boldsymbol{\omega}$, if there is a $\mathbf{F}_\omega$ then that can change the Π_ω. In the above slide, there are the equations for the variable energy fluxes.

So, in the inertia range this is dropped, this is dropped, and flux can change because of $\mathbf{F}_u$, correct, these we done in the past. So, there is a diffusion term. So, that will diffuse $\boldsymbol{\omega}$, ok. So, we need this for MHD, but for the next slide I will not really, I will make it simple. Now, this is general framework. So, let us think of what happens to passive vector.

(Refer Slide Time: 13:47)

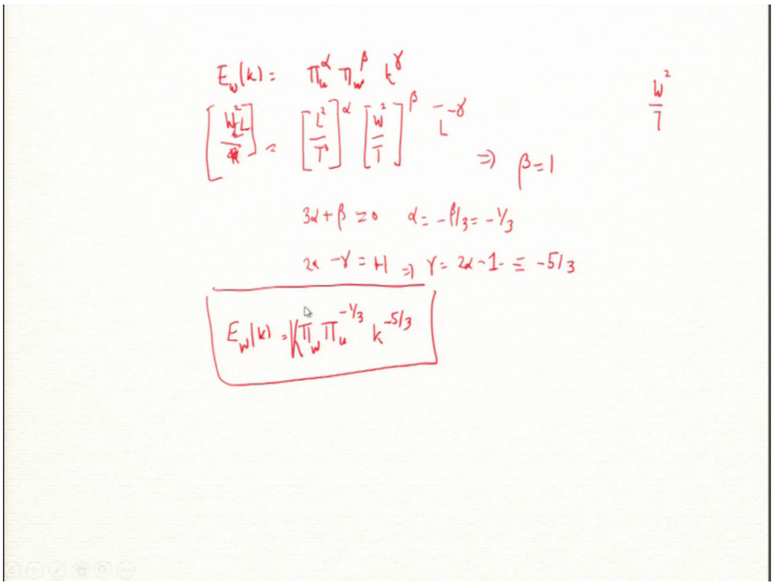


So, what happens to passive vector? $\mathbf{F}_u$ is on is not a function on $\boldsymbol{\omega}$ and let us assume it active only at large scales. Also assume that $\mathbf{F}_\omega$,and the injection by it are active at large scales. So, inertia range there is 0. So, if you look at the equation from the previous slide, what happens to $\frac{d\Pi_u}{dk}$in the inertia range? There is no dissipation, there is no injection, so this is 0. $\frac{d\Pi_\omega}{dk}$, what happens to that? 0 too. So, both Π_u and Π_ω are constants in the inertia range.

So, what is the spectrum for a kinetic energy? Same is Kolmogorov because $\boldsymbol{\omega}$ does not do anything. So, E_u this is same as Kolmogorov. What can you say about $E_\omega(k)$. So, it can depend on function of Π_u, where Π_u is crossing this regime, right, inertial range is seeing Π_u. Inertial range also says Π_ω, it also sees k. So, its function of three quantities.

Please refer to the slide below for the derivation of scaling of $E_\omega(k)$

(Refer Slide Time: 15:44)



Thank you.