Physics of Turbulence
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Lecture - 34
Two-dimensional Turbulence

So today we will discuss Turbulence in 2D flows ok.

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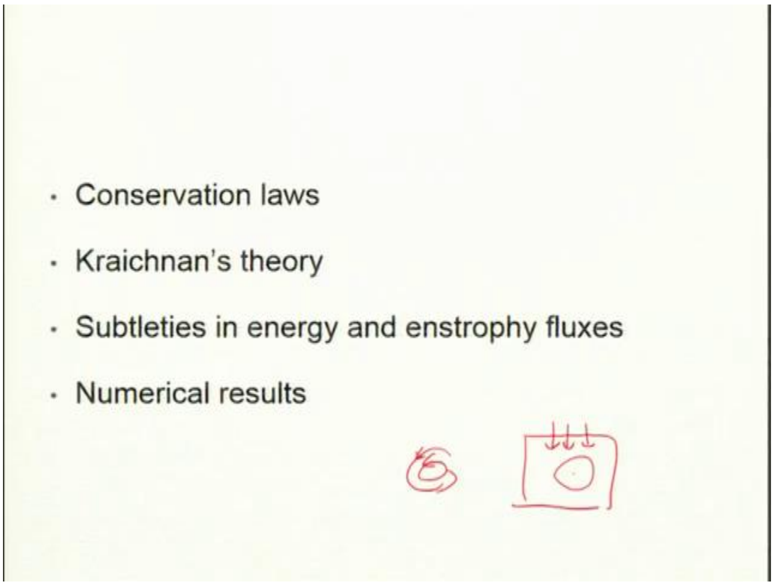

So, my outline is so conservation laws. So, there are which we discussed before, but Kraichnan theory and then there are some subtleties in energy and fluxes, numerical results. But 2D flow is it real, I mean do you really see it in nature, it turns out real 2D flow difficult to get; but there are like Soap film know these people have done experiments. In fact, this one lab in IIT itself, Sanjay Kumar's lab in Europe they do experiments in Soap films. So, you can flow from the top and it is coming down and you can make turbulence

So, Soap film is 2D is velocity only in x y plane, but in it is in real flows like hurricane is almost 2D. So, you also seen this velocity field is like that, the main velocity field and the fluctuations within it. So, there are some u_z components, but they are much small compared to u_x , u_y also that atmospheric flows in general. So, there is horizontal velocity is around you know, it would be 10 kilometre per hour, the wind blowing right now outside. It would be bit less, but during the monsoons it is quite high; but vertical velocity is much smaller. So, that is also considered to be quasi 2D, and lot of ideas we discussed for 2D flows are applicable to those flows.

So, the 2 D flow is very clearly important role in many many systems, like in astrophysics strongly rotating stars or galaxies that typically 2D. And, I am going to show you that 2 D flows show strong structure formation, very strong formation and the reason will become clear after this presentation ok.

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So, what are the conservation laws which we did it before? So, can you name for the conservation laws for 2 D?

Student: Energy conservation.

And.

Student: Enstrophy.

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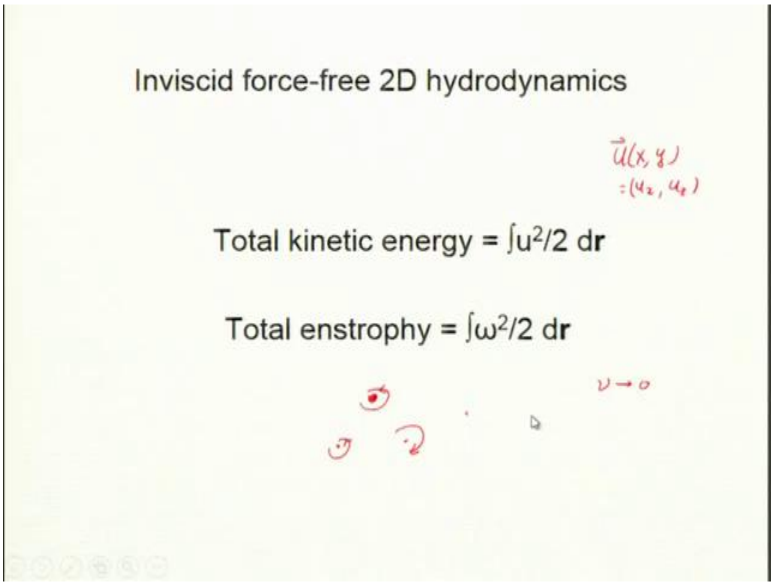

Enstrophy. So, one thing is energy total kinetic energy is conserved and enstrophy is conserved. From yesterdays lecture also you might have seen, the 2 D flows the $(\mathbf{u} \cdot \nabla)\boldsymbol{\omega}$ term essentially advects vorticity. It does not increase or decrease, this was stretching of vorticity in 2 D. So, this vortex are you should also keep in mind that vortex are 2 D lines, infinite lines there is no change along z. So, please remember that **u** is only function of x and y, and **u** is also only u_x and u_y component there is no u_z component ok, is only function of x and y.

So, if there is a vortex column it will be same along z. An analogy which I am not going to describe here, I think I did probably make a remark that current carrying wire, the infinity current carrying wire has magnetic field; the equations are exactly the same for velocity field and for 2D velocity field and this current carrying wires. So, this is approximately good analogy. So, you can have many current carrying wires and they will also attract repel; attract repel is not very correct analogy that is different which I will not discuss right now. But other than that is basically these are wires which are infinite wires and they are interacting. So, that is good analogy, but there is no change along z ok.

So, the field lines are they are strong vortices; for if viscosity is tending to 0, then they are tending to 0 very tiny, but they must be small, but non-zero. Then you get very small this vortex and velocity field is there could be like that also ok. So, they are come in cyclonic, anti-cyclonic and they are point vortices, viscosity going to 0. Viscosity is not equal to 0, but finite; then this they get a flux, there is a core has some size this called vortex core, anyway that is not what I will discuss right now.

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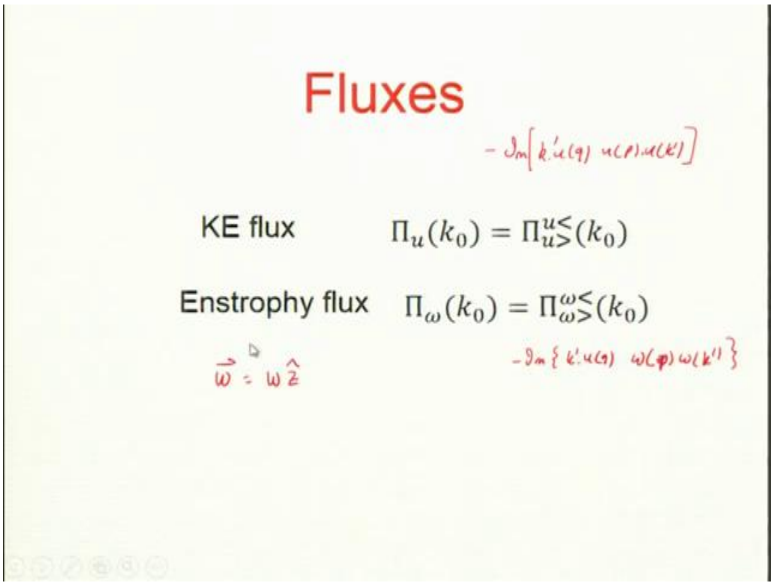

So, given two conserve quantities you can have fluxes. In fact, all are quadratic quantities u^2, θ^2, helicity. Product of two variables you can define flux that comes from that, you have three products product of three variables and that defines flux. So, we have energy flux and enstrophy flux ok.

So, energy flux is I already, the formula is exactly the same which is $S^{uu}(\mathbf{k}'|\mathbf{p}|\mathbf{q})$ will $-\Im[\{\mathbf{k}' \cdot \mathbf{u}(\mathbf{q})\}\{\mathbf{u}(\mathbf{p}) \ \cdot \mathbf{u}(\mathbf{k}')\}]$, I am not putting vectors $\mathbf{k}'$ minus. So, this is for u to u transfer. And if I just sum over all the modes inside this sphere to modes outside this sphere you get kinetic energy flux. You also enstrophy flux, but this is only from $\omega^{<}$ to $\omega^{>}$, since there is no stretching, there is no u to ω transfer there is only ω to ω transfer; and that is coming from $S^{\omega\omega}(\mathbf{k}'|\mathbf{p}|\mathbf{q})$ is $-\Im[\{\mathbf{k}' \cdot \mathbf{u}(\mathbf{q})\}\{\boldsymbol{\omega}(\mathbf{p}) \ \cdot \boldsymbol{\omega}(\mathbf{k}')\}]$, $\boldsymbol{\omega}$ is a vector ω is a scalar.

Now this we did before $\boldsymbol{\omega} = \omega\hat{z}$, along z ok. So, I do not need a scalar product sorry, $\omega(\mathbf{p})\ \omega(\mathbf{k}')$. So, $S(\mathbf{k}|\mathbf{p}|\mathbf{q})$ by sum over modes inside to modes outside, I get these fluxes, good. So, we want to see whether in steady flows, well turns out in 2 D there is no steady flow. So, which will also become clear after this talk, that if you have somewhat give us some time for the flow to organize then it has certain properties; the flux is one show some properties and that is what I would like to discuss today.

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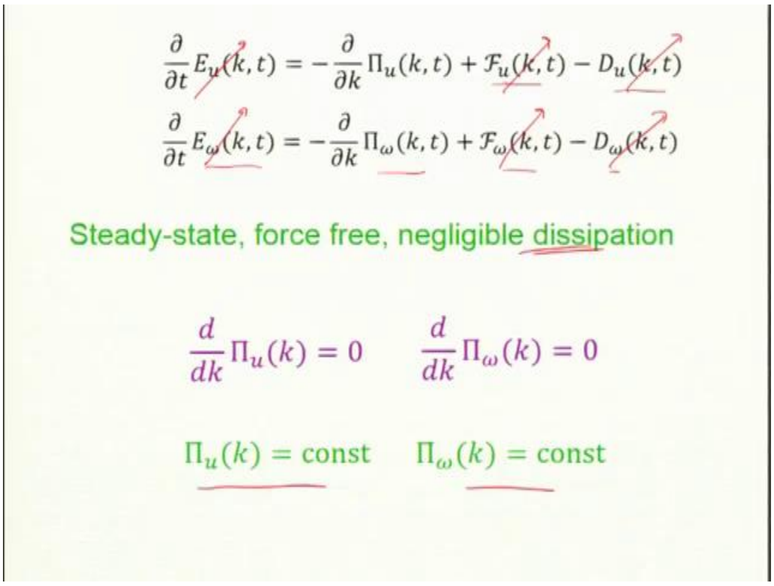

So, these are standard equation for the energy, you know this we did before. So, energy of a shell can change regards the fluxes, external force, and dissipation. Enstrophy can also change by enstrophy flux, so this is important know. Now let us assume that is quasi steady at least, is not changing much. So, we will drop these, these terms then we can get some steady flow. And study is important, you know do not keep time dependency; that is like to it is already complex problem and you make it more complicated. So, study is force free, negligible dissipation.

So, force free is I also turn it off and dissipation is weak; that means, I turn this off. So, we get basically $d\Pi_u/ dk$ = 0 for kinetic energy flux. So, flux Π_u , kinetic energy flux must be constant; it follows from this equation when I am not doing an assumption. Of course, assumption in this one, and steady state is an assumption, actually turns out steady state is an assumption. And, from the second equation I will get enstrophy flux to be constant, but kind of steady flow we will show this. So, now the question is, these two flux conserved quantities and let us see by the both of your positive and negative, where does it seen sign if it all.

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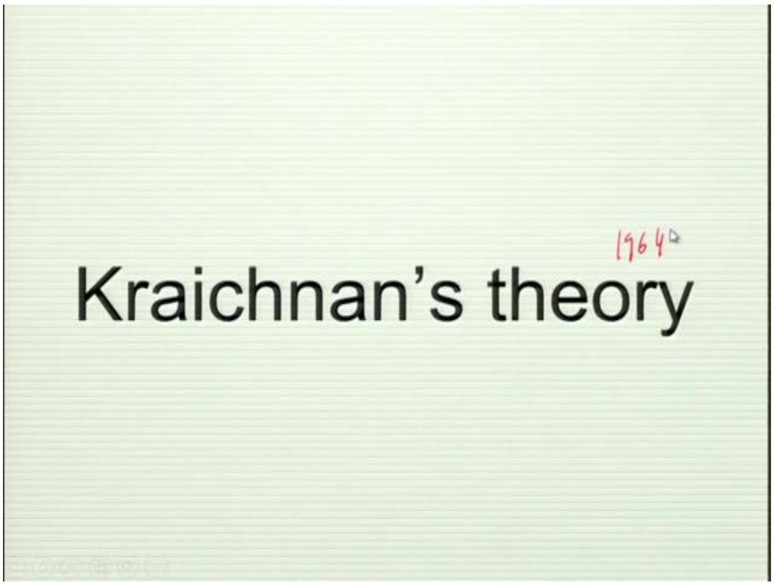

So, this is what was discovered by Kraichnan. In fact, way back in 1964 or 62, I mean very I mean this like 60 year old theory, 50 year old 55.

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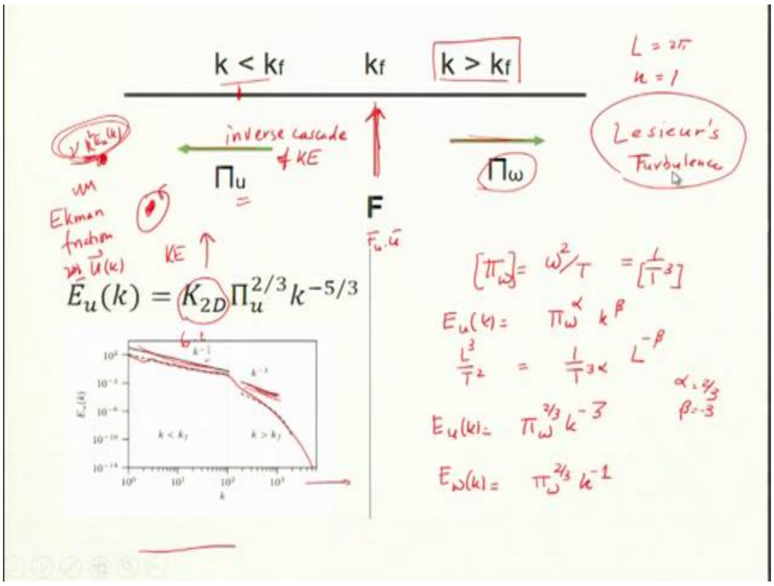

So, what is Kraichnan's theory say? So, these are wave number line. So, I forced at some intermediate scale, not at large scale; in 2D we force intermediate scale. So, like ocean, is huge. So, ocean you, ocean could be 1000 of kilometres, but your forcing could be your large scale is 1000 kilometre; but your forcing could be 10 kilometres ok, one hundred thousand size. So, my wave number is not normalize to one, well my box size is 2π then my lowest wave number is 1, right; 2π/L is 1. But I will say I will force at 50, 100, 10; 50 is more like it; so, you to get more space for both left and right.

So, we will have wavenumber less than k_f and wavenumber greater than k_f. So, according Kraichnan theory which is slightly detailed field theory and so on, which I will not prove it; but there are ways to show that it is reasonable, what is being told is reasonable by just the triad interactions ok. But I will not discuss it today. So, I will not say why, how you motivated, but is doable there are intuitive theory; if you want to look at you can look at by Lesieur's book, Lesieur's fluid mechanics, turbulence book. You will discuss this stuff. So, I have just state the result.

So, you want to conserve both kinetic energy and enstrophy, you can discuss it is off; that will force you to have kinetic energy flux going backward. So, this thing you know, in 3D kinetic energy was going forward; that means, large scale to small scale, but here kinetic energy is going from small scale to large scale. So, what will that do, is something goes from small scale large scale; that means, large scale will become stronger.

So, joint hurricanes are created by this. In fact, mechanism is this. So, energy is going from small scale to large scale, see if I force somewhere normally want energy to go to smaller scale you know, cascade to heat in 3D; or in 2D it goes other way round, it just becomes larger a large scale. And if that is how hurricanes are born, 2D structures are form and huge structures is if I will discuss maybe bit later. A rotating turbulence we get huge cyclones in our simulations, this big vortex going zooming fast ok.

So, these primarily because of the inverse cascade, so this call inverse cascade of energy, of kinetic energy; forward is from large scale to small scale or small wave number to large wave number; but this other way round. Now what happens to $k > k_f$this region. So, this region according to Kraichnan, enstrophy goes forward. So, kinetic energy is not going forward, kinetic energy move backward; but this region Π_ω is dominant, I will discuss bit later today. That there is a Π_ω here too Π_u, but that is weak; here Π_ω is strong here Π_u is strong, this we can see in many different ways.

But I will not prove it today, but you require some kind of you know field theory for rigorous proof, Kraichnan paper is field theoretic which I will not discuss in this lecture. Now, so this is a picture. So, enstrophy goes forward, kinetic energy goes backwards. Now these were the picture is. So, if I supply energy here. So, given force we supply energy, energy supplies $\mathbf{F_u} \cdot \mathbf{u}$ right that is energy supply, force times velocity that is power. So, it will go backward. So, this dissipation is weak here right, because dissipation is $k^2 E_u(k)$, well if a strong E_u; then there is dissipation $\nu k^2 \mathbf{u}(\mathbf{k})$, it is not the dissipation is 0.

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But because k is small, it becomes weaker. So, the dissipation is weak. So, that is why getting steady state is difficult, because energy is piling up at large scale. In fact, it keeps piling up and our computer simulations break, the energy just from keep going; and when energy velocity will very large, then computer is not able to time step and it just blows up. So, kinetic energy tends to go up in getting steady state is somewhat difficult for 2 D; but it is possible, in our code we do something, well our code we do not know anything, we just let it run it reaches quasi steady state, it becomes kind of, but it keeps increasing energy it does not blow up.

But people normally tend to put some viscosity friction here, additional friction. So, put some additional friction at large scale and that also has a name. So, this called Ekman friction, friction at large scale; that is not neatly squared this is not of this type, but it could be constant, is just proportion to velocity field $\mathbf{u}(\mathbf{k})$, $-\mathbf{u}(\mathbf{k})$, not $k^2 \mathbf{u}(\mathbf{k})$. Now, what is this spectrum? So, this is a flux, now let us imagine that flux is these two fluxes are constant; a kinetic energy flux is constant and enstrophy flux is constant. So, what do I expect for this spectrum, in the left we can, you can guess.

Student: (Refer Time: 15:05).

Left must be Kolmogorov, because Kolmogorov was derived truly from dimensional argument five third. So, I assume that any wave number here in between, the spectrum there will depends on the flux and wave number; if you do the dimensional matching is same as 3D derivation. So, in the left space spectrum is just Kolmogorov. What do you expect? So, this is, but the Kolmogorov constant can be different. So, the integrals involved you know, dimensional matching does not say anything about the constant. So, in 3D it is around 1.6, but in 2 D this is around 6, well 6 point something.

What about right, right side I have to do the dimensional let us apply the same dimensional argument; but the dimension of Π_ω is different in dimension of Π_u. So, what is dimension of Π_ω? It is ω^2 by time and what is dimension of ω, is 1 by time.

Student: 1 by time.

Right ω is $\nabla \times \mathbf{u}$**,** so, 1 by time. So, the dimension of 1 by T^3. So, now, I will I am looking for $E_u(k)$, I will say that is $\Pi_\omega^\alpha k^\beta$. So, $E_u(k)$ is dimension of L^3/T^2 right; that we did it before, this has $\left(\frac{1}{T^3}\right)^\alpha$, k has dimension of L^{-1}. So, that is very straightforward, α is 2/3 and β is -3. So, my spectrum in the right hand side is $\Pi_\omega^{2/3} k^{-3}$. So, the spectrum in the right hand side, k^{-3}, so kinetic energy.

What about E_ω? E_ω is I multiply k^2,$\boldsymbol{\omega} = \nabla \times \mathbf{u}$. So, if I multiply k^2 is going to be k^{-1}. So, enstrophy spectrum is k^{-1}, and kinetic energy spectrum is k^{-3} ok. So, let us see what you get in. So, the sketch is this, this from simulation which she is doing some of the simulation. So, the -5/3 in the left is kind of nice line minus five third, this five third is here ok. Right side is not quite -3 right, because minus 3 is this line and we are getting steeper and that is because of dissipation.

So, there is a dissipation of enstrophy and the dissipation is steepening; dissipation will always take energy out there, somebody constant flux is giving you some spectrum. If you increase dissipation, if you add dissipation then energy will be steeper; because somebody is just eating up the energy you know. So, there is not -3, if you want minus 3 which we are trying to do that we need to increase the range in that direction. We need a bigger grid or you force somewhere here, you shift the forcing range; then you get enough dissipation range sorry enough inertia range and that is where we expect minus 3 ok.

So, this is Kraichnan theory. So, I did not prove it the derivation of fluxes, but you can look at this book ok.

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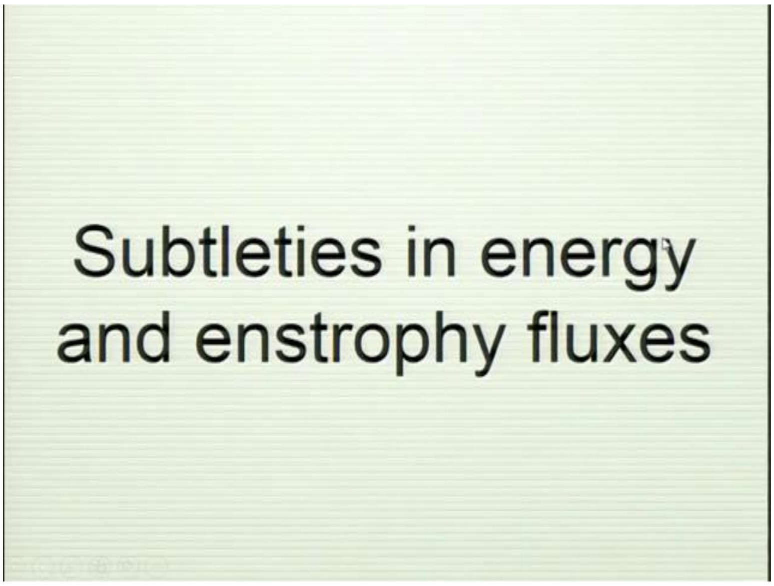

Now, there is certain problems.

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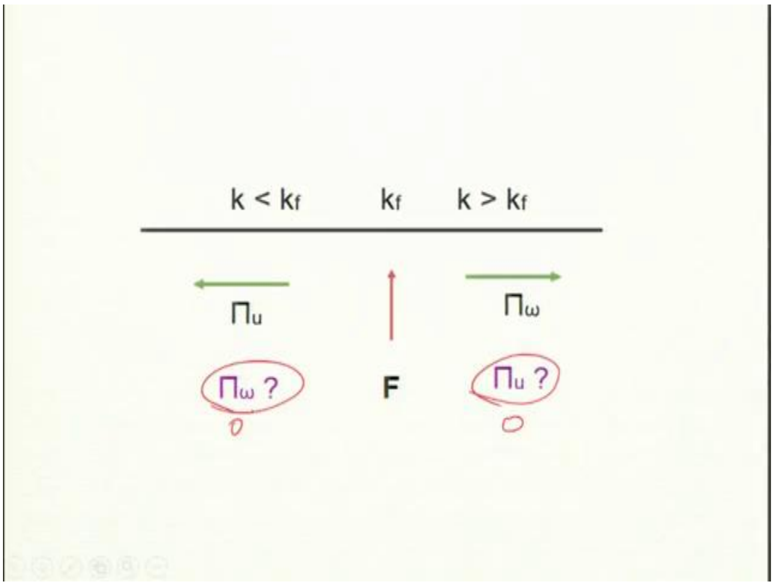

Now, of course, Kraichnan theory tells you about Π_u and Π_ω; but question is I would like to know what is Π_u in the right hand side? So, what is Π_u, and what is Π_ω in the left? So, there are quite a few papers with several, Π_u is 0 here, and Π_ω is 0 here or very small and that is not quite correct ok. In fact, both of them are not constants simultaneously; one way to see this is the following ok. So, this is what we would like to answer, what are this Π_ω in the left and Π_u in the right.

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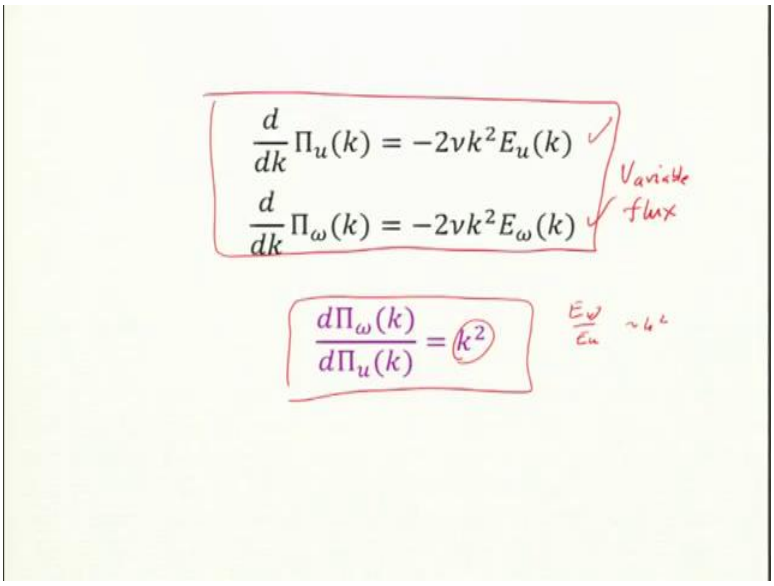

So, let us look at these two equation which I did it before, assuming steady state that is a difficulty; but we will assume steady state. Now if I take the ratio of these two, if I take the ratio what will I get. So, by the way this will cancel νk^2, but I get E_ω/E_u, and what is the ratio of E_ω/E_u; k^2. So, this is the ratio. So, the ratio is k^2. So, both of them can not be constant at the same time. So, one of them should be decreasing, when one guy is constant. So, something like that should happen. So, this is telling us that we should start with these formulas and try to derive the other one.

In fact, for me I am very fond of these equations, under steady state use the flux, variable flux equation we call this variable flux know. So, the name is variable flux and here dissipation is changing the flux; whereas the force is there in a small band. So, force is 0 for the initial range, both left and in the right. So, let us try to solve them. So, let us work out the flux and spectrum already showed you. So, let us look at the fluxes, both the fluxes E_u and E_ω to left of k_f.

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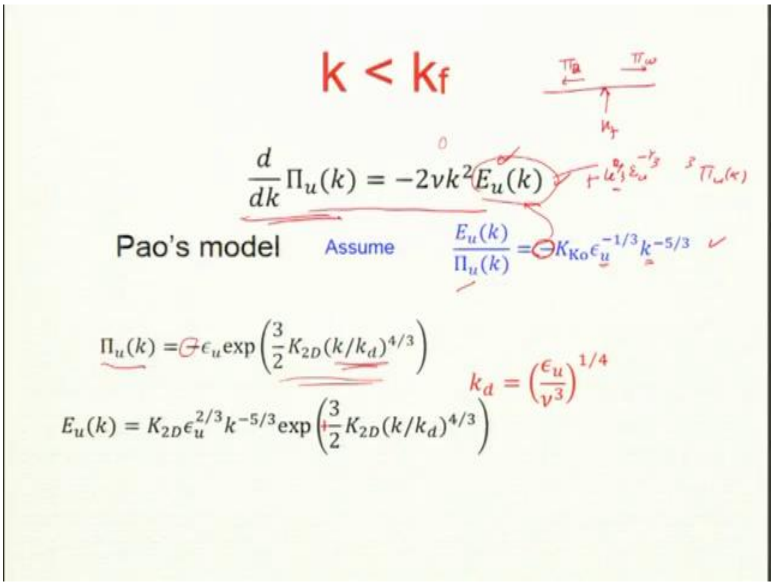

So, we are forcing it k_f, to left of k_f and to the right of k_f ok. Here Π_u is constant; here Π_ω is constant. So, if Π_u is. So, $k < k_f$, Π_u is constant. So, this is. So, this we satisfy this relation right; but as I said I am interested in both dissipation, and well I am interested in solving this more general equation. So, if you make it 0, then I get Π_u constant; but if I have the dissipated term, then I will get more general formula which will be not constant, but it will be some function of k.

So, the two unknowns Π_u and E_u. So, then we will apply Pao's from Pao's model and according to. So, I will just modify Pao's model in 3 D. So, what is the Pao's model Π_u/E_u, or E_u/Π_u is only function of k and dissipation rate or flux is independent of ν or forcing ok. So, this is. So, E_u/Π_u is only function of ϵ_u and k and of course, proportionality constant is also there. So, this basically follows from the spectrum and assumption ok, this is an assumption we could depend on ν know when, but it does not depend on ν this assumption was made.

If I, now I have two equations this equation, this equation I can solve for both. Now please remember the minus sign here, because a spectrum is positive, but flux is negative. So, put a minus sign, because minus minus will become plus, this minus and minus when I substitute E_u here, so I will get function of Π_u right. So, I replace this by $\epsilon_u^{-\frac{1}{3}} k^{-\frac{5}{3}}$, this is k^2 already and Π_u. So, I get 1 D, first order ODE in Π_u ok, and this k^2 and $k^{-\frac{5}{3}}$ gives you $k^{\frac{1}{3}}$; but is a plus sign, remember this plus sign I am not writing ν and so on.

So, plus sign makes Π_u increase with k, not decrease with k. So, is easily solvable is 1 D. So, I will escape all the algebra. So, Π_u is exponential, because is one third integrated this will get k four third and this minus sign here. So, because I know the flux is negative ok. So, this is the minus sign. And so, k_d is much bigger than k ok. So, this exponential term is not very large, is it is one actually, order one; because k_d is large. So, k/k_d is approximately 0, but it shows some effect, fit is better with this.

Now, what about spectrum, once I know Π_u I can substitute and I get E_u, so straightforward. So, it is again minus five third, but there is a correction exponential correction, but it comes the plus sign; in 3D it is comes as a negative sign. And what is k_d? k_d is usual, because is the derivation is exactly same as 3D, except the minus sign has become plus sign. So, this is a plus sign inside and this because of this minus sign ok. So, this is a formula. So, I will show you in computer simulation this seems to be a better fit.

What about Π_ω? So, I solve for Π_ω using similar equation is this. So, we also have pi equation for Π_ω. So, what is the equation for Π_ω, this equation.

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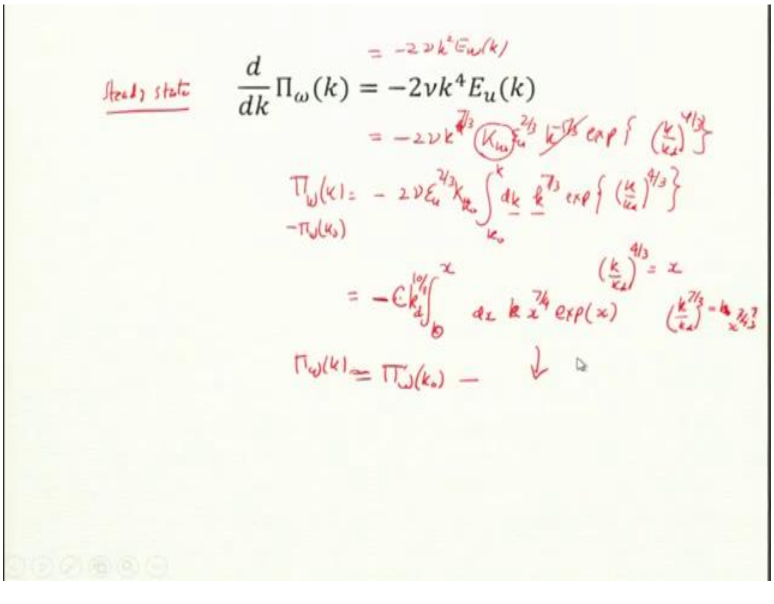

So, let us try to find out what is Π_ω. So, equation is $d\Pi_\omega/dk$ is $-2\nu k^4 E_\omega(k)$, but $E_\omega(k)$ is $k^2 E_u(k)$. So, you get this equation. Now see I know u k, so I substitute, so $-2\nu k^4$. So, Kolmogorov constant, $\epsilon_u^{2/3}$ two third know; actually this is not Kolmogorov constant, this where Kolmogorov constant, but for 2 D. So, is around 6 not 1.5, $k^{-5/3}$ exponential apart from constant is $\left(\frac{k}{k_d}\right)^{4/3}$ ok.

Now, this five third will correct this four to seven third, now I integrate this. So, it is best to do in a computer $d\Pi/dk$ is -2ν, all the constants come out K u k Kolmogorov integral d k k seven third exponential k by k d four third, well this is if you want mathematical formula. So, what is the formula, which function is this; right side function where do you want to look in a table. Earlier this kind of integral where there was no computers, so people used to look in a table. So, which table we should look at.

Student: (Refer Time: 27:03).

Well you will go from k_0 to k. So, I must write pi omega half k k 0 ok; but k_0 can be assumed to be small ok. So, this is if you see it is a gamma function, you can relate into gamma function.

Student: So exponential.

Right. So, exponential is there, but gamma is a positive argument; but gamma can be positive and negative. So, you have to make a change of variable. So, I just want to make you give you a idea. So, four third we should write as x. So, the right hand side is the constant C k know. So, we basically worry about x. So, replace k four third by x. So, it become $\exp(x)$ and $k^{7/3}$ will be. So, I have to say what is k seven third, is going to be k_0. So, k by k by k naught is x.

So, these together will come out outside will come out, all remove get all the dimension out, these another trick for do integral. All the dimensions outside, so I should remain take the k, k has dimension of k_d. So, k the natural unit for k is k_d. So, they will come out and make it k towards 10/3 and everything now is k/k_d. So, $\left(\frac{k}{k_d}\right)^{7/3}$ will be actually tell. So, the one k, yeah this is what is going to come; this is going to give us x to the power three quarter into 3 by 7 by 3 7 by 4, I get $x^{\frac{7}{4}}dx$ and this goes up to x so, 0 to x.

Now, this we can see in a table for gamma function, but computer can do it better and I would like to. So, we do not know, I mean this is what we want to; if assuming steady state which is not extremely clear whether it is steady state, earlier state we should get these behaviour, this is a minus sign here. So, $\Pi_\omega(k)$, now I am not, well we are not sure what is Π_ω at small value.

Normally expect with 0, what is a 2 D; the big vortex sitting there, it may give some vortex flux. So, these are something which we are trying to investigate ok. So, this one, so what is this form? So, I will show you numerically what you get, but this model is what we like to fit, if it fits; but this is being discussed or this is being investigated that is what we are doing right now clear.

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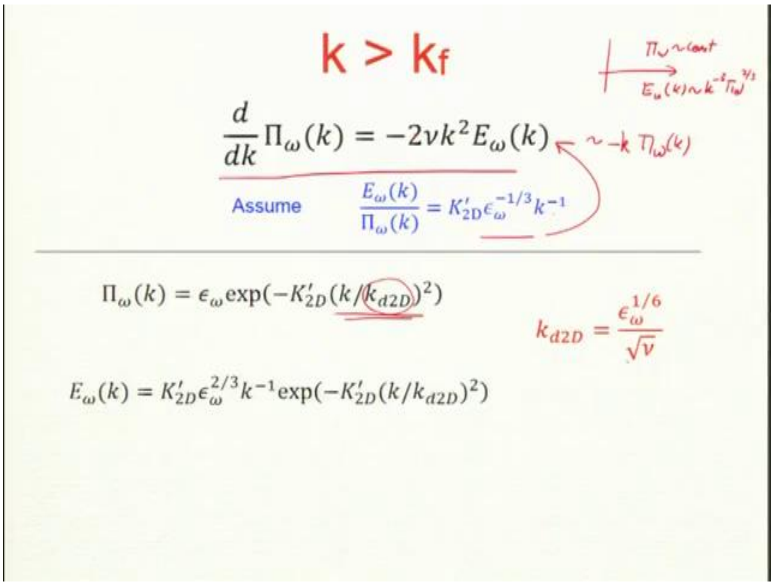

So, this for $k < k_f$; for $k > k_f$ too we are sure about one thing that Π_ω is constant approximately. If there is enough range for enstrophy flux; but dissipation will also start playing a role so, the equation to solve is this equation right. Now let us assume Pao's model again works, you know also. So, is in theory this what you try, you try to see whether you are model you can extend that model to something else.

So, in mathematics by the way there is another. So, if you, if you talk to on computer science or computer science specially; you want to solve a problem then you see a problem which already solved, and then use that solution to solve this problem ok. So, there are tricks solve a general problem then use it takes special case, or solve a special problem generalize it. So, one idea if you know how to solve one problem, then you try to use a solution elsewhere; or a code works then you use a code recycle it for something else.

And it uses the same idea which seems to work, it is quite nice for this model that we use Pao again. So, E_ω/Π_ω is independent of ν, is only function of ϵ_ω and k now. Since E_ω is k^{-1} right E_u is $\Pi_u(k)k^{-3}$. We proved that spectrum in the right side. So, this $k > k_f$, in this region Π_ω is constant and $E_u(k)$ is $\Pi_u(k)^{2/3}k^{-3}$k minus 3 5 omega two third. So, E E_ω will be k^{-1}, because we multiply k^2. So, this is what we will get, for from dimensional analysis from Pao's model.

Now, I substitutive it here, so I will get only 1 D, ordinary differential equation first order. So, which is straightforward and now see the power; power is k^{-1} will here, come here and k into k k in to k minus 1 is k. So, I will get basically this. So, if I integrate this, I will get k^2. So, now, exponential k squared with a minus sign. So, this one is minus. So, the solution is Π_ω is exp $\left(-K'_{2d}\left(\frac{k}{k_{d2D}}\right)^2\right)$. So, this is a constants ok, which I will show you right now, and is and this Π_ω is positive flux; that means, goes from large scale to small scale.

And E_ω will be once I know Π_ω then I know E_ω, and once I know E_ω I know u which is k minus 3 and what is k_{d2D}, k_{d2D} is this it comes by non-dimensional is this, well basically it comes from this equation ok. Now, what about Π_u? So, we can compute Π_u by same trick which we did before. So, I know E_ω I know E_u.

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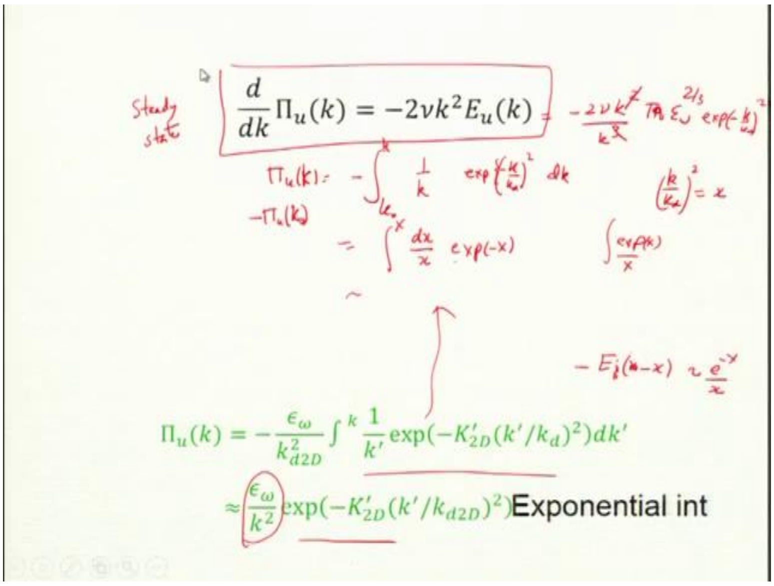

So, I will use this equation. So, this is general, only thing we assume is steady state; there no forcing, we are beyond forcing. So, integrate this. So, how will I integrate, this is $-2\nu k^2$, now E_u is k^{-3}. So, $\epsilon_\omega^{2/3}$ and they are constants which I am ignoring. So, this becomes $1/k$. So, integrate this. So, Π_u is k is integral well sorry, I forgot exp $(-\left(\frac{k}{k_d}\right)^2)$. So, $\frac{1}{k}$ exp $(-\left(\frac{k}{k_d}\right)^2)dk$, $-\Pi_u(k_0)$. So, $-\Pi_u(k_0)$ is this and this k_0 to k.

This function we can again relate it to some known function ok. So, one thing is to make a change a variable $\left(\frac{k}{k_d}\right)^2$ is x, similar idea. So, this becomes. So, dk/k is dimension less. So, you can easily check that these dx/x exp $(-x)$. So, now this also this is name for this function know, this is not well this is not called gamma function is called exponential. So, this integral is this ok, I mean I already made some more simplification, k is k is squared is x ok.

Now, this does fit well because there is identity. Now I let me just tell you that, this call integral exponential integral, it has a name called exponential integral and $\mathrm{E}_i(x)$, well this exponential integral, this is an identity well it is a asymptotic, it is not a and it is called asymptotic for large experts of behaviour. So, $\mathrm{E}_i(-x)$; so, in the tables it is given for $\exp(x)/x$, not for $-x$. So, I made a change of variable. So, x to $-x$ so, this is minus of this is $\exp(-x)/x$ ok. So, if this function goes is $1/x$, but x is already k^2. So, $1/k^2$ ok. So, this is what we are asymptotically claiming therefore, large x it will be this will be this formula ok

So, Π_u is not Kraichnan theory is silent, Π_u it does not tell you Kraichnan pi u it only says Π_ω is constant. So, it using Pao's formula and this equation to compute Π_u and now let us. So, is that clear. So, this derivation is just algebra; but we now have all, energy spectrum of course, in that in turn gives you enstrophy spectrum as well as Π_u and Π_ω in both the regimes. Now we will try to see whether it fits with the simulation data.

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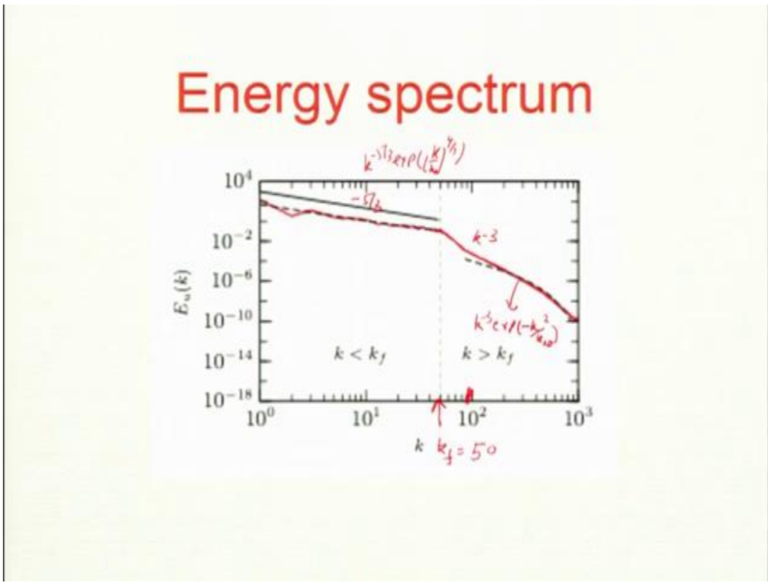

So, this is a spectrum. So, we are forcing it k equal to 100, k_f is 100, around here k_f is. So, we are falling somewhere here, this why this line is somewhere here well 10, 20, 30, 40 actually this is not 100 is this is a new run this is a 2001, 2, 3, 4 40 is force at 40 change of year is 40 not 100, 100 was some other run.

Student: It is 50.

50, 20, 30, 40, 50, sorry yeah 50; now to the left this line is the dash line is the fit ok. So, it is $k^{-5/3}\exp\left(\left(\frac{k}{k_d}\right)^{\frac{4}{3}}\right)$. You can see that viscosity small, so k_d is large and it is not changing much this is five third line; right side since you are forcing at 50 and maximum wave number is around 1000, there is not enough range for k^{-3} spectrum. Right side expected k^{-3} spectrum you know, but I do not get a power law, it is dropping exponentially.

And this one is $k^{-3}\exp\left(-\left(\frac{k}{k_{d2D}}\right)^2\right)$. So, exponential is dominating, there is minus 3 parts, but there is a domination of exponential part. So, it steepens further ok; so, $k > k_f$ is dominated by the exponential. We need to make a run where the force somewhere here, let us say k equal to 10; then you will get a large range for minus 3 regime and we should expect minus 3 ok, is that clear to everyone. So, minus 3, to get minus 3 you need simulation which has enough scope of getting minus 3.

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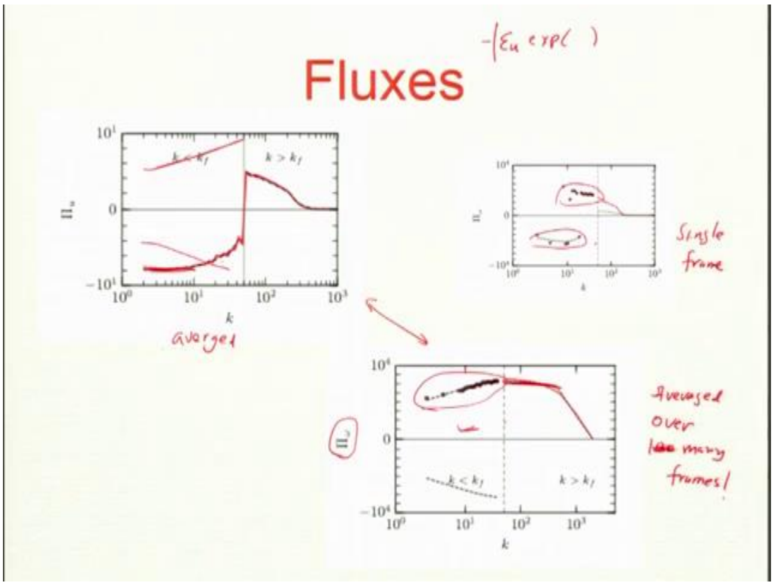

So, let us look at the spectrum now, a flux, so that spectrum. So, flux Π_u is not constant all over, but it is fitting nicely Π_u this one. So, this is what is a Π_u spectrum and we are getting constant here, Π_u I expected to go increase know negative why is it pi constant minus. So, it is minus of that ok. So, is it is constant. So, is ϵ_u minus exponential of that. So, it should have, in a case this part is constant; but it should be, if I take a mod it should increase with k we need to check, so this part.

So, mod means this if I take a mode if I come here it should have increased. So, it should have gone through that ok. So, this part is needs to be seen, this right side is that Π_ω is constant here; right side Π_ω, Π_ω is expected to be constant in the right. And Π_u is decreasing in k, $1/k$ constant by k squared moderate by exponential. In fact, even though we do not get k^{-3} spectrum, but we are reasonably getting Π_ω constant, and this part is that part ok.

Now, Π_ω to the right is looks ok, but to the left is very strange; left what has both positive and negative parts. So, either it is some steady state assumption is not correct. So, this is summed over, now this plot and this plot are different; this for a single frame and this averaged who had many frames where around 100 frames, where I am not sure well many frame.

Student: And left one is also average.

This is also averaged.

Student: Average.

So, these two are average, and this is single frame you wanted to see whether it is fluctuating vortex indeed fluctuating and we are getting positive according to theory it should be negative this ϵ_u ; unless there is a big flux at $k = 0$. So, this part we do not understand ok.

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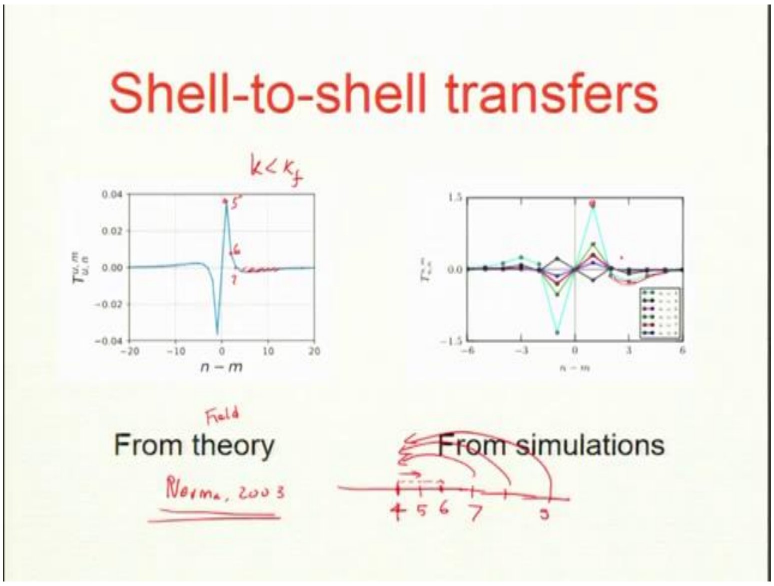

So, we need to see what is happening here. Shell to shell is also surprising, but we know from theory. So, this was what I did in Pramana 2003 I think; so, energy in 2D, so this is for $k < k_f$. So, though energy is going backward all full energy, but there is a local forward cascade. So, what is local forward cascade? So, if I look at shell 4 it gives positive energy to shell 5 it 4 gives, but 6 what is 4 to 6, now it is distant; actually 4 to 6 also positive, but small, but 4 to 7 is inverse.

So, 7 is giving to 4, 8 it is giving to 4, 9 is giving to 4. So, all these people are giving energy backward. In fact, there lot of energy mode which are shells which are giving energy backward from. So, this is next shell, this next to next. So, this will be 5, this is 6 if m is 4 ok, and this is 7, 7, 8 all the way up to something like 18, 15 or so, here for this particular this from analytical theory, this called field theory. We compute this stuff, which I will not tell you how to compute them, but this gives you negative.

So, though I give energy, so if I look at right side I give to labour; but I get lot of it from the small shops. So, I had to give to my neighbour, but I get lot of from the right side. So, on the whole money is flowing from right to left; so, for each shell is just flowing backward, if I sum up all of it. So, this submission you can see in this theory, if I assign the book as well I believe I did that.

Now, this is what is plotted from simulation. So, simulation is n to $n+1$ is positive, but in simulation right away these are negative ok; except the black which is negative with the for the next itself, rest are all this is positive then negative, red is positive than negative, this for different m, $m-n$ sorry different m. So, this is not visible here, but for I. So, this is $n-m$ know. So, I choose different m, then I get different different plots. So, for m equal to 5 I get 6 7 like that, m equals 6 I get 7 8 9 10 like that. So, in the right this called data collapse.

So, this theory is not, where the simulation is not very resolved and probably the lot of fluctuations ok. This 2 D seems like the steady state is not, assumption of steady state is not very very robust. So, that is why here there are not collapsing to single plot, single plot collapse we should have all of them should have fit on the same plot; then would have been stronger theory. So, this theory is not very solid.

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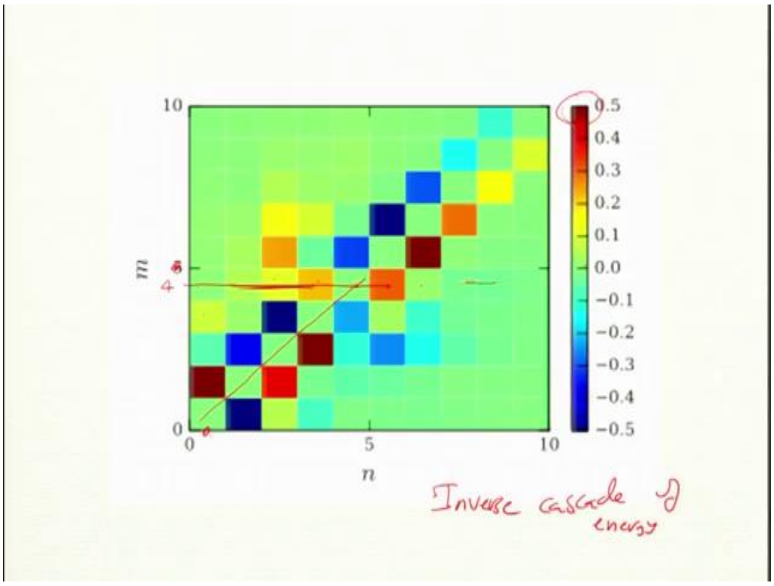

So, we can also look at density plot. So, what is the density plot give you. So, if you look at shell, let us say this one. So, this shell diagonal, so which one is this shell 0 1 2 3 4, so the shell 4. So, this index is not ok, shell 4. So, shell 4 to 4 is 0, I do not give money to myself. So, 4 to 5 is positive, 4 to 6 is almost 0, green is 0, red is positive; but then it search getting negative this blue. So, it is negative.

So, like what we saw. So, I gives to next neighbour, but I get from more distance neighbours and the negative side is also telling you. So, these guys getting from negative, not the giving to negative. So, 4 is giving to 3, 4 is giving to 2. So, I give to my left; that means, the things are going flowing backward that is inverse cascade. So, energy is flowing to n equal to 1, all energy is going basically towards n equal to 1, so this consistent with inverse cascade of energy, kinetic energy ok, so u^2.

Now, we want to do the same thing for enstrophy. So, enstrophy should forward, but not clear whether it be there be non-local component, local component. So, this energy transfers give you lot of insights; how energy is flowing, and who is getting more energy ok. So, shell to shell give us that information ok. So, stop.

Thank you.