Physics of Turbulence
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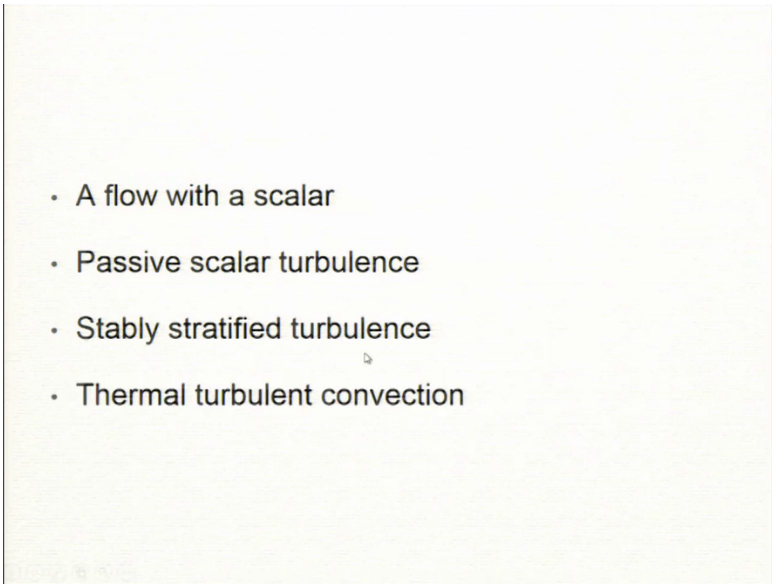
Lecture - 36
Flow with a Scalar

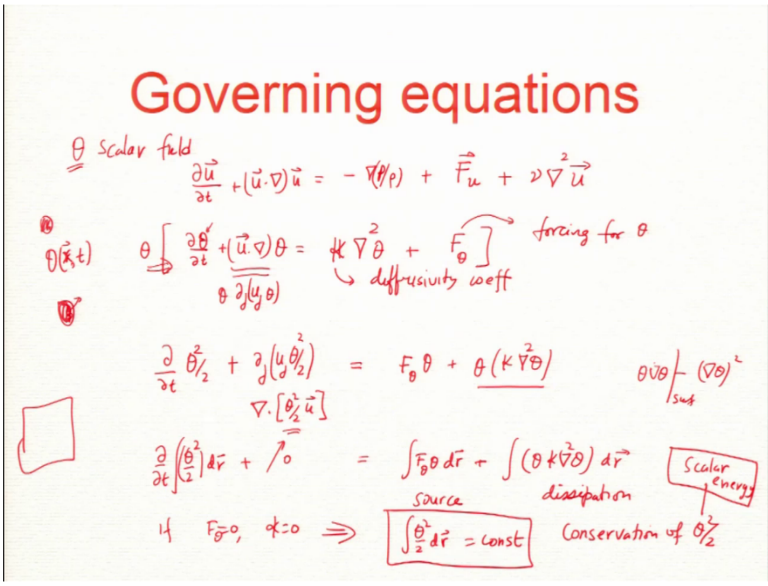
Ready.

So, we will start a new topic. Now, so far we discussed flow with only velocity field, right. So, it was water flowing or air flowing, but now we will add some more complexity and put a scalar in along with the velocity field. So, what does it mean? We can put ink in flowing water. So, that is scalar you know, the density of ink; so, that we will go around with water. So, that is called scalar which is only magnitude, but it does not, it is it does not have a direction. So, you can also have magnetic field in the flow and that is a vector, so that is flow with the vector which I will do later, ok. So, we do MHD with that formalism.

So, scalar temperature is a scalar and also convection which you already done. So, I will cover them slightly fast, you know, I mean I do not have time, so I will do it only the basic ideas I will convey in the lectures. So, let us formulate our equations. And I will work with the only real a Fourier space except I will give you the corresponding 4-5th law, you can also derive structure function for scalar fields and that proof I will not give, but I will state the results, ok.



So, what I will do is a flow with a scalar. So, I will just basically put the formalism then passive scalar turbulence. I will define what passive scalar is. So, then we will do stably stratified flows like earth atmosphere is stable, if the density is heavy at the bottom and lighter at the top and then we will work with convection, ok. So, these are the 4 topics I will cover, today and tomorrow. So, let us start with scalar turbulence.



So, our equations are. So, we have a new field, ok, so I am going to call as θ, another field θ which is scalar field. It is in the same flow. So, velocity will advect the θ. So, we need to write equation for both velocity field as well as for θ. So, velocity field is a same equation as before, that is the velocity of the elements of fluid. So, we already done this before, $\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u}$. So, we will assume incompressibility.

Density is constant plus force $\mathbf{F}_u$. So, u subscript means velocity. Now, we have to be careful. Now, there will be two forces, one for θ as one for **u** and then viscous term. So, $\nu\nabla^2\boldsymbol{u}$. So, this we already done. So, I am going to use the results of the past. Of course, we need to when I say a solid in scalar equations then I need to solve both, **u** as well as term scalar.

Now, what is the equation for θ? So, θ is advected with the flow, ok. So, we will have, so theta is function of x and t or **r** and t. So, told time derivative of θ. So, I am not going to derive for full thing, but you can derive like what you did for velocity partial. So, you can take a patch of scalar and how it flows with the velocity field. And so, how will this patch a partial move?

And it moves velocity advects it and also changes by diffusion. So, the advection part is this one θ and the diffusion part. So, this must be, so the proof I can only sketch, so this θ, this part is the θ element θ total θ inside a small packet and that can change by advection and by diffusion. So, this is changed by advection, right., the flux, theta flux going out of the sphere and then we have advection.

So, κ is my diffusivity coefficient for θ. So, if I mean density diffuses, temperature diffuses, so this these are diffusion coefficient, ok. And there could be force on this a scalar force, ok, so somebody increases θ, no injection, I can inject ink or I can heat the fluid, so that I am supplying temperature thing. So, we cue up θ, it is very useful to cue up θ, ok. So, this is forcing for θ. The way temperature velocity is forced by studying it you can also inject, you inject θ by, so F_θ is positive. It can also be injection at different scales, need not be only large scale, ok.

So, something which changes θ is F_θ, ok. Now, should not think like force, force is not necessarily mass times acceleration. So, something which change it is I call it force. Somehow it is a mathematical framework.

Now, this equation has some interesting properties. So, we can write down equation of θ^2. So, let us write down equation for θ^2, ok. So, I can multiply this by θ whole equation, no. So, this will give you $\frac{d\left(\frac{\theta^2}{2}\right)}{dt}$. What about the other one? **u** is still incompressible, ok. So, I can write this equation, this term is $(\partial_j u_j)\theta$, yes.

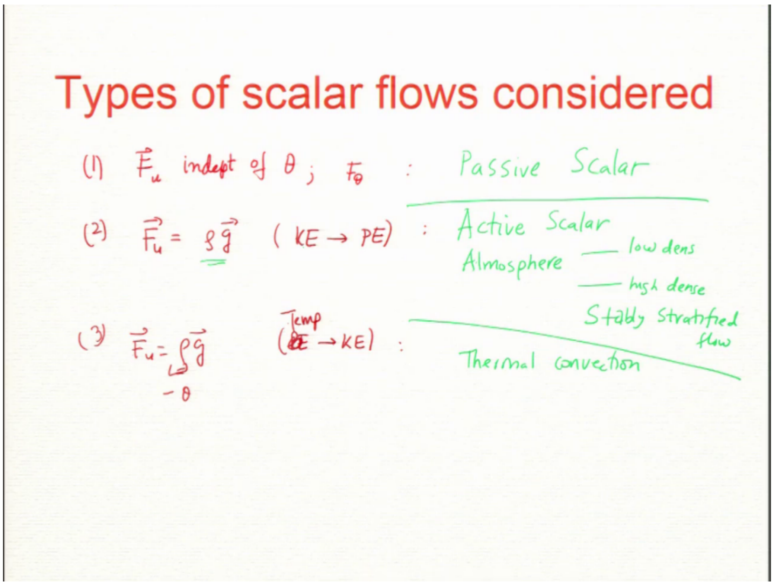
Now, I put theta here, so you can easily show that this $\partial_j\left(u_j\frac{\theta^2}{2}\right)$, by product proof. So, this is $\nabla\cdot\left(\frac{\theta^2}{2}\boldsymbol{u}\right)$, yes. Right hand side will be θF_θ, so this is the injection of injection of θ^2 like we have force injection no of that gives you increase u^2, similarly F_θ then θ will increase or decrease θ^2. And the other term is $\kappa(\nabla^2\theta)\theta$.

It is like you can do by parts this part, right, I mean $\theta(\nabla^2\theta)$ can be written as $\theta(\nabla\theta)$ – $\nabla(\theta^2)$. I can now, $\int d\boldsymbol{r}$. If I integrate over the whole volume, so there is the whole volume box $\frac{d}{dt}\int\frac{\theta^2}{2}d\boldsymbol{r}$. What about this? What is this for periodic boundary condition integral of this?

If you have periodic boundary condition I apply gauss theorem, so it will be surface integral. So, left surface, right surface will cancel, top surface bottom surface is cancelled. So, this term gives you 0 by gauss theorem. Now, right hand side is of course, $\int F_\theta\, d\mathbf{r}$ $+\int D_\theta\, d\mathbf{r}$. Now, this term is the dissipation term. So, θ^2 can be killed. But like of course, ink does not get killed, ink is diffused. So, we use the same analogy as fluid, so my linear mod basically kinetic energy is diffused at some level and it is no more fluid velocity, no. So, you can see that it very small scale you cannot call it fluid velocity it is gone into thermal heat or the heat will heat energy.

Similarly, κ, it becomes very small then I do not call coherent motion, I say θ^2 has dissipated, ok. So, this is a dissipation, in the same language, ok. So, this is a dissipation term and this is source term. If the source in dissipation has 0, so F_θ is 0 and κ is 0. Then what happens to $\theta^2/2$? It is constant in time, ok. So, if F_θ is 0, κ is 0 implies $\int\frac{\theta^2}{2}d\boldsymbol{r}=\boldsymbol{const}$. So, this is a conservation of θ^2. So, I am going to call it you know scalar energy is it is a generic word which I will going to use for all temperature, density or ink you know. So, we will call it scalar it is not energy really, it is a quadratic quantity, but it is convenient to call it scalar energy.

So, density square is not same as kinetic energy, ok. So, this is only for convenience or for mathematical similarity. So, we will call it a scalar energy, ok. So, these are conventional follow, fine. So, we can derive some more properties, but this is one property. This will suffice for our thing.



Now, what types of flows I will consider in the set of lectures. So, one thing is you can say I forgotten scalar, no. So, this actually I focus on both, that relations. So, one thing is type one is passive scalar where F_u, there is a F_u, but it is independent of θ; that means, the scalar does not affect the velocity field. So, that is why it is called passive scalar. So, if they are light dust particles then it does not do anything to the velocity, it just travels with the velocity field you know. So, velocity dictates the future of θ, but not vice versa. θ does not do anything to velocity field.

Of course, its particle is heavy then it will affect the velocity field, but light particles do not. So, you can think of mass less particle injected in the velocity field it does not. In fact, this is used for experiments, so there is something called tracer particles which are injected inside the velocity field, inside the fluid and it does not change the velocity field, but it can be used for visualization of the flow, ok. So, this yes so, but F_θ can be anything F_θ is present, but F_θ can be.

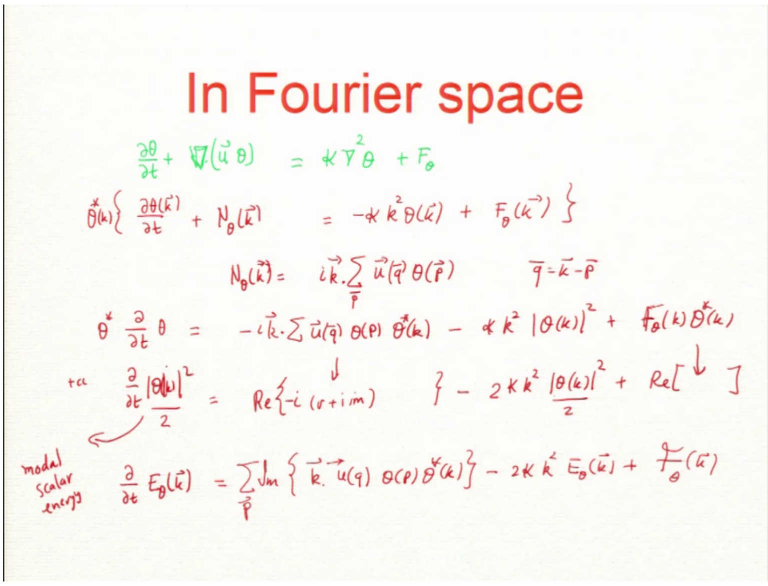
F_θ is rather θ is driven by **u**, so F_θ could also be function of **u,** ok, but we will assume that F_θ is only at large scale some kind of force which is only in large scale, ok. So, this is in the class of passive scalar. So, so these called passive scalar, ok. And the very important topic for obvious reason that they use for diffusion of pollutants you know or your sewer two or things or you know this is most of the time you can treat as passive scalar, at least a lot of places to work with this passive scalar.

Now, second so $\mathbf{F}_u(\theta)$, but it will so I am going to make in at a stable atmosphere, in this atmosphere. So, density is a scalar form no. So, but I here now, density and velocity field are together, right fluid is moving. So, it has density and velocity field, but because of gravity, so force is ρg, so ρ couples with gravity, ok. Now, this ρg will affect the velocity field, right because velocity field can be decreased or increased I mean basically for in the atmosphere gravity I am going to show you that it takes the energy away, kinetic energy away. So, there is a transfer from kinetic energy to potential energy, ok. So, these I will show you, ok. I will I will prove it.

And like here the velocity field is function of scalar. So, it is not a passive scalar any more, right. So, we call it also active scalar. So, this is called active scalar; that means, the velocity field is affected active scalar, and this particular example for atmosphere which is where high density below and low density above, ok. So, for this it is called stably stratified flow, stably stratified flow, ok. So, this is one class of active scalar where the flow will have properly different than chronograph theory, ok. So, I am going to show you that chronograph theory we will not work for this problem.

So, we need to modify the spectrum. So, third theory is in fact, it is the same force rho g, but variety in terms of temperature. So, density is inversely proportional to temperature, no higher hotter the fluid lower the density. So, this is minus theta, temperature fluctuation, ok. So, here it turns out potential energy goes to kinetic or theta drives a kinetic energy, right I mean when heat the what heat water kinetic energy is increases by heating. So, put, so or lesser a thermal a temperature pushes kinetic energy, and this is thermal convection, ok.

So, the difference between these two systems here and here is the energy it flow direction, in one of them kinetic energy is lost to potential, in second potential pushes the kinetic energy, and that changes the physics quite a bit, ok. So, I will cover these 3 types of flow in two days, ok.



So, let us write down the equation Fourier space, ok. So, with flux and so on are very conveniently described in a Fourier space. So, we write down the equation Fourier space. So, velocity field is already written including with the F, right. So, F will be $\mathbf{F_u}$, so no problem. You need to worry about θ.

So, in real space what are the equation for θ? It was $\nabla \cdot (\boldsymbol{u}\theta)$, right. So, because **u** is incompressible, so there, lot is 0 is that is $\kappa\nabla^2\theta$+F_θ. So, what happens in Fourier space? So, its expert for you know you guys are now experts, this one. What about this one, $\kappa\nabla^2\theta$?

$\kappa k^2\theta(k)$. This one, $F_\theta(k)$, whatever function is, I just do Fourier transform and I get this. So, we write generically well I am basically $F_\theta(k)$. This one.

It is a non-linear term, so it becomes convolution. So, in a note I call it $F_\theta(\mathbf{k})$, $N_\theta(\mathbf{k})$ is a scalar, ok. It is not a vector. And what is $N_\theta(\mathbf{k})$?. So, divergence will give I(**k**).

u(**p**) or e, I will call it $\Sigma_p(\mathbf{u}(\mathbf{q})\ \theta(\mathbf{p}))$. And what is **q**? **q** = **k** - **p**, ok. So, this is equation for N_θ or this is form of N_θ.

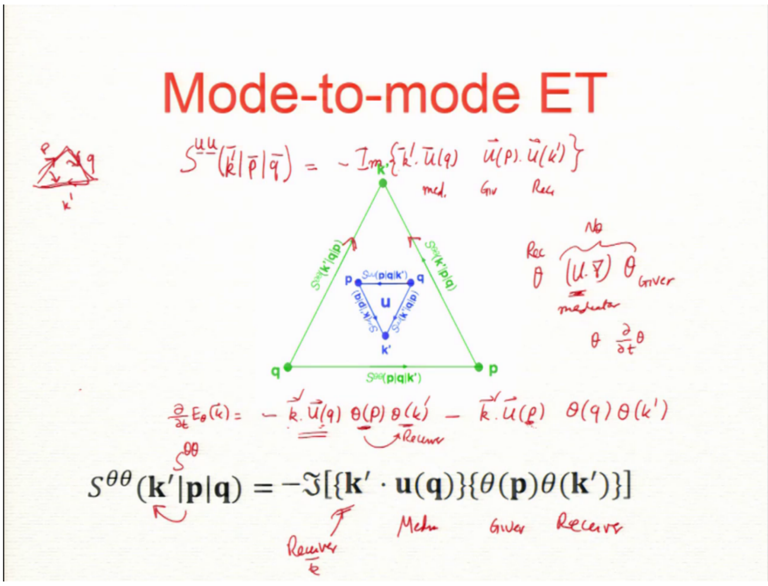
You can also write down for θ^2,$|\theta|^2$. So, θ is complex, no. So, what will I do with this to get that? I am multiplying this by $\theta^*(\mathbf{k})$, correct and then add complex conjugate to it. So, resulting equation are adding complex conjugate that will give you a $|\theta|^2$. So, let us write down for $\theta\theta^*$. So, that gives you $\frac{\partial(\theta\theta^*)}{\partial\theta}$. So, I will take you to the right-hand side, ok, so let us take it to the right-hand side it is N_θ; so, $-\iota\{\boldsymbol{k}\cdot u(\boldsymbol{q})\}\{\theta(\boldsymbol{p})\theta^*(\boldsymbol{k})\}$, right.

And the other term is $-\kappa k^2|\theta(k)|^2$. And third one is force term is $F_\theta(k)\theta^*(k)$. Add complex conjugate to this, so this becomes $\frac{d}{dt}\left(\frac{|\theta(k)|^2}{2}\right)$, right at constant, so by product rule. Now, here I will have exactly complex conjugate of this, right.

So, it is a real part of this object, twice, no, a plus a star is twice real of a, imaginary part cancels. Now, this will become -$2\kappa k^2\left(\frac{|\theta(k)|^2}{2}\right) + \mathfrak{R}[F_\theta(\boldsymbol{k})\theta^*(\boldsymbol{k})]$. So, I am doing some derivation just to make you familiar to this and this I divide by 2 and multiply by 2. So, this called modal energy, modal scalar energy, modal scalar energy and I defined I denote it by $E_\theta(\mathbf{k})$, ok. $\boldsymbol{\varepsilon}$

Now, there is a minus i sitting here, so I want to give it -i. So, this is real +i imaginary. So, real of -i is 0, right. So, only the imaginary part will contribute and i^2 will give you -1, so -1 if you plus. So, its $\mathfrak{I}\{\boldsymbol{k}\cdot\boldsymbol{u}(\boldsymbol{q})\}\{\theta(\boldsymbol{p})\theta^*(\boldsymbol{k})\}$ and this is $-2\kappa k^2 E_\theta(k)$ and this is same as n. So, we just keep it I am going to call that notation F, the math cal. This is not force, force is English F, ok, now this math cal F calligraphic theta of k. So, this is the injection rate, ok, I am injection rate 2($\theta^2/2$, ok.

Now, this is looks very similar to mode to mode transfer, no. There instead of $\{\boldsymbol{u}(\boldsymbol{p})\cdot\boldsymbol{u}^*(\boldsymbol{k})\}$I have now$\{\theta(p)\theta^*(k)\}$. Yes. So, these in fact, is very similar and we can define mode to ode energy transfer from this, make sense. So, so we can do some more work, but I think since I want to bit a speed up, I will write down detailed here. I mean you can also define how theta changes the helicity or the velocity field, ok. Now, all that part I will skip, I will not focus on you. But now let us focus on E_θ mode to mode flux and what does flux do.



So, mode to mode scalar energy transfer because, what about mode to mode kinetic energy transfer. Will it be the same or different? When I put ink what about; so, there is a energy transfer from $\mathbf{u}(\mathbf{p})$ to $\mathbf{u}(\mathbf{k})$, would you use the same formula which I did before or will I use different formula?

Same. How many of you think same and how many of you think not same? some are un-committed where (Refer Time: 24:12), right, I mean. So, what, how do you derive this mode to mode transfer formula? It is from the non-liner term, right $(\boldsymbol{u} \cdot \nabla)\boldsymbol{u}$ term is responsible for energy transfer.

Now, for with passive scalar what is the non-linear term for the velocity field? It is exactly same non-linear term, is $(\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u}$ velocity equation does not change. So, the mode to mode transfer for the kinetic energy exactly is exactly the same, ok. So, chronograph flux, chronograph flux I did not means the flux is constant, the formula remains the same, ok. So, let us write that first because this is important S^{uu}. So, uu means velocity to velocity.

Now, you have to be careful velocity to velocity not kept it in earlier notation as well uu, u to u. So, $\boldsymbol{k}'$, so we had made this vector no $\boldsymbol{k}' \boldsymbol{p}\ \boldsymbol{q}$. So, $\boldsymbol{k}' + \boldsymbol{p} + \boldsymbol{q} = \mathbf{0}$, but $\boldsymbol{k}' = -\boldsymbol{k}$, ok. This we did in great detail, ok. So, in this course if you remember this formula, I will be very happy I mean this is, after one year I will ask you, do you remember the mode to mode. So, you should remember this. And we derived this is minus imaginary.

So, $\{\boldsymbol{k}' \cdot \boldsymbol{u}(\boldsymbol{q})\}\{\boldsymbol{u}(\boldsymbol{p}) \cdot \boldsymbol{u}(\boldsymbol{k}')\}$. So, $-\boldsymbol{k}'$ means $\mathbf{k}$. So, if you look at your just a slide before, so the $-\boldsymbol{k}'$ gives you just $\mathbf{k}$ $\boldsymbol{k}'$ a $\mathbf{k}$ just $\mathbf{k}$, ok. So, that is, but this is convenient with the $\boldsymbol{k}'$

. Now, they this $\boldsymbol{u}(\boldsymbol{k}') = \boldsymbol{u}(-\boldsymbol{k})$, $\boldsymbol{u}(\boldsymbol{k}') = \boldsymbol{u}^*(\boldsymbol{k})$, ok. But this is symmetric, I can change $\boldsymbol{k}'$to **p** in and **p** to $\boldsymbol{k}'$ I can follow this. So, this is the giver, the receiver and mediator, modes and this is a media, receiver, wave number, ok.

Now, what do you expect for the scalar now? Now, in the in my book I tried to derive the formula, I derived it. But we will not go through the full derivation, but I will only give the formula, ok. So, here we had the velocity field which is blue, for this these are velocity with blue, ok. The, this transfer are uu transfers, which is I already wrote. Green are the scalar fields.

Now, if you look at what I wrote in the previous slide energy equation for scalar. So, let us write the equation for the energy. It has $\frac{dE_\theta(\boldsymbol{k})}{dt} = \sum_p [\{\boldsymbol{k} \cdot \boldsymbol{u}(\boldsymbol{q})\}\{\theta(\boldsymbol{p})\theta^*(\boldsymbol{k})\}]$, ok. So, let us turn out the viscosity and a diffusivity and force F_θ there off. So, these are term.

Now, if you focus on a single triad, this single traid, this one then what will I get? So, this sum will become only two terms. Now, I am going to make a change of notation here, ok. So, $\boldsymbol{k}'$ means -**k**. So, this is the $-\boldsymbol{k}'$ and this was $\boldsymbol{k}'$ becomes that. Now, since it is a sum, so I will have one more term $-\{\boldsymbol{k}' \cdot \boldsymbol{u}(\boldsymbol{p})\}\{\boldsymbol{\theta}(\boldsymbol{q})\boldsymbol{\theta}(\boldsymbol{k}')\}$. So, this is only two terms. Now, E_θ(k) can change by a scalar energy transfer from **p** to **k** or actually we will have it here. So, **k** is getting from **q** and from **p**, right both the sources. Now, among these the sum is given to me for identify which way is coming from **q**, which is coming from **p**.

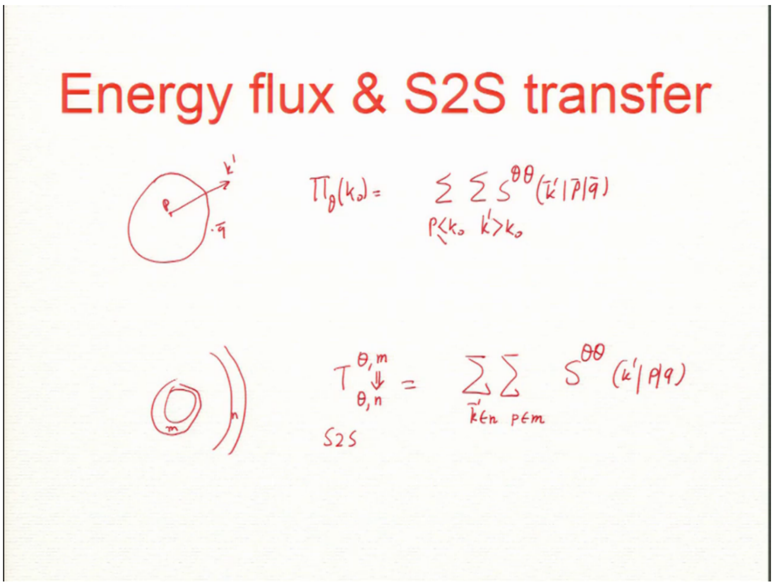
Now, one physical argument which I gave in the class before is the candidate this is **p** to $\boldsymbol{k}'$. Why? Because if I look at my energy area space, so it is $(\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{\theta}$ these are dynamics parts the advection part, I multiply this by θ. So, what does **u** do? **u** only advects, I mean uu is the different different category **u** is not participating in transfers of θ and . So, this must be the mediator no. Yes or no?

Student: Yes, sir.

This, in fact is different category different vector field different field. This is the receiver, right because it is coming into these my N_θ, this is a giver because I wrote $\theta \frac{d\theta}{dt}$ right. So, this is my receiver and this is the giver, ok. So, this is the good interpretation. So, since this is a mediator, so the gradient acts on **u**. So, $\{\boldsymbol{k}' \cdot \boldsymbol{u}(\boldsymbol{q})\}$ the one multiplying $\boldsymbol{k}'$ like these my receiver and I was saying **p** is a giver. So, this must be the giver, so this is giver and receiver and this guy **u** must be mediator.

Now, there after two of them, but this is this is a **p** giving. So, this cannot be the mediator $\mathbf{u}(\mathbf{p})$, $\theta(\mathbf{p})$ will be the mediator, theta **p** is the giver. If I say **p** to; so I am I am fixing it here, ok, I mean I am interested in this one $S^{\theta\theta}$. So, I actually I have it in my next slide next click. So, $S^{\theta\theta}$ **p** to $\boldsymbol{k}'$ if I am looking for it then this must be the giver and this must be the receiver, and **u** because $\theta\theta$ are exchange and the $\mathbf{u}(\mathbf{q})$ must be the mediator. So, this is what I have it here, ok.

So, this is a mode to mode scalar energy transfer in a trail, mode to mode, and using these I can now define flux, right. So, this is just set of. So, this is physical argument. You can also construct mathematical argument very similar to what we did for well I did not do in great detail, but we can construct mathematical argument as well as from tenser algebra, ok. Are you happy with this? So, this is a mode to mode energy transfer. Giver is here, receiver and mediator. So, mediator is velocity field and that dots with receiver wave number, and this is the minus sign because you see the minus sign here.

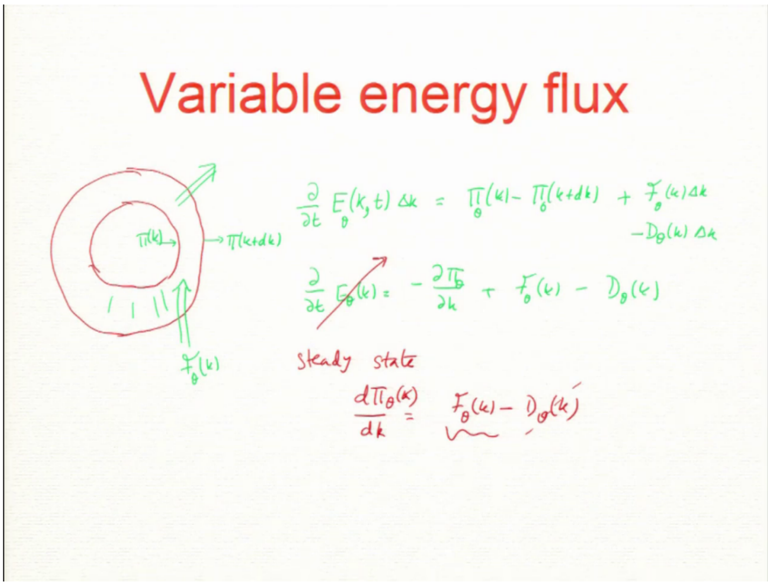


Now, once I know mode to mode energy transfer. I can define flux. So, what is the flux? The flux is θ^2 coming out from the mode to mode this sphere to the modes outside this sphere, that is the definition. You say scalar quantity is not like vector quantity of, it is a multi-scale energy transfer, ok so, the modes inside giving mode to modes outside.

So, **p** is here $\boldsymbol{k'}$ must be outside and **q** will be of course, there because without **q** transfer cannot take place. Though it is mode to mode, the mediator is important, mediator must be present, otherwise no transfer, ok. So, but I know the formula s theta $\boldsymbol{k'}\ \boldsymbol{p}\ \boldsymbol{q}$. So, I set I need a sum, right. So, this we need a $\prod_{\theta}(\boldsymbol{k_o})$. So, **p** must be inside. So, $\mathbf{p} \leq \mathbf{k}_o$, receiver is outside $\boldsymbol{k'} > \boldsymbol{k_o}$, ok. So, this will give you the flux and you can compute it by pseudo spectrum method, by Fourier transform you can do it.

Now, we can also define shell to shell transfer which I will not use for this lecture, but you can also define. So, we can make two shells like this, another shell which is bigger, shell m to shell n. So, imagine I want, energy transfer from shell m to shell n. Now, only scalar energy transfer. We already done uu transfer uu transfer will be exactly same what we did before, because non-linear term is exactly the same for velocity field.

So, theta field scalar energy transfer will be, so the notation is theta comma m. So, this is the m is giver, so m is super script above and receiver is theta comma n equal to no I do not call it S, I call it **T**, ok. So, these shell to shell transfer, shell m to shell n. So, from top to bottom this is a notation which we follow. And these again two sums a $\theta\theta\ \boldsymbol{k'}\boldsymbol{p}\ \boldsymbol{q}$. So, $\boldsymbol{k'}$ belongs to n, and giver is m, ok; so, straightforward. You can do variable energy flux. In fact, this is important concept which ties up convection scalar, passive scalar and stably stratified flows, ok. So, this is the my combination point. This is the meeting point.



So, we derived for kinetic energy you know. So, a flux in the shell or sorry energy in a shell can change by what? This is the flux like this coming out. So, this is called inner flux pi, so, I am my maybe I will try to improve this. So, let us make these two circles bit better, two put those.

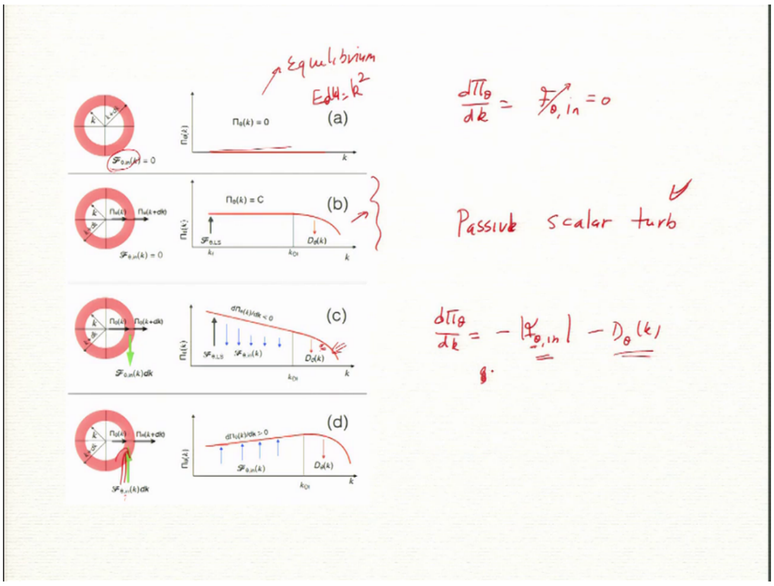
Now, I am looking at scalar energy of the shell, ok. How does it change? So, d/dt is very similar like what we did for velocity field, $E_\theta(k)$ I should multiply by Δk you know. So, this is the energy change in the thing, but I will divide by Δk everywhere.

So, it can change by, change in the flux. So, this is ∏(k + dk), this ∏(k). So, what comes in is ∏(k), what goes is ∏(k) minus k, so this will be equal to ∏(k)-∏(k + dk), correct. So, n is plus, right that is a notation that will increase E(k), it can also change by external force. If I inject then I can change. So, this is $F_\theta(k)$, math cal, math cal and you can decrease by dissipation.

So, this is plus $F_\theta(k)\Delta k$ - $D_\theta(k)\Delta k$. Δk in dk as same from here, is that clear. So, this straight form of from energy. Now, divide by Δk everywhere and that will give me the equation $\frac{dE_\theta(k)}{dt} = -\frac{\partial \prod}{\partial k}$. Why minus sign? Because this is a k+dk is with a with a minus sign plus F_θ, so $\prod_\theta$, $\prod_\theta + F_\theta(\boldsymbol{k}) - D_\theta(\boldsymbol{k})$.

Remember this, this is as exactly the same for velocity field, except **u** was replaced by theta. Now, what happens on the steady state? Steady state, so this goes to 0. So, and is only function of k now. So, I use total derivative. So, $\frac{d\prod_\theta}{dk} = F_\theta(\boldsymbol{k}) - D_\theta(\boldsymbol{k})$ ok. So, what I want if it side is because I am going to use it for my later lectures. So, these my equation for how flux will change with **k**.

Now, there are various scenarios. So, let us imagine the we focused in your inertia range. In the inertia range my D_θ also will be negligible it is there, but it is small compared to F_θ, ok. Kolmogorov theory assumed F_θ at to be 0 in the inertia range, ok. So, there are now various scenarios. So, that is in my next slide.



So, these where, so you see this F_θ is the green one, ok. Now, here F_θ is 0 F_θ comma in, i n is inertial. So, it is slightly tiny for you, but this is a F theta injection rate in the inertia range. It is 0. So, what happens to the flux? So, we have remember now this previous slide D_θ is F_θ inertial, D_θ I am ignoring. If this is 0 then what happens to F $\prod_\theta$? Constant. Now, constant can be two points, one is 0 or one is nonzero. We will assume it to be positive, it goes from large scale to small scale. So, this is 0, it is the equilibrium theory which I mentioned that all these modes are kind of sleepy they do not give any energy to each other or very little energy and they are all equal energy. It is like thermo dynamics, ok. That is the analogy I had discussed.

So, equilibrium corresponds to thermodynamics it is uninteresting for our set of lectures, and I expect this to be k^2. I have not seen papers discussing this in, but I am not really looked in literature, but passive scalar with $\prod_\theta$ being 0, which will be there if you make κ is 0, ok. So, these is what something which you should you try.

So, Anadu has some data also. So, we have kind of bit of data. So, we could try to see whether we get k^2 in 3D for θ^2 as well. Remember, every mode if you have same energy then the shell spectrum must be k^2, right because area of shell is k^2. So, a prediction is $E_\theta(k)$ is k^2 for equilibrium scenario, ok.

Now, these one scenario which follows from variable energy from, flux which is I like it very beautifully, I mean this is. Now, if it is positive and constant, then here this is the passive scalar regime constant flux. So, I will leave it today. So, I hope you will give me the time. So, passive scalar we will, I will derive the spectrum for passive scalar. If constant flux the derivation is very similar to what it is for Kolmogorov theory for hydro and it works nicely you get 5/3 for theta spectrum. But I will derive it, so. So, the passive scalar turbulence, we call it passive scalar turbulence because non-linearity must be strong. So, otherwise their own be cascaded.

Now, this one is $\prod_{\theta}$ is a $-|F_{\theta}|$ of in. So, my forcing is sucking the energy out, kinetic energy, whether we are talking about where we are talking about not kinetic energy, we talked about theta energy yeah. So, this something which I will not probably use it in the same language, but you see we can also define for how $\prod_{\theta}$ changes with **k**. See, if it is there then $\prod_{\theta}$ will decrees, right I mean clearly because this is negative. So, $\prod_{\theta}$ will decrease for the scalar θ is the flux of theta $\prod_{\theta}$, flux of theta should decrease and if it is positive then flux must increase, ok.

But please remember this is for $\prod_{\theta}$ is not $\prod_u$. So, to determine $\prod_u$ I had replaced this force injection by F_u, ok. So, this can be concluded I am going to use only this one for passive scalar this one, but I will not use these two in my set of lectures, ok, but I will use F_u.

Thank you.