Physics of Turbulence
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Lecture - 39
Turbulent Thermal Convection

So now, we will go to the third system. So, we did passive scalar, stably stratified flows now we will go to thermal convection. So, this is another discussion of scalar; temperature being coupled with velocity field; so, Turbulent Thermal Convection.

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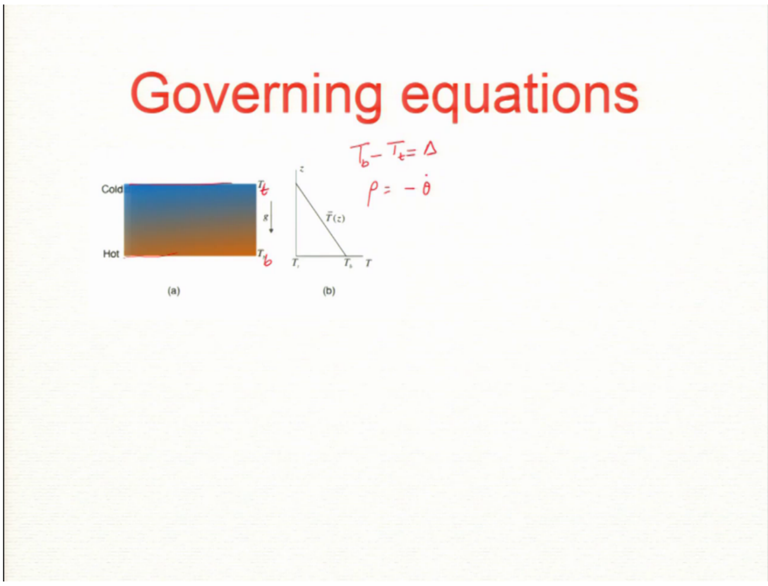

So, system is very similar except. So, let us look at the internal temperature. So, hot below know convection means you know heat from below, but cold at the top. So, this is slightly specialised is not like heating water in a container in a pan, but you also put a lid at the top.

So, hot plate below and cold plate above. So, there are two plates. So, this is somewhat simpler because you do not want to deal with the surface at the top. So, it is to simplify our life in analysis is assume that this is a plate at the top as well. So, this is the temperature difference. So, T_b, and T_t are the temperature of bottom and above plate respectively. $\Delta = T_b - T_t$ is the temperature difference. So, T_b is bigger than T_t what about the density, which is denser? Above is denser right because there the temperature is less. So, it is a system which is opposite of stably stratification that is why it is unstable. So, the top guys come down and the bottom guys go up. So, the hot looms they go up why you guys like it, which is wants to go up and the top one which is colder it is comes down and that is creates this roll and some of you already done in your project and like.

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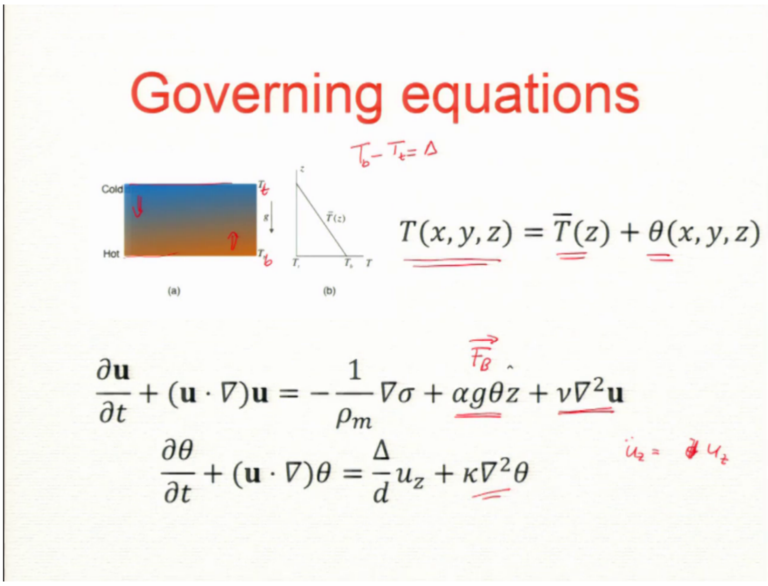

So, total temperature I write as this linear profile of temperature plus fluctuation ($\bar{T}(z) + \theta$) ok. Same idea like for density we had $\bar{\rho}(z) + \rho$ now we do the same thing here as well ok. So, θ is 0 at the top and bottom. So, θ is a fluctuation over a linear profile, now equation will be very similar to what we wrote for density. So, $\mathbf{F}_B = \alpha g\theta\hat{z}$ is the Force by Buoyancy, where α is the expansion thermal expansion coefficient.

Now, if I do the linearization and stability analysis what will I get? So, some of it I we already done know linear stability, it is a growing solution. So, this is unstable, but stability is hampered by the viscous term right. So, this we done in the pass. So, you agree that this is unstable and naturally precisely because hot thing is going up and cold is going down. So, it will make it unstable ok.

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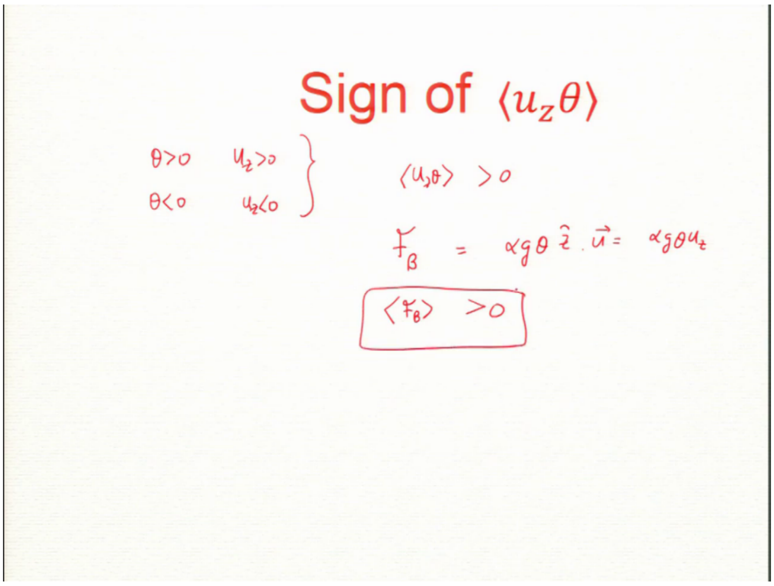

So, what is sign of u_z? I think some of you correctly said. So, u_z so, hot fluid θ positive u_z is positive right. So, θ here positive means is hotter than the ambient., if it is hotter then it will go up. So, θ and u_z have the same sign. So, $< u_z\theta >$ must be positive.

If theta is less than 0 means temperature is colder than the mean at that level; so, it will come down. So, u_z is negative. So, this implies $< u_z\theta >$ is greater than 0. So, what is sign of energy kinetic energy injection rate $(\mathcal{F}_B) = \alpha g\theta u_z$? If I do the average it will come out to be positive. It would have situation when it becomes negative for short while, but overall average is positive ok.

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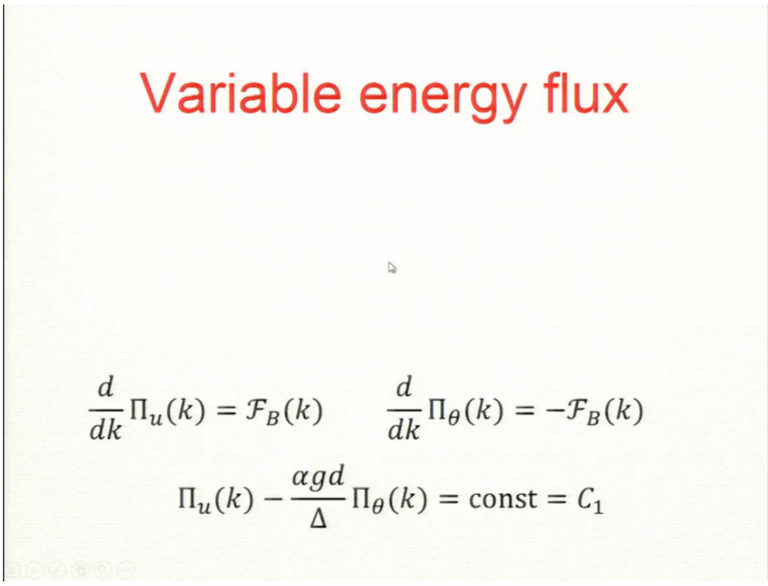

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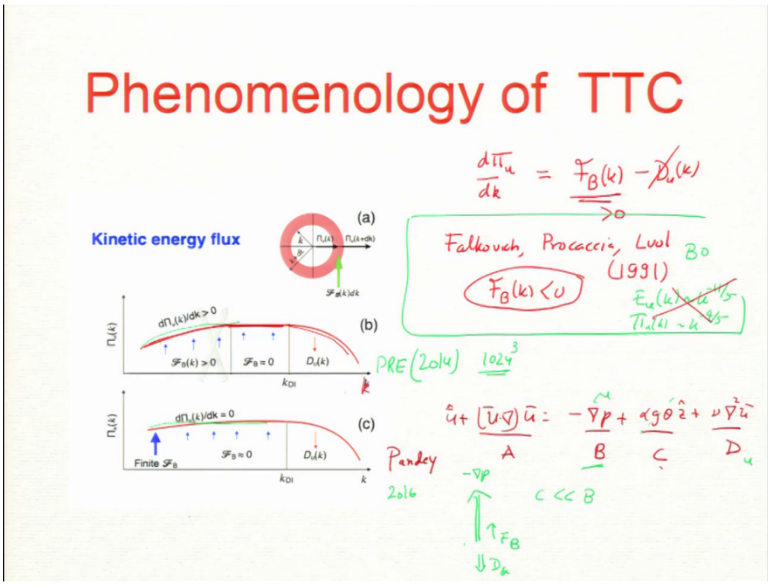

So, what should happen with the flux? So, remember this $\frac{d\Pi_u}{dk}$ in the inertia range by the way for thermal convection we do not apply any external forcing; buoyancy is good enough to drag the flow.

If you just wait for some time it will become steady. So, $\frac{d\Pi_u}{dk} = \mathcal{F}_B$ in the inertia range, I drop the viscous term (Refer Time: 06:08). So, this is positive. So, my flux must increase with k ok. However, Buoyancy is weak to increase the flux noticeably and remain constant throughout the inertia range .

But people did not have this idea. In fact, their names like Falkovich, Procaccia, Lvol. So, the people in 1991; there is two papers where their argued in by using field theory in somewhat complex way, but their calculation has one glitch which was very important glitch that claimed that $\mathcal{F}_B$ is negative. It is following from influence from Bolgiano Obukhov not the physical intuition. So, physical intuition is like everybody will agree that u_z and θ are in the same direction. So, $\mathcal{F}_B$ is positive.

So, it was a mistake and as a result they had argued. In fact, big community everybody is believed that Bolgiano Obukhov theory should also work for thermal convection. So, spectrum must be minus $k^{-11/5}$.

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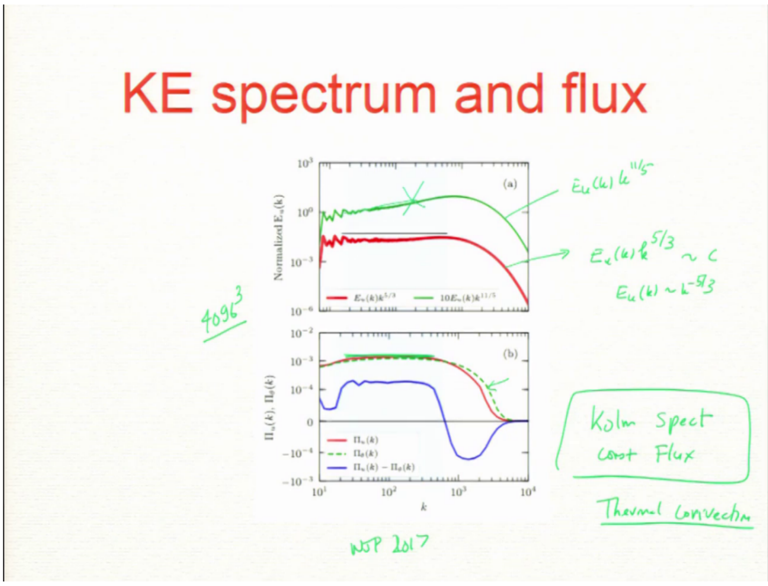

So, this is normalized spectrum $E_u(k)$. So, we you multiply $E_u(k)$, this one is multiplied by $k^{5/3}$ and it is flat. So, $E_u(k)$ vary s $k^{-5/3}$. This one is multiplied by $k^{11/5}$ and the compensated plot is increasing; that means, $E_u(k)$ is not going as$k^{-11/5}$.

So, it is very different than stably stratified flows and it better be because their nature of energy induction into kinetic energy is different.

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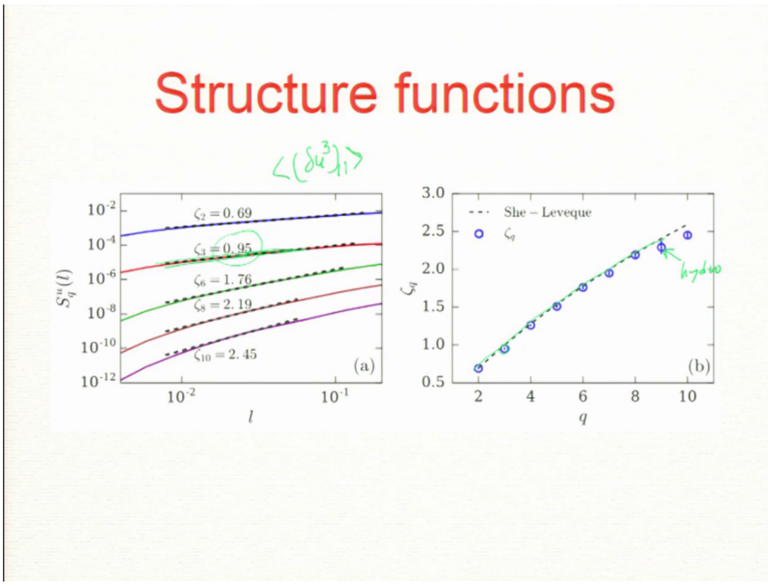

And so, we have done this structure function. So, what do you expect for structure function? So, you look at S_3 .So, S_3 is what is according to Kolmogorov theory.

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So, she Leveque is good for hydro hydrodynamic turbulence it fits very well, but we see RBC is also fitting very well with the She-Leveque model; that means, this structure function is telling us that hydrodynamic turbulence and thermal convection are similar property of turbulence and which is as expected I mean basically in same lines pressure gradient is driving the flow.

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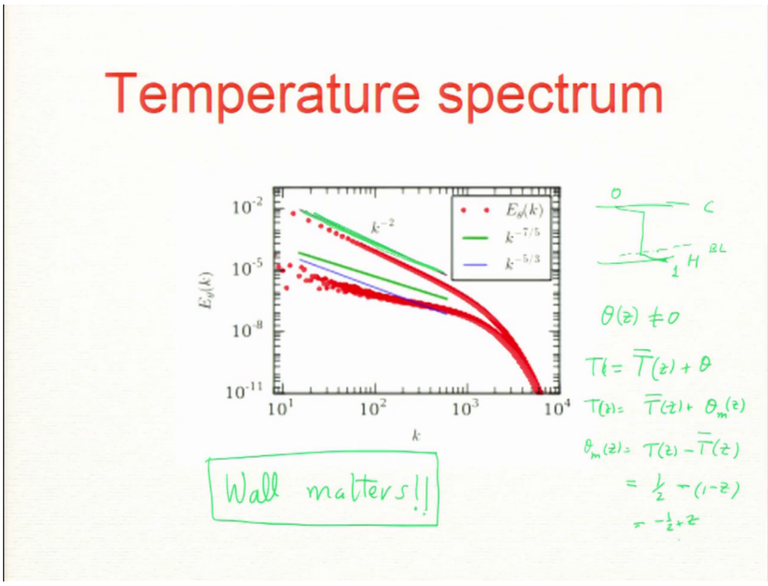

Now, we can also look. So, this is a technical part. So, we will look for temperature spectrum. Now I will not explain this, we get this funny structure these one two lines. Now these two lines bothered me for six months I did not know why these two lines are coming. This coming in the simulation result and you always suspect there is something wrong with the simulation and it turns out this is very physical; this is coming from physical region this is correct. And why it is correct? Because the two walls hot wall and cold wall and the temperature has mean profile.

So, θ if I do the average your planar average. So, theta overall if you do over the whole volume it is 0 if you do the planar average it is not 0 this is not equal to 0 ok. So, in fact, it is easy to see. So, let me just write this. So, $T(x, y, z) = \bar{T}(z) + \theta(x, y, z)$. If I do the planar average means integrate over x y. So, this becomes $T(z) = \bar{T}(z) + \theta_m(z)$

So, $T(z)$ for thermal convection, it looks like this (please see the above slide). So, this is hot plate and cold plate. So, temperature decreases a rapidly, it is constant the middle and then decreases the again ok. So, I need the plate this sharp drop in temperature now this is called boundary layer ok. So, boundary layer temperature drops by half. If the whole drop is one unit; so, half at the bottom and half at the top and in between is constant.

So, what do you expect for $\theta_m(z) = T(z) - \bar{T}(z)$? So, in the bulk, assume this to be one and this is 0 then this $T(z)$ is $1/2$ and $\bar{T}(z) = 1 - z$ So, $\theta = z - 1/2$. If I do the Fourier transform, I am not going to get 0. So, this is the Fourier transform this gives you that top line.

So, this we show it in our paper. So, the mean profile of θ_m is giving me the top line in the remaining modes Fourier modes are giving you the bottom line ok. So, this is what I say that wall matters in turbulent flows you put a wall then you expect something different. So, Kolmogorov theory it is of course, very beautiful, but it does not explain lots of phenomena. So, wall so, my phrase is wall matters for turbulence. You cannot say homogeneous and isotropic flow is the theory and rest all are like minus theory.

So, this is you know we have this grand unification, or we have like universal theory. So, I gave a colloquium which I do not know some of you are not there. So, the one theory is not going to explained everything instead I am strongly disbeliever of that idea and one theory you can explain everything. So, Kolmogorov theory is nice, but is only works for homogeneous isotropic turbulence of course, this gives the idea about how do you use flux, what does flux do, but one idea is to explained everything like what we have in particle physics that does not work ok. Here wall is important ok.

Thank you.