Physics of Turbulence
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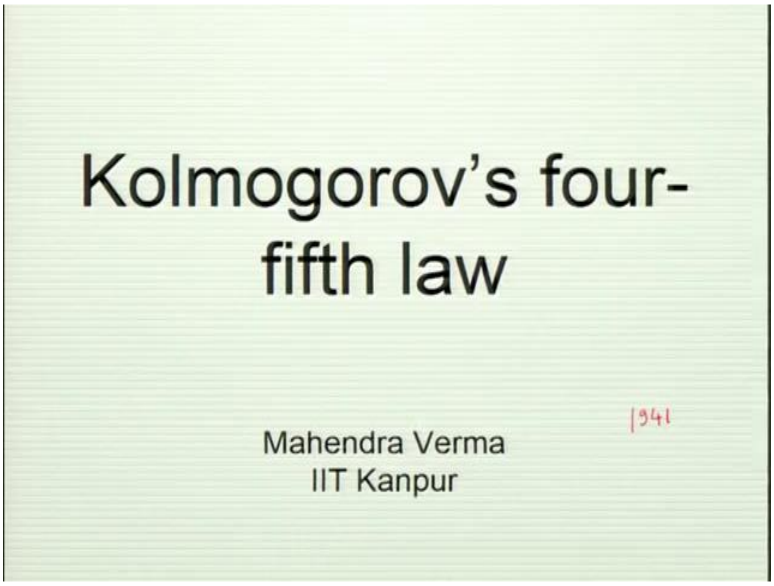
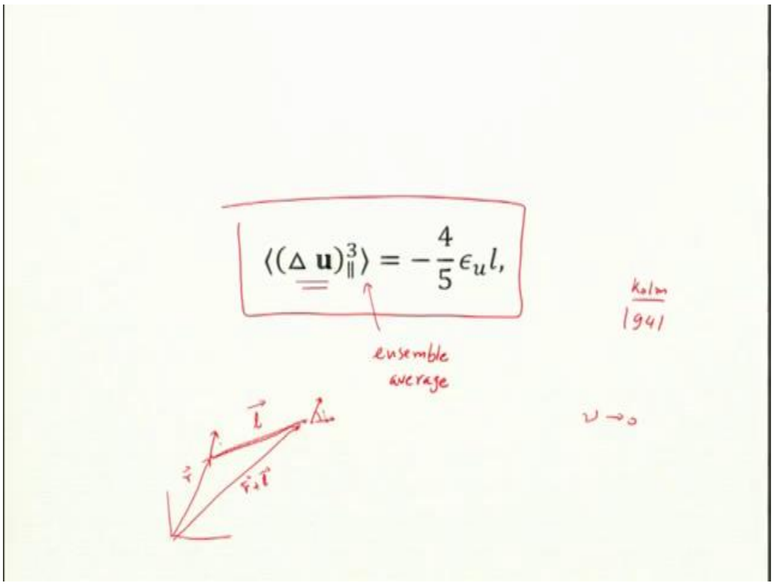
Lecture - 30
Kolmogorov's 4/5 law
Isotropic Tensors & Correlations



So today's lecture will be on Kolmogorov's four-fifth law. This law is in the real space. So, far we focused on furious space description, but now we will go to real space description and this was derived in 1941 ok. The Kolmogorov wrote three papers; the first two papers are related toward today. In fact, this is a second paper the dissipation. So, I gave you all the paper. So, everybody has those three papers, it is a second paper where he derives, what I am going to derive show today.



Of course those papers are very cryptic, he just writes one line, but to derive it takes quite a bit of effort, which I am going to do today ok. So, this is there in Kolmogorov's second paper 1941 and it is in a Russian journal, but it has been translated in English. So, what does it mean? So, we are take two points in real space. Now, this is in real space. So, we are going to going to real space which I separated by vector l ok. So, if I take a coordinate system, this is **r** and this is **r** +**l**. So, difference between two points will be **l**. So, there will be velocity at these two points V 1 and V 2.

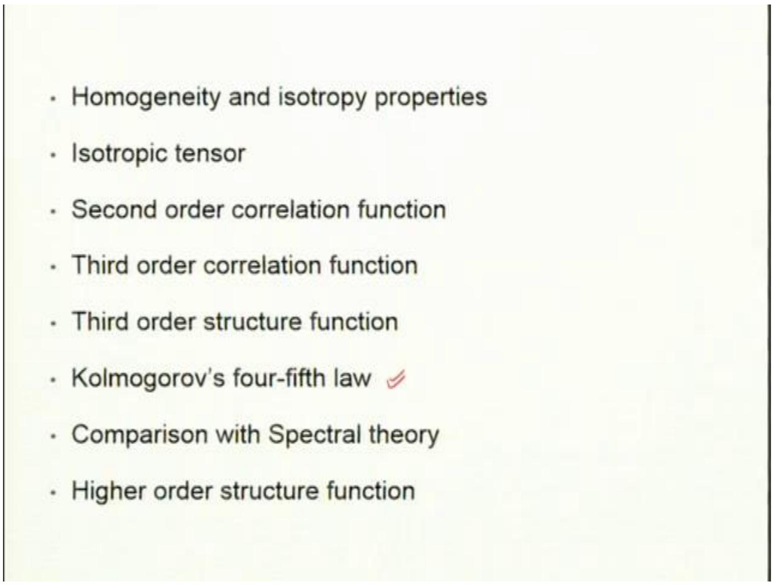
So, take the difference between the velocities at two points with Δ **u** take the projection of Δ **u** along **l**. So, that is what is meant by parallel. So, I take the projection. So, I take the projection like this till here and difference, perform average. Now, this average is supposed to be ensemble average means, I take many copies of turbulent flow started with similar initial condition, not same condition similar initial condition and then do this average over many copies this called ensemble average.

See, if I do this average then what I will get is -4/5 ε_u ,which is the dissipation rate times the distance between the two points, this l is the distance scalar of course, this is under certain assumptions like viscosity going to 0 limit steady state.

So, I will put similar assumption which I did before, same assumption, but this is a real space and this is connected with 5/3 which is easy to see, how is it connected?

So, one thing is, there are some details you know. So, assuming that it is a factor structure or. So, cube order is l. So, what about what do you expect about $(\Delta u)^2$. Now, of course it will be parallel only know, but a spectrum is connected with velocity difference between two points allow all directions. So, I think let us do the bit later ok. So, I think I do not want to get into so, this connected with five-third you can derive it ok.

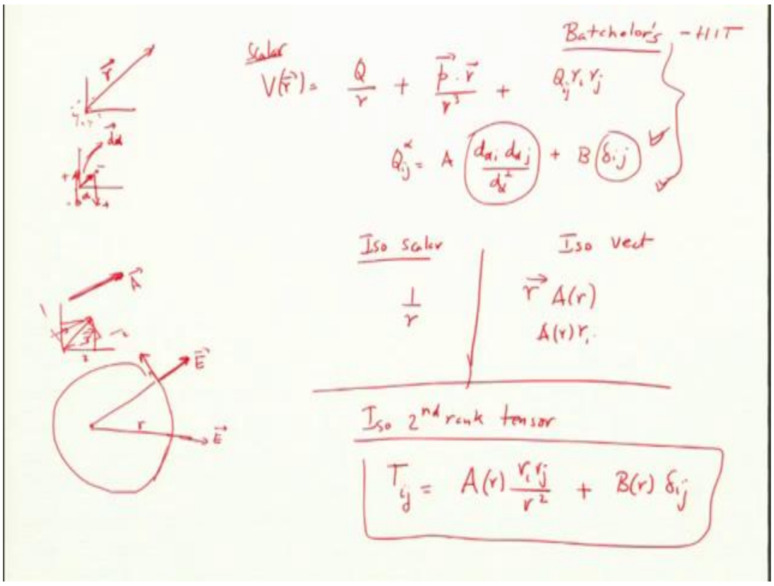
So, I will derive it may be in the next class ok. So, this is easily derivable. So, my objective today is to derive this ok. Now, I will skip some of it, I will give those notes where all the details are there. So, this bit of tensor algebra, but I will give the main steps because these steps are quite involved. So, I cannot do it on this computer ok.



So, the steps are first, I have to define what is homogeneity and isotropy, isotropic tensor, second order, third order correlation function, third order structure function. So, this all has to be done ok, then this law comes as four-fifth law and then I will compare with spectral theory and I will describe about the higher order structure function, which is generalization of Kolmogorov structure function.

So, let us start with what is homogeneity and isotropy, this assumed by Kolmogorov in his paper.

So, before going to isotropy and homogeneity, I have to talk about something about isotropic tensor, because we need this for the derivation. So, they are in fact, so it is a, so the two ways to look at it, but let me just give the simpler version. So, I think everybody has done electrodynamics in your undergraduate classes know it is so, I am just going to invoke that algebra. So, I have some charge, you know charge and I want to look for potential from this origin.



So, I make it this thing. So, charge they do not single charge, single charge I know the potential it is 1/r, but here many-many charges, then I want to find the potential. So, if you recall V(r) so, you expand for all charges. So, how does it work? First is; some of all the charges which would be $\frac{Q}{4\pi\varepsilon_{or}}$, I am not writing in CGS.

Now, second thing is well the total is Q, but they may not be a point source. So, this is scatter. So, imagine that they are two charges, which are so, one example you know that there is a plus and minus separate by distance d it is a dipole. So, dipole total charge will be 0, but I have a potential know. So, how do you write that part? So, there is a dipole electric dipole. So, which is **p**?

So, now I have to construct a scalar given **p**. So, **p** is a vector. So, how do I construct the scalar? So, now, they have another vector call **r** vector ok, I take dot-product this with r divide by some function of r. Now, we know that it goes r^2 right $1/r^2$. So, this one r dimension is already there. So, there will be r^3. Apart from free factors of coefficient like 2, 3 and so on and so forth. So, this is how we construct for dipole.

Now, third is, I am here plus minus, but net dipole I will not to make it 0. So, there is a minus here and plus here am I net I pull the 0, but still it has a dielectric field and potential. So, this is a quarter-pole. So, how do I make the quarter-pole. So, it is a higher order term. Now, that comes as $Q_{ij}(r)$. So, **r** is a vector which is component. So, r_i r_j so, this is Q now, Q_{ij}. Now, how do we construct Q_{ij}? Q_{ij} is second rank denser, it has two indices. So, because it is it is not a vector anymore like p is a vector know, dipole moment is a vector, it is not a vector, it is bigger object, because p is 0 for this. So, Q_{ij} is written as following.

So, there are so, we have basically for each charge there is a vector. So, he calls like, if you look at Griffins book, each charge is separated from the origin by distance d**l**, $\mathbf{d_i}$ vector. From the $\mathbf{d_i}$ vector I construct the following tensor ok. Now, I am not going to worry about the coefficients that you have to do more work, but it will be A. So, there is a vector **d,** this is vector **d** here you know. So, I am going to less. So, focus on d_α, I do not want to say α is that label of the charge particle α equal to first charge, second charge, third charge. So, $d_{\alpha i}$ $d_{\alpha j}$, this is a second rank. Now, they it requires i j contraction plus B δ_{ij}. So, delta function is when i equal to j it is 1 otherwise it is 0 ok.

Now, for convenience is also derived by d_α^2. So, this one and this one are the same dimension right, because this is dimensionless d d by d^2, it is also dimensionless. A and

B are some function of r, it is a number, it is a scalar quantity. So, this is a second rank tensor. Now, this for single charge, but you can do for more charges by summing up. Now, we will see that A B will be computed in books by like Griffins, but this is second rank, this is of this form. Apart from the numbers which is I think 2 by 3 minus 1 by 3 something like that apart from the numbers. This is the form. This has to come from mathematics ok. This also we will work for magnetic potential, I mean any of this.

I need, this is to be scalar and scalar will be expanded like that, this is I mean pure tensor algebra. Now, why is it call isotropic look? Now, this is a tricky part any vector let us say constant vector a under rotation satisfies certain property. Now, this you might have done right. You have done this now, what is reference of vector. So, in mathematics vector is not called with objects having length as at magnitude and directions. That is not a proper definition. So, what is proper definition of vector?

So, what is it?

It transforms exactly like a position vector **r**. So, if I rotate my coordinate system to like this, my A component of A vector will not be same as Ax Ay, it will become Ax', Ay', but that will transform exactly like, x comma y, because position vector also will change instead of x y now, it has become x' y' like this ok. Anyway, this I will not get into it ok. I mean this is this is big derivation. So, this how you define vector, Tensors also defined just like that, second rank, third rank, this is a definition, but what is the isotropic? Isotropic is that if I rotate my.

So, for charge particle point charge particle my vector field e vector, a different position not different right, it changes direction. So, I if take same r it changes direction **E,** but it is magnitude remain same and also it is direction is radialy outward. So, it is like isotropic, though it is not exactly same, because the vector has change here the a vector I need A vector under transformation, we will transform in particular way, but isotropic tensors have even more stronger constraints that this objects should have certain property ok.

Now, for isotropic vector **E**, Now, I am going to so, these one way to look at it these one part of it. So, the source, I would like you to see if you are interested is book on homogeneous isotropic turbulence homogeneous isotropic turbulence ok. Now, so, isotropic vector is easy, isotropic scalar is easy to visualize. Isotropic scalar is if I go around, I should get the same value, if I am. This is **r** away from the source.

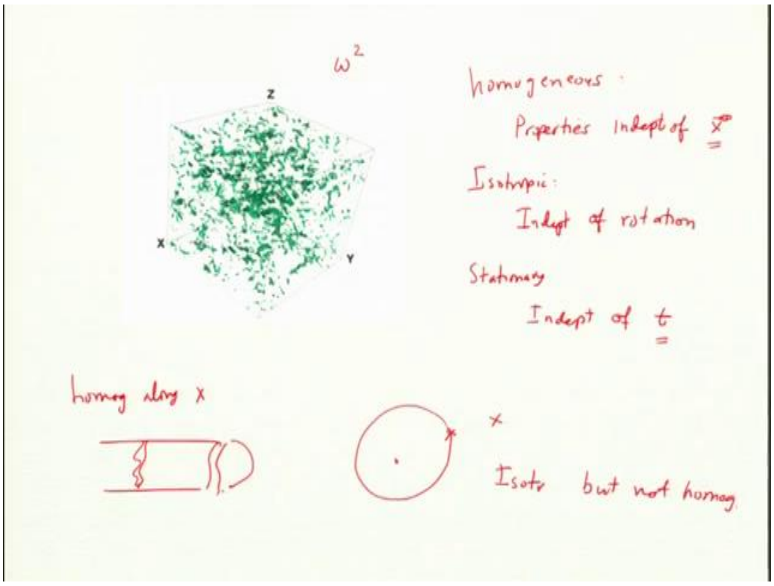
So, isotropic potential is 1/r is an isotropic scalar is that clear isotropic scalar. Scalar is by the way temperature is scalar field, because when I rotate my coordinate system, I get the same value at that position but temperature need not be constant or readily symmetric. This addition constraint from the center of the source a by temperature is same, it all points away from it, all points on a sphere of radius r. This is an additional constraint on this scalar, is that clear? and that temperature field which is only function of r is isotropic. For a point source that will be the well under condition, it possible that it may develop the temperature may spread, but it may spread symmetrically, if it is point source in the center.

Now, we can also think of isotropic vector. So, electric field, if I have many-many charges, electric field is have will have different and different directions right, but they still a vector, but isotropic vector is, if I go from one position to other position in this sphere, my magnitude remain same, but it is pointing outward ok. So, that will be isotropic vector and that will be **r** vector multiplied by A_r right this is the form. There is one more form which I will not discuss, it could be like tangential and we can construct like, but this is what you should think of today or A_{rri}.

So, these isotropic vectors, isotropic scalar, what about isotropic tensor second rank? Can you make a guess?

Now, this is not easy to visualize the vectors and scalars are easy to visualize, but second rank is not so easy to visualize ok. So, I can use this idea. So, construct a second rank tensor, which isotropic, only function of distance r. So, that will be T_{ij} is a of r_i r_j by r^2 let us keep that plus B(r) δ_{ij}. So, this is an isotropic second rank tensor motivated by this discussion on potential ok.

So, this I cannot visualize, second rank tensor I mean it has two components and rather two indices, not two components. It has nine components in 3 d, is that clear? So, I am going to use this result I need this. That is why I am giving you this background ok. So, everybody is happy with it. More discussion on bachelor book because I will not finish otherwise.

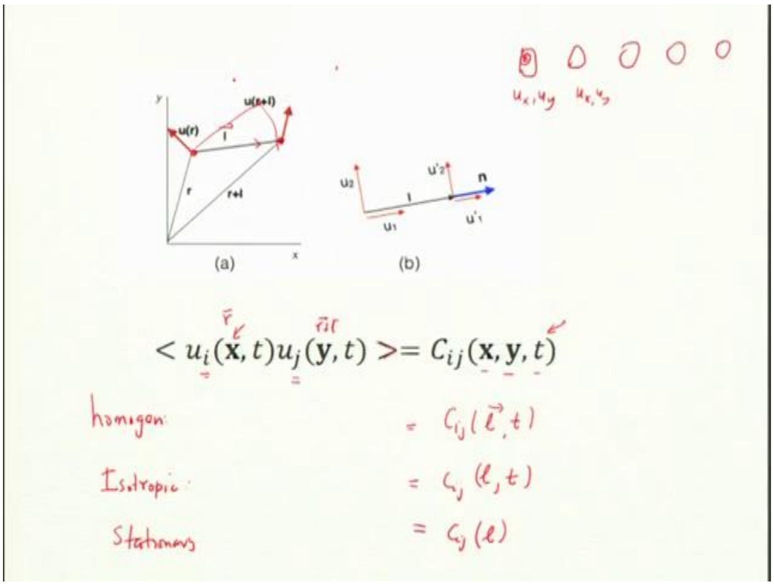


For homogeneity and isotropy of turbulence, this is from stimulation on a periodic box, which we did in our lab. So, this is verticity squared, it is a scalar quantity is magnitude a different positions are shown. Now, we can see that like is not coming from point source sitting in the middle. This is homogeneous in the sense no position is preferred.

So, homogeneity means, population is equally distributed on this planet, it is not, that is why it is not homogeneous, it is something is some position cannot be identified the unique position, then it is homogeneous ok. The inverse is supposed to be homogeneous ok, there is no heaven or something like that at least in visible inverse. So, this is homogeneous in the sense, there is no position which is unique position in this space. What about isotropy, if I rotate this box, then I get very similar structure.

So, that is an isotropy. So, we have homogeneous which is the properties are independent of x of position. I will be more specific in the next slide. Isotropic is independent rotation and we also need something called stationary that means, is independent of time. So, if I look at it now or later, it is looking approximately same, by the way this is our dynamic system.

So, you will not get exactly same phenomena or same distribution it will change, but it will randomly changed in this position ok. So, independent of time so, homogeneity means independent of space and a stationary means independent of time, but of course, I visually seen one thing, but now you will not quantify it and quantification is done by correlation tensor, velocity-velocity correlation is second order correlation ok.



So, this where we are now we are making good progress but if you are not getting my point please stop. Is that clear to everyone isotropy homogeneity?

Student: Sir, just give, if you have an example.

Yes, for example, potential of a charge particle, same a point charge is an isotropic but not homogeneous right, because we can let us go back. So, we have point source. So, go to any direction it will get a same potential, but potential here and here are different. It goes 1/r. So, these isotropic, but not homogeneous and their situation it is homogeneous, but not be isotropic. It could be like constant field, it is homogeneous along that direction, but it is not isotropic, because when you rotate, I will not get it ok.

Student: flowing sir, what that is anisotropic and inhomogeneous along both.

It is so, along the direction of flow.

It is homogeneous along that direction. So, this homogeneous I said about every direction, but you can say let say homogeneity along x. What is it mean? If I travel along x I get the same field. So, if I whatever get here is same as here ok. So, that is homogeneity along x but it is not homogeneity along y, because this parabolic profile so, I think change along y, but we will assume that we are homogeneous isotropic and which for turbulence and which is quite a good approximation away from the worked, without external field like gravity or magnetic field, if you are have if you do not have those fields, then you shake it up in periodic box it is homogeneous and isotropic ok.

So, now we are going to look at second order correlation first. So, I am going to focus on two points here **r** and (**r** + **l)**. So, this is our notation. So, **r** is a vector (**r** + **l**) is vector the difference between two is also vector **l**. Now, I measure velocity at this two position **u**(**r**) and **u**(**r** + **l**). Now of course, both are vectors. So, we can have correlation which is product, but it is not really a distance, because this is a vector.

So, you cannot has just multiply right. So, multiply x-component with x-component, x-component with y-component and x-component with z-component and so on. So, the short hand notation is u_i u_j, i takes value 1 2 3 and j takes value 1 2 3, but you see this is the first position x. So, I will call this is **r** and this is (**r** + **l**).

Of course, in general it will depends it is this correlation function, let us assume same time I am assuming right now same time. So, it will depend on x y and t right. These are the arguments. So, this is the general property of the tensor. Now, I am doing average velocity field fluctuates. So, I want to average it out. So, that is only function of (x y t) is not property of the sample right I mean. So, we wait for sometime average then you find its function of (x y t).

Now, I am going to in temperature average that is why the t is kept here is call ensemble average. If I average in the same box then t will go away. So, it takes many-many copies and at a given time the average, is that clear to everyone, I mean not clear.

I start with initial conditions, system one, system two, system three, system four, system five change the initial conditions ok, but total energy is the same, random start. So, this will give you some u_x and u_y, this will give you u_x u_y at a given time. So, all of them will give you velocity field. Now, take velocity fields u_x and u_x of **u** (**r** + **l**) for each box at given time, then average ok. So, this is at given time, but from different-different systems and this call ensemble. Ensemble means many-many systems, is that ok.

Now, if it is homogeneous then what do you think?

So, what we will depend on, it cannot depend on x and y together.

On both x and y, it will depend on the difference between the two, if this position were shifted by let us say I take it here, but I translate them just translate. So, translation is not changing the direction these two points I just translate it, in any direction, but do not change the orientation.

Student: Yes.

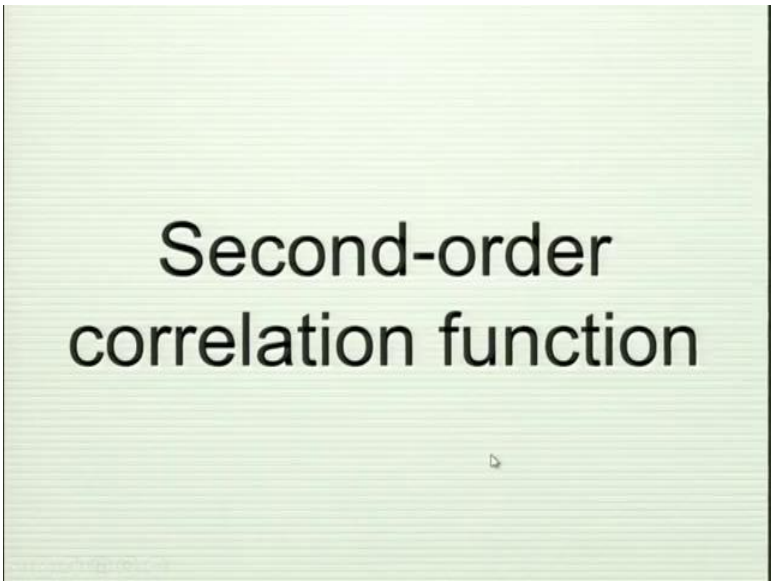
So, it will depend only on the difference between the two. So, this becomes for homogeneous, this is C_{ij}, it does not depend on x and y separately. It depends only on vector **l,** vector **l** nah it will depend l_x l_y l_z. So, this is homogeneous. So, first put homogeneity now, of then you put isotropic on top of it I want homogeneous and isotropic both. So, what do I do?

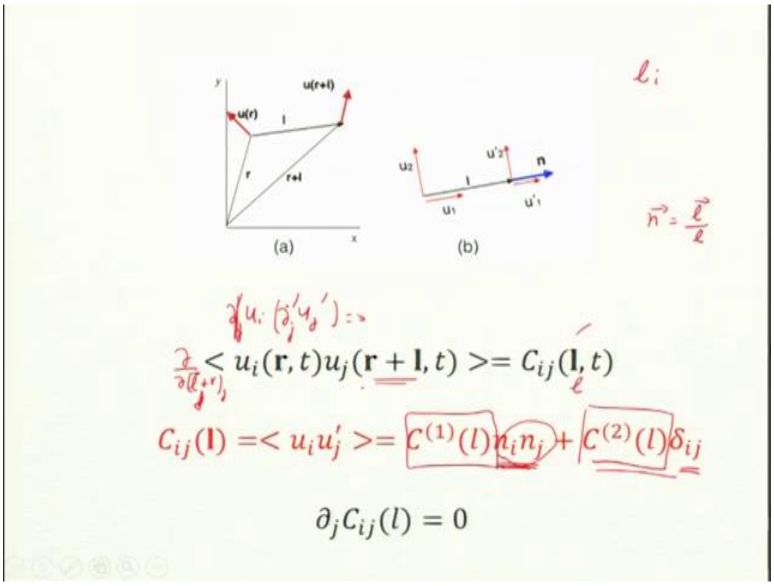
So, it says that not only this l but this l by keeping the distance equal if I rotate it so, this point is fixed but my second point is rotated here, I should get same correlation. Of course, it is a tensor. So, my components will change, but I should get well I mean, I am going to show you. Now, that it should be only function of l, it cannot depend on direction. I will be more specific in the next slide after this. So, magnitude of l and if it is stationary then at also will disappear it is only function of l. C_{ij} and l is magnitude of vector **l**.

Yes.

Student: If it is stationary, then we do not have to do all ensemble average right, we can take into.

Yes, stationary then you can take different times of the same system and that is what we typically do in simulations. You assume time average as equal ensemble average ok. So, this is what is assume by Kolmogorov ok.





So, I am putting the same picture again and this is same, but now, I put vector **l** here and isotropic means it becomes small l my isotropy remember, I said it is an isotropic. So, my second rank tensor has this is independent of direction of l.

So, if I rotate it I get the same thing so; that means, n what is vector **n**? Vector **n** is unit vector **l**/l this is my definition. So, short hand I do not want to write l_i l_j I write n_i n_j. So, this is what I motivated know so, isotropic tensor.

So, since it is does not depend on well basically, if I rotate I get the, my system should be the same so; that means, it is n_i n_j and δ_{ij} for vector, vector is easy to see it should be just l_i, because it depends only on **l** vector, but second rank tensor will be l_i l_j. This is coming from l_i l_j plus δ_{ij} ok. So, this is my motivation y z isotropic since C_{ij} is isotropic.

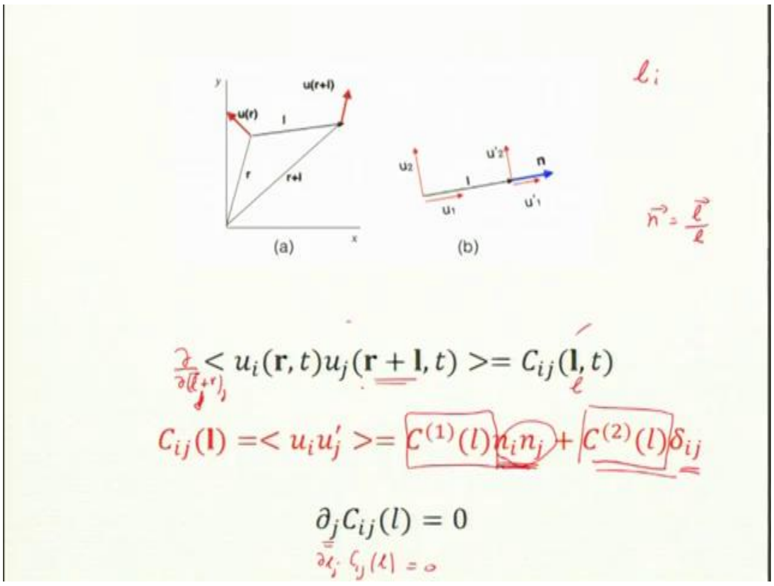
Student: Right.

Is not a same number for all i j; that means, it is scalar, but since it has a tensor it has to be of this form and this $C^{(1)}$ and $C^{(2)}$ is only function of magnitude l. If I know the magnitude, I know this.

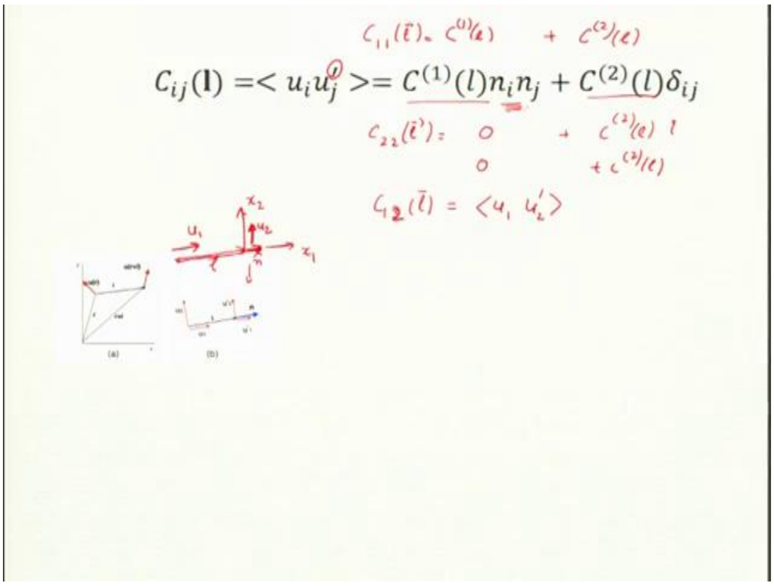
Now, I will make it more specific. So, this is a form and now you can see the beauty of this, this expression. So, we will go to the next slide ok. I one more point since **u** is incompressible. So, if I take the derivative of this with related to well I can take with related to l vector l_i l_j sorry, l_j. So, it will act on this one, it will not act on this right and since, I am doing over l_j summing over j. It should be at 0; yes or no?

So, these implies this equal to u_i u_i, I am going to call it prime because (**r** + **l**). So, this is a notation going to come very soon. So, (**r** + **l**) we will denote by prime we should writing (r + l) you write prime. Now, derivative with related to l_j is same as with related to r +l_j is a second variable which I am taking derivative∂_j'. So, u_i is a constant for it. So, it basically we acts on this is l_j and this is 0 ok.

So, incompressibility on the velocity one of the velocity, if I take the derivative divergence I should get 0. So, I am taking divergence on this if I do derivative with l now finally, by the way the point is when I do the ensemble average r disappears, r does not appear here. So, it is only function of l. So, I take the derivative with related to l, r is not there anymore.



So, that is why this is, when I say this I means ∂_{lj} $C_{ij}(l)$ this is 0 is that and I need this condition. So, this is helpful, I am going to only state the results what we derive from it, but we use this for our algebra ok.



Now, I made this picture smaller. So, these are my same equation C_{11} C_{21} n_i n_j. Now, I choose my coordinate system like this. So, my **n** vector of course, it is a long l right n is vector **l**/l. So, this small picture everybody see that is a two timing or.

Student: Ensemble average.

Visible, so, this is vector **l** and **n** vector will be unit vector along that fine. Now, choose my coordinate system, this is x axis x1, this is a y axis x2 and third one is towards me, above the page, the x1 x2 x3 is a three component. I choose local coordinate system with x1 x2 x3, x1 is along n.

Is that clear?

Student: Sir, C_{ij} is a function of l it is; that means, it is no longer isotropic right, because it is still a function of the vector **l**.

So, you are right. So, I can make it **l** small l.

Student: Sir, that is small l; Ok, but I by the way here these are function of l actually. So, it strictly speaking i should put keep l.

Student: No.

No, I should keep l actually. So, the l is coming here. So, it will become clear in a minute ok. So, I am taking the component in and see what happens.

So, x1 x2 x3 now, I am going to substitute that particular component. So, if I look for correlation C_{11}. So, actually let us let us do the algebra before I put my results C_{11} or vector l, what is it?

So, C_{11} is along x direction or along x1 direction. So, it will be C_{11} and what about n1 n1; what is the n 1 n 1.

What is n 1?

Student: 1.

1.

The n2 is 0 and n3 is 0, it has no n has n is a long vector.

Student: x.

x.

Student: Yeah.

So, x component, it has no y component, z component right. My **n** is along this direction. So, this become both of them, become 1, what about this one.

Student: Delta.

So, δ_{11}

Student: 1 is.

Is 1 ok? So, C_{11} is $C^{(1)} + C^{(2)}$, what about C_{22} ?, So, $C_{22}(l)$.

Student: $C^{(2)}$.

So, C_{22} what is this guy, n2, n2.

Student: 1, 1. n2, n2.

Student: 0, 0.

So, n2 is 0. So, this part is 0; what about this one?

Student: It is C_{12} there.

This is 1. So, $C^{(2)}$ will give you only $C^{(2)}$ of C_{13}.

Student: $C^{(2)}$.

So, C_{13} is coming towards u it is just, these again 0 and $C^{(2)}$ of l, because a isotropy.

The correlation so, it is a vector **l** isotropic means in the perpendicular plane, it should be same. So, along x2 and along x3 my correlations are the same. So, the two vectors, know this is vector here, velocity vector to velocity vector here. So, I am taking the component correlation my velocity like this that is C_{11}, my velocity like this it is C_{22}.

Student: Ok.

Velocity like this is C_{33} is that clear and C_{22} and C_{33} must be equal. What about C_{13} or C_{12}, what is this?

Student: 0.

Why is it 0?

From the formula it is 0, but can you argue why it is 0? It should be 0 for isotropic tensors.

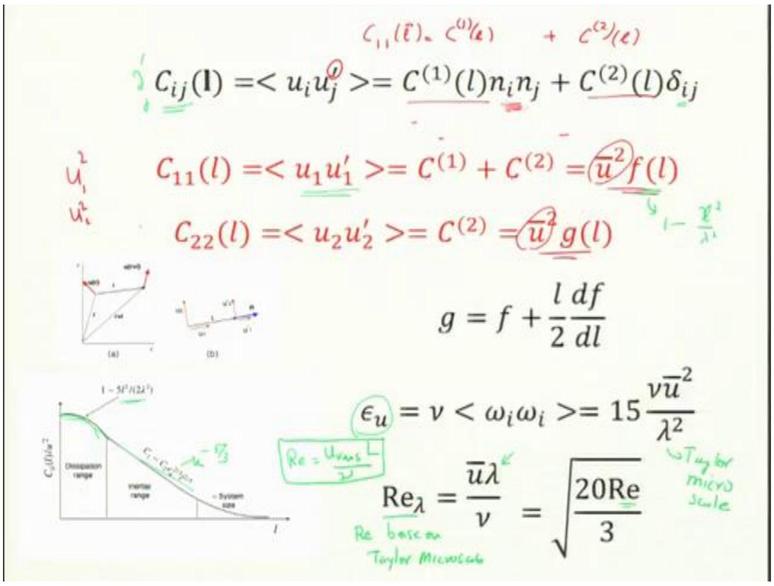
So, look this velocity along these direction **u1** velocity along these direction, I want u2 ok. These are correlation of C_{12} means u1 u2'.

Student: Ok.

Now, prime I have already introduce the notation nah prime means r + l, short hand otherwise it just too much to write. So, now, if u1 is fixed well at a given time what is the probability of finding u2 upward and u2 downward, half-half is so, it is random. So, sometimes u2 will be like this, sometimes u2 will be like this, that is why this u1 u2' prime is 0.

So in fact, any of these things, we have velocity like this either scalar or a vector in the perpendicular like scalar multiply, because that can take both positive and negative value you will get 0 or a vector component oppose in the perpendicular direction that can take this and this 0. So, all the cross terms are 0 is that ok. So, these are property of I mean these are all captured in my formula here, this formula by the way these true for not only this choice of coordinate system, but true for any coordinate system, I choose. I have to just transform by rotation, I could a chosen my x coordinate system to be like this like that in fine, just to do the transformation.

So, this n2 need not be 0, if I do like this n2 will not be 0 right ok.



So, this is what I have in my stuff. So, this $C^{(1)} + C^{(2)}$. So, I should so, this $C^{(2)}$ now, we write like this u^2 f(l) and u^2 g(l). So, u^2 is the u_1 u_1 multiplication at the same position ok, this is just u_1 u_1. So, $u_1{}^2$ it is$\overline{u^2}$. Should it be same as $u_2{}^2$?

Student: No.

Yes or no, why cannot I say, that is isotropic.

Student: Yes.

So, u_1^2 must be same asu_2^2. So, is isotropic. So, take component of velocity magnitude in that direction.

Or y direction or z direction should be same. So, that is why I have the same thing sitting here and sitting there fine. Now, so, this one if I remove this magnitude u^2 then this function will be dimensionless and for l equal to 0, it becomes 1 right, because u_1 equal to u_1'. So, f (l) is one for l equal to 0 and it decreases. In fact, correlation will decrease when you increase l right. You go away from the same point correlation expected to decrease.

Student: No.

Is highest correlation? So, it is maxima at l equal to 0 ok. Now, this part I am skipping, this derivation I am skipping. This one function you can expand it. It is$1 - \frac{l^2}{\lambda^2}$, it goes this form and g also goes with this form. Now, are f and g, are they related to each other?

Now, they related, because they are constrained that $\partial_j' C_{ij}$ is 0, if I take the derivative of this with related l I should get incompressible condition k can and I will give it a relation, which is there in the notes, I will skipped. It is quite a bit of algebra and this gives a relation g and f are related like this.

So, take the l derivative. So, this is a derivative of l and we can plot both f and g, but I plot it together. So, C_{ii} is the contract it, it should be so, these becomes 3 right n_i n_j sorry, n_i n_i is what 1.

Student: 1.

So, n_i n_i is 1 and this gives you 3 δ_{11} δ_{22} δ_{33} ok. So, this part I will skip please, look at the notes and do it. So, this is C_{ii} by u^2 and it has this part is parabolic $1 - \frac{51^2}{2\lambda^2}$ it comes by algebra.

Now, this part goes $l^{2/3}$, it is from Kolmogorov theory, because it is five-third will if you do the Fourier transform you get two-third. This I will do in the next class and it vanishes finally, for very large l. So, there are two important things is parabolic and this is a power law.

These from Kolmogorov k minus five-third and this is by maxima at l equal to 0. Now, this lambda is very specific it has specific name is what Taylor micro scale ok. This is coming from this definition. So, lambda is called Taylor micro scale and that is connected with the dissipation rate.

Now, this is also quite a bit of algebra. So, I am connecting ε_u by vorticity-vorticity given correlation function, I can compute vorticity correlations and that gives you this it is doable, but you have to just dig in and you have to do the algebra and once, I know lambda from dissipation rate and viscosity and u again computed Reynolds number fine and Reynolds number that is called so, this is lambda is called Taylor micro scale and this is called Reynolds number based on Taylor micro scale. So, this is multiplied by lambda$\frac{\bar{u}\lambda}{\nu}$. So, instead of system size that is Reynolds number.

So, Re, remember what is Re? R e is$\frac{u_{rms}}{\nu} l$ there is definition of Re yes or no, but Re based on Taylor micro scale is $\bar{u}$ which is not u_{rms} $\bar{u}$ and this $\bar{u}$ and u_{rms} are different ok.

Now, this I will again ask you to look at the notes. So, $\bar{u}$ is just a pre factor of λ/ν and that is connected with Re_λ by this is $\sqrt{\frac{20Re}{3}}$ ok. This can be shown. This is there in my notes. Now, this is about second order correlation function. Now, we are long way to go on, I mean there is we just do is that any question on second order correlation function.

So, we have f and g now, C_{11} C_{22} and so, their second order correlations we can compute given velocity field. It is isotropic. So, it depends only on l but remember, it is not a number. It is a tensor.

So, we have C_{11} and C_{22} they are different right, they are different here f and g are different functions. So, it is not a single number, but these are beauty of this tensor which captures all of it in one formula ok, respecting isotropy. So, it tells you the $C^{(1)}$ $C^{(2)}$ $C^{(1)}$ $C^{(1)}$ comma C_{12} is 0 C_{11} is number, C_{22} is same as C_{33} all that follows from here.

Student: Sir, this correlation is always 0, only is this applicable in homogeneous isotropic.

Homogeneous isotropic, anisotropy not all; some of it will go through but not all ok. So, with magnetic field this will not work; ok so.

Thank you.